Detailed investigation of exchange effects in $^3He+\alpha$ and $\alpha+^{16}O$ scattering

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The effects of intercluster antisymmetrization in $^3He+\alpha$ and $\alpha+^{16}O$ systems are examined. The result indicates that the characteristic quantities of the Born-equivalent local potentials can be reliably employed to predict the relative importance of the various nucleon-exchange terms. Based on these quantities and by studying the cross section or phase-shift behavior, it is noted that many exchange terms are of only minor significance when the energy is high or when the absorption is strong. Furthermore, one finds that, at an energy higher than about 25 MeV/nucleon, the features of the cross-section angular-distribution curve, which has a distinct V shape, can be simply explained. Here our investigation shows that the contribution to the cross section in the decreasing part of the V-shaped curve comes essentially from the direct term and the one-exchange terms (mainly the knockon exchange term), while the contribution to the cross section in the increasing part of this curve comes essentially from the core-exchange terms.

[NUCLEAR REACTIONS ^3He (α,α), $^{16}O(\alpha,\alpha)$. Effects of antisymmetrization with resonating-group method.]

I. INTRODUCTION

In this investigation, we continue our study, initiated a few years ago,[1,2] of the effects of intercluster antisymmetrization in a two-cluster system which consists of clusters A and B with nucleon numbers equal to N_A and N_B $(N_A > N_B)$, respectively. The purpose here is to extend the analyses carried out in previous investigations[1-3] and thereby gain further knowledge concerning the general features of the exchange-kernel function $K(\vec{R}',\vec{R}'')$ which characterizes the nonlocal part of the effective $A+B$ internuclear interaction present in a resonating-group formulation.[4,5]

By adopting for the internal functions of the clusters A and B translationally-invariant shell-model functions in harmonic-oscillator wells having width parameters α_A and α_B, respectively, the kernel function $K(\vec{R}',\vec{R}'')$ has the form

$$K(\vec{R}',\vec{R}'')=\sum_x \sum_q K_{xq}(\vec{R}',\vec{R}'') , \tag{1}$$

where

$$K_{xq}(\vec{R}',\vec{R}'')=P_{xq}(\vec{R}',\vec{R}'')\exp(-A_{xq}\vec{R}'^2-C_{xq}\vec{R}'\cdot\vec{R}''-B_{xq}\vec{R}''^2) + P_{xq}(\vec{R}'',\vec{R}')\exp(-B_{xq}\vec{R}'^2-C_{xq}\vec{R}'\cdot\vec{R}''-A_{xq}\vec{R}''^2) , \tag{2}$$

with P_{xq} being a polynomial in $\vec{R}'^2$, $\vec{R}'\cdot\vec{R}''$, and $\vec{R}''^2$. In the above equations, the index x denotes the number of nucleons interchanged between clusters A and B $(1 \le x \le N_B)$ and the index q denotes the interaction type (for details, see Ref. 2). For each value of x from 1 up to (N_B-1), there are five interaction types which are denoted by the index $q=a$, b, c, d, and e, while for $x=N_B$ (core exchange) there are only three interaction types, namely, $q=a$, c, and d.

With the utilization of the complex-generator-coordinate technique,[5,6] the general expressions of A_{xq}, B_{xq}, and C_{xq}, for any $A+B$ system, can be derived. The result shows that the quantities A_{xq} and B_{xq} are, as expected, always positive, but the quantity C_{xq} can acquire either a positive or a negative value. This latter fact can be advantageously employed to further classify the nucleon-exchange terms into two classes:

(i) Class-A terms with $C_{xq} < 0$. For these terms,

the Born scattering amplitudes are forward peaked and can be exactly reproduced by equivalent local, energy-dependent potentials having a Wigner character.

(ii) Class-B terms with $C_{xq}>0$. For these terms, the Born scattering amplitudes are backward-peaked and can be exactly reproduced by equivalent local, energy-dependent potentials having a Majorana character.

The classification into class-A and class-B exchange terms is particularly useful at higher energies greater than about 25 MeV/nucleon. At these energies, the cross-section angular-distribution curve normally exhibits a distinct V shape with its tip occurring at an angle θ_m which is a measure of the relative importance of the class-B exchange terms. When these terms make important contributions, θ_m will be relatively small (i.e., close to about 90°), and vice versa. For $\theta<\theta_m$, the cross-section curve has a decreasing trend and the contribution comes essentially from the direct and class-A terms, while for $\theta>\theta_m$, it has an increasing trend and the contribution comes essentially from the class-B terms. Therefore, by just visually inspecting the experimental angular-distribution curves at such energies, one can already learn whether there are significant class-B contributions or not, although such an inspection will not yield much information concerning the importance of class-A terms. For example, the experimental values of θ_m observed in $^3\mathrm{He}+\alpha$ scattering at 35 MeV/nucleon (Ref. 7) and $\alpha+^6\mathrm{Li}$ scattering at 41.5 MeV/nucleon (Ref. 8) are equal to about 90° and 105°, respectively, indicating that the class-B terms are important and must be properly taken into consideration in both these cases.

To obtain a rather definitive understanding of exchange effects, we have considered in Ref. 2 two specific examples, namely, the case of $^3\mathrm{He}+\alpha$ scattering where the nucleon-number difference (N_A-N_B) is small and the case of $\alpha+^{16}\mathrm{O}$ scattering where this difference is comparatively large. In that study, effects of exchange terms with different values of x on the scattering phase shift and cross-section angular distribution were examined. In the present investigation, we shall extend the study in these two cases by further investigating the influence of various interaction types and the effects of antisymmetrization when there is absorption present. The main purpose is, of course, to see whether in a given problem certain exchange terms may be reasonably omitted from the resonating-group calculation without significantly impairing the quality of the result. This will be useful information to have, because in a complicated scattering problem, such as $^{16}\mathrm{O}+^{20}\mathrm{Ne}$ or $^6\mathrm{Li}+^{208}\mathrm{Pb}$ scattering, a straightforward application of the resonating-group method, even in the single-channel approximation, would be quite formidable from the computational viewpoint.

In the next section, we briefly review the procedure of analysis described in Ref. 2 and discuss the significance of the characteristic quantities introduced in previous investigations. Detailed studies of exchange effects in the $^3\mathrm{He}+\alpha$ and $\alpha+^{16}\mathrm{O}$ systems are then described, respectively, in Secs. III and IV. Finally, in Sec. V, we make concluding remarks and discuss the essential findings of this investigation.

II. BASIC PROPERTIES OF THE KERNEL FUNCTION

A. Characteristic quantities of equivalent potentials

For clarity in discussion, we shall adopt the assumption of setting α_A equal to α_B (i.e., $\alpha_A=\alpha_B=\alpha$). As has been found in recent investigations,[3,9] this is a reasonable assumption to make, since we are only interested in learning the main characteristics of nucleon-exchange effects. Also, for simplicity, we shall omit all charge effects by letting the charge of the proton be infinitesimally small.

As is seen from Eqs. (1) and (2), the kernel function consists of a sum of terms, each containing an exponential factor multiplied by a polynomial factor. The exponential factors, depending on x and the nucleon numbers of the nuclei involved, collectively determine the general features of antisymmetrization, while the polynomial factors, which depend additionally on the dynamical structures of the interacting clusters, contain information concerning more specific features such as blocking and clustering effects. At present, we have some general, but not detailed, understanding about the roles played by the exponential factors, and some meager information about the structures of the polynomial factors.[10] Thus, it is still necessary to examine specific nuclear systems in order to gain a systematic and progressively more reliable understanding of the effects of antisymmetrization. It is our hope that in the future one would at least be able to determine analytically the general properties of those terms in the polynomial factors which have higher powers in $\vec{R}'^2$, $\vec{R}'\cdot\vec{R}''$, and $\vec{R}''^2$.

Using the analytical expressions of A_{xq}, B_{xq}, and

C_{xq}, one can extract semiquantitative information about exchange effects from the kernel function. The procedure used is carefully described in Ref. 2. Briefly, what one does is compute, in the Born approximation, the scattering amplitude corresponding to each nucleon-exchange term, and then construct an equivalent, local, energy-dependent Wigner or Majorana potential which yields exactly the same scattering amplitude as the nucleon-exchange term under consideration. This equivalent potential is characterized by two quantities, a characteristic range R_{xq} and a characteristic wave number k_{xq}. The information one seeks about antisymmetrization effects can then be obtained by studying in detail the properties of these characteristic quantities.

The general expressions for R_{xq} and k_{xq} are given in Refs. 3 and 5.[11] As is seen, these quantities depend in a simple way on just two parameters, the reduced nucleon number μ_0 given by

$$\mu_0 = N_A N_B / (N_A + N_B) , \tag{3}$$

and the force parameter λ given by

$$\lambda = \kappa / (\alpha + 2\kappa) , \tag{4}$$

with κ being the range parameter of the nucleon-nucleon potential.[12] This latter parameter (i.e., λ) has a range between 0 and $\frac{1}{2}$. In a realistic situation where κ is approximately equal to α, λ has a value around $\frac{1}{3}$.

Although the analysis procedure described above is based on the Born approximation, it should be mentioned that explicit studies in a number of nuclear systems[2] seem to indicate that the information obtained may have semiquantitative validity even when the energy is relatively low. Recently, this belief was further strengthened by the Wentzel-Kramers-Brillouin (WKB) investigation of Horiuchi and Aoki[13,14] on the kernel function. These authors reported that, with such a semiclassical study which is useful in heavy-ion systems at rather low energies, the conclusions reached are basically the same as those obtained with the Born-approximation analysis of our investigation.

B. Extraction of semiquantitative information from properties of characteristic quantities

Based on the values of k_{xq} and R_{xq}, one anticipates that the relative importance of the various nucleon-exchange terms may be determined from the following considerations:

(i) Since in the expression for the equivalent potential the exponential factor $\exp[-(k/k_{xq})^2]$ appears (k is the asymptotic wave number), it is reasonable to expect that, at relatively high energies, equivalent potentials with large characteristic wave numbers may make dominant contributions.

(ii) Since the absorptive potential is known to have a range similar to that of the nuclear direct potential, one may reasonably expect that, in the case where absorption is strong, equivalent potentials with longer characteristic ranges may be more important.

These considerations suggest that it may be useful to define two alternative characteristic quantities, i.e., the characteristic energy E_{xq} and the characteristic weight ζ_{xq}. These quantities are defined as

$$E_{xq} = \frac{\hbar^2}{2M\mu_0} k_{xq}^2 , \tag{5}$$

with M being the nucleon mass, and

$$\zeta_{xq} = (k_{xq} R_{xq})^2 . \tag{6}$$

This latter parameter (i.e., ζ_{xq}) may be considered as providing a qualitative measure of the relative importance of the corresponding nucleon-exchange term.

The above discussion indicates that, at relatively high energies and with absorption taken into account, it may be a good approximation to include in the calculation only those nucleon-exchange terms which have large values for the characteristic weight. In the following sections, we shall carefully examine the ^{3}He+α and $\alpha+{}^{16}$O cases to see if this type of simplification is indeed reasonable or not.

Among all class-A nucleon-exchange terms, the type-1c term (i.e., $x=1$, type c) has been shown to have both the largest characteristic weight and the largest characteristic energy.[2] Therefore, this particular term must always be properly accounted for in order to yield satisfactory results. In the folding-potential model of Satchler and Love,[15] the assumption has in fact been made that this term is the only nucleon-exchange term which needs to be considered, even when the energy per nucleon is quite low. This seems rather doubtful to us, and we shall examine it below to test its validity.

III. ^{3}He+α SYSTEM

As a guide for our analysis, we compute first the various characteristic quantities with $\alpha=\kappa=0.46$

fm^{-2}. The results are shown in Table I. Here one sees that the type-1c and three-exchange terms have especially large values for the characteristic energy. In addition, it is noted that, among the class-A and class-B terms, the type-1c term and the type-3a and 3d terms have, respectively, the largest characteristic weights.

To obtain an indication that, especially at relatively high energies, the values of the characteristic quantities can be used as a reliable guide, we compute at 100 MeV the values of the quantities Δ_l^{xq}, defined as

$$\Delta_l^{xq}=\delta_l^{xq}-\delta_l^D , \tag{7}$$

where δ_l^D represents the phase shift calculated with the direct potential V_D alone, and δ_l^{xq} represents the phase shift calculated with V_D plus the type-xq nucleon-exchange term but with all other exchange terms omitted. The resultant values, obtained using the nucleon-nucleon potential of Ref. 12 with a Serber mixture, are listed in Table II together with the values of δ_l^D. From this table, one notes the following interesting features:

(i) The magnitudes of Δ_l^{xq} correlate closely with the magnitudes of the characteristic weight ζ_{xq}.

(ii) The class-A and class-B exchange terms do have the expected odd-even l-dependent behavior.

(iii) At such a high energy, the type-1b and two-exchange terms make a very small contribution to the intercluster interaction.

(iv) For $x=1$ and 3, the type-a and type-d terms, which have the same characteristic weight, seem to have opposite influences on the phase shifts. This is a feature which has also been observed in other systems,[14] and for which we have no convincing explanation at this moment. In the present case, this has the consequence of making the type-1c and type-3c terms somewhat more significant.

TABLE I. Characteristic quantities for the ^{3}He+α system.

x	Interaction type q	Class	R_{xq} (fm)	k_{xq} (fm^{-1})	E_{xq} (MeV)	ζ_{xq}
1	a	A	1.60	1.38	23	4.9
	b	A	0.91	1.38	23	1.6
	c	A	1.71	2.71	89	21.4
	d	A	1.44	1.54	29	4.9
2	a	B	0.93	1.05	13	1.0
	b	B	1.19	1.66	33	3.9
	c	A	0.31	1.11	15	0.1
	d	B	0.85	1.15	16	1.0
3	a	B	2.95	2.35	67	48.0
	c	B	1.60	2.35	67	14.1
	d	B	2.45	2.83	97	48.0

The above discussion indicates that, at sufficiently high energies, certain nucleon-exchange terms may be omitted without causing major deterioration in the quality of the result. In the weak- or no-absorption case, one may omit those terms which have characteristic energies much lower than the scattering energy under consideration. In the ^{3}He+α case, this means that at an energy of, say, 60 MeV (35 MeV/nucleon), only the type-1c and three-exchange terms may need to be included in the calculation.[16] When the absorption is relatively strong, one may further utilize the information obtained by examining the values of the characteristic range or the characteristic weight. Under such a situation, it is reasonable to expect that, in the ^{3}He+α case at 60 MeV, one may drop the type-3c term as well.

To confirm the above assertions, we compare in Fig. 1 the 60-MeV differential cross sections calculated with the full kernel [referred to as the

TABLE II. Values of Δ_l^{xq} and δ_l^D, in degrees, for the ^{3}He+α system at 100 MeV.

l	Δ_l^{1a}	Δ_l^{1b}	Δ_l^{1c}	Δ_l^{1d}	Δ_l^{2a}	Δ_l^{2b}	Δ_l^{2c}	Δ_l^{2d}	Δ_l^{3a}	Δ_l^{3c}	Δ_l^{3d}	δ_l^D
0	1.8	−0.1	16.2	−3.6	0	−0.4	0	−0.1	26.1	−3.8	−43.0	109.2
1	1.9	−0.1	16.6	−3.6	0	0.4	0	0	−29.7	3.5	28.7	105.9
2	1.9	−0.1	15.5	−4.0	0	−0.4	0	−0.1	26.0	−3.6	−43.0	99.6
3	1.9	−0.1	14.7	−4.1	0	0.3	0	0	−28.0	3.0	28.4	90.5
4	1.9	−0.1	13.1	−4.0	0	−0.3	0	0	25.0	−2.8	−40.5	79.1
5	1.8	0	11.1	−3.4	0	0.2	0	0	−23.9	2.2	27.5	65.8
6	1.4	0	8.5	−2.5	0	−0.1	0	0	21.6	−1.5	−31.5	51.8
7	0.9	0	5.5	−1.6	0	0	0	0	−16.2	0.9	22.6	38.3
8	0.4	0	3.0	−0.8	0	−0.1	0	0	13.8	−0.5	−18.2	26.6
9	0.2	0	1.5	−0.3	0	0	0	0	−8.3	0.2	13.1	17.5
10	0.1	0	0.7	−0.1	0	0	0	0	6.3	0	−8.1	11.0
11	0	0	0.2	−0.1	0	0	0	0	−3.5	0	5.4	6.8

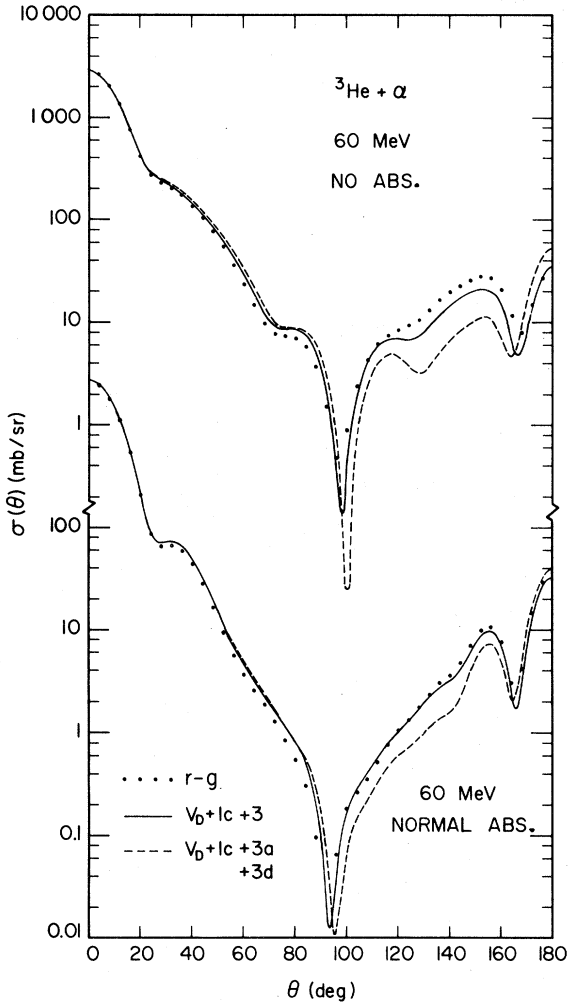

FIG. 1. Comparison of ^{3}He+α differential cross sections at 60 MeV in the no-absorption and normal-absorption cases, obtained with the *r-g*, V_D+1c+3, and $V_D+1c+3a+3d$ calculations.

resonating-group or *r-g* calculation (solid dots)], with only type-1*c* plus three-exchange terms [referred to as the V_D+1c+3 calculation (solid curves)], and with only type-1*c* plus type-3*a* and 3*d* exchange terms [referred to as the $V_D+1c+3a+3d$ calculation (dashed curves)]. In the top and bottom parts of this figure, the results shown are obtained without and with absorption, respectively. To account for absorptive effects, we have included in the calculation an imaginary potential of volume Woods-Saxon form with radius and diffuseness parameters chosen, respectively, as 3.2 and 0.5 fm. The depth parameter is taken to be 10 MeV which yields for the total reaction cross section a normal value of about 400 mb, close to the value expected at this energy[17] (hence, the use of the label "normal absorption" in this figure). Here one sees that the V_D+1c+3 results are indeed quite similar to the resonating-group results. The $V_D+1c+3a+3d$ result is somewhat poor at backward angles in the no-absorption calculation, but does become more satisfactory when absorption effects are taken into account.

The situation is different at lower energies. In Fig. 2, we show a comparison of the V_D+1c+3 result with the *r-g* result at 16 and 30 MeV (i.e., 9.3 and 17.5 MeV/nucleon). From this figure one notes that, in the no-absorption case, the agreement is fair at 30 MeV but becomes rather poor at 16 MeV. When normal absorption is included in the calculation, the agreement does somewhat improve at 30

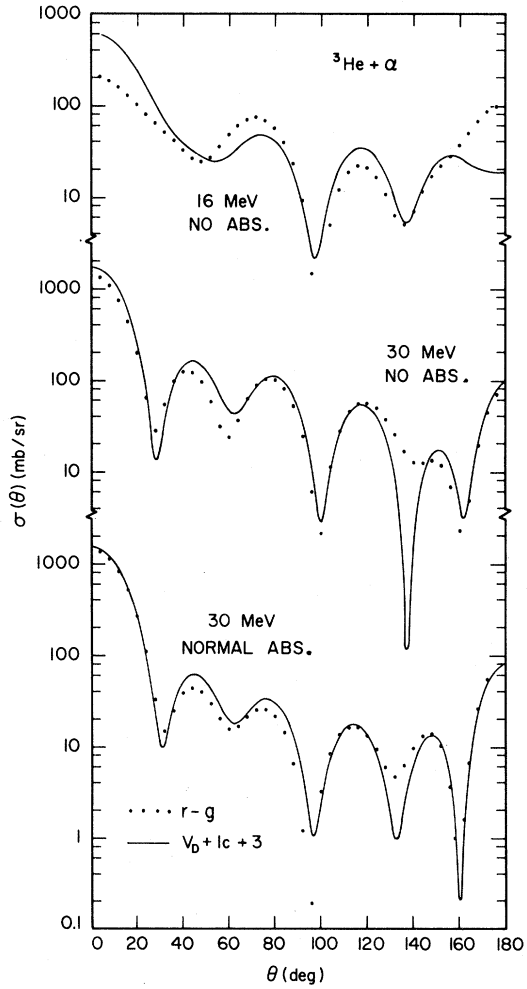

FIG. 2. Comparison of ^{3}He+α differential cross sections at 16 and 30 MeV in the no-absorption and normal-absorption cases, obtained with the *r-g* and V_D+1c+3 calculations.

MeV, but the 16-MeV result (not shown) remains unsatisfactory.

On the other hand, it is found that, even at the rather low energy of 16 MeV, the V_D+1+3 result is quite satisfactory. This indicates that the omission of two-exchange terms does not appreciably impair the quality of the result. The reason for this is quite simple. From Table I, it is seen that, among the class-A and class-B terms, the two-exchange terms have by far the smallest characteristic weights. Thus, the unsatisfactory nature of the V_D+1c+3 calculation at low energies is a consequence of the omission of the rest of the one-exchange terms, in particular, the type-1a and 1d exchange terms.

The energy-dependent nature of the effectiveness of the V_D+1c+3 calculation is summarized in Fig. 3. In this figure, we compare the V_D+1c+3 and r-g phase shifts calculated with no absorption for $l=0-5$ in the energy range of 10–60 MeV. Here one notes that the agreement does become progressively better as the energy increases. At 60 MeV, the differences in the phase-shift values are quite small in odd-l states and are only a few degrees in even-l states.

At relatively high energies, the distinctly different roles played by the class-A and class-B exchange terms, as mentioned in the Introduction, are demonstrated in Fig. 4. Here we show the V_D+1c and r-g results at 60 MeV, obtained with normal absorption. As is seen, the cross-section behavior at forward angles is reasonably reproduced by the V_D+1c calculation, but the strong rise at backward angles does not materialize at all. This indicates, therefore, that the behavior of the cross-section angular distribution in the forward direction can be mainly accounted for by the direct potential plus the type-1c exchange term, while that in the backward direction can be mainly accounted for by the three-exchange terms (essentially the type-3a and 3d exchange terms).

To conclude this section, we comment on the folding-potential model of Satchler and Love.[15] In constructing this model, these authors contended that, among all class-A terms, the knockon term (i.e., type-1c term) is the only exchange term which need be considered and this term can be effectively represented by an energy-independent local poten-

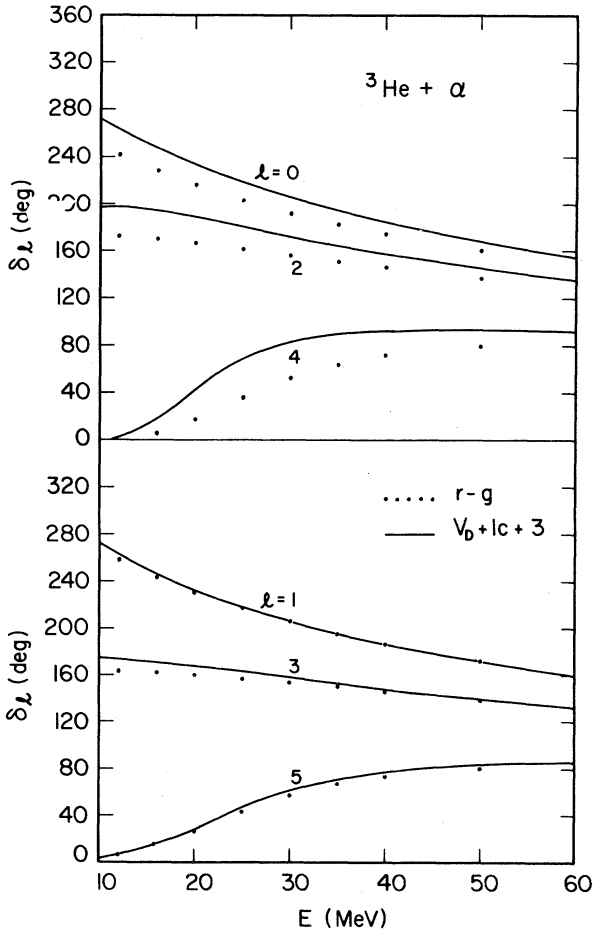

FIG. 3. Comparison of ^{3}He+α, $l=0-5$ phase shifts obtained with the r-g and V_D+1c+3 calculations. Absorptive effects are not included in these calculations.

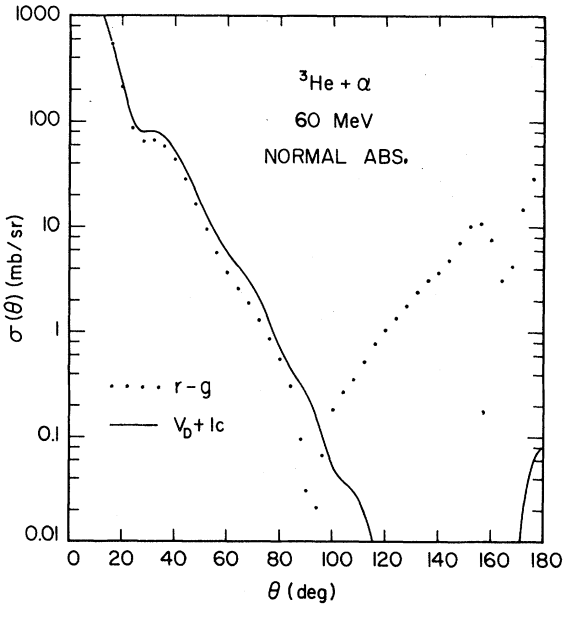

FIG. 4. Comparison of ^{3}He + α differential cross sections at 60 MeV in the normal-absorption case, obtained with the r-g and V_D+1c calculations.

tial having a spatial shape assumed to be the same as that of the folded matter distribution. At the energies which they are concerned with (i.e., $\lesssim 15$ MeV/nucleon), our study here shows that it is in fact not sufficient to include only the type-1c term, but is necessary to take other one-exchange terms also into consideration (with the possible exception of the type-1b term). On the other hand, their assumption about the shape of the effective exchange potential is reasonable, since from Table I one may note that the type-1a, 1c, and 1d terms in the ^{3}He+α case have a similar characteristic range of around 1.6 fm which is significantly shorter than the characteristic range of 2.28 fm for the direct potential. In fact, as long as this model is to be applied only in a restricted energy region, even the assumption of energy-independence for the exchange potential is not entirely unrealistic, since for the dominant type-1c term the characteristic energy has a large value of about 50 MeV/nucleon for all scattering systems.[2] Thus, we are of the opinion that the folding model of Satchler and Love could be a useful model in the low-energy region, although one should interpret the exchange contributions as coming not only from the knockon term but also from other one-exchange terms as well.

IV. $\alpha + ^{16}$O SYSTEM

Proceeding in the same way as in the ^{3}He+α case, we compute the $\alpha + ^{16}$O characteristic quantities with $\alpha = 0.32$ fm^{-2} and $\kappa = 0.46$ fm^{-2}. These are listed in Table III. Here one sees that, because the value of $(N_A - N_B)$ in this system is much larger than that in the ^{3}He + α system, only the class-A terms have large characteristic energies and characteristic weights. At high energies, this means that, except at very large backward angles, the class-B terms are expected to make an insignificant contribution to the scattering differential cross section.

Based on the characteristic quantities listed in Table III, we can make the following assertions:

(i) At moderate energies around 20 MeV (i.e., about 6 MeV/nucleon) where the absorption is not too strong, both one- and two-exchange terms must be included in the calculation. On the other hand, even under the condition of weak absorption, one may further omit the two-exchange terms if the calculation is performed at an energy higher than about 25 MeV/nucleon.

(ii) In the strong-absorption case, one may need to consider only the one-exchange terms even at energies around 20 MeV, because the two-exchange terms have much smaller characteristic ranges or weights.

For the verification of item (i), we show in Fig. 5, $\alpha + ^{16}$O, $l = 0$, 5, and 10 phase shifts at energies from 20 to 80 MeV obtained with r-g (solid dots), $V_D + 1$ (solid curves), and $V_D + 1 + 2$ (dashed curves) calculations. To obtain these phase shifts, we omit absorptive effects and use the nucleon-nucleon potential of Ref. 12 with $w = 0.334$, $m = 0.481$, $b = 0.076$, and $h = 0.109$. As is seen, there is, as expected, very good agreement between the $V_D + 1 + 2$ and the r-g results. The $V_D + 1$ result is not satisfactory at low energies, but does become progres-

TABLE III. Characteristic quantities for the $\alpha + ^{16}$O system.

x	Interaction type q	Class	R_{xq} (fm)	k_{xq} (fm^{-1})	E_{xq} (MeV)	ζ_{xq}
1	a	A	2.26	2.35	36	28.2
	b	A	1.56	2.35	36	13.4
	c	A	2.29	4.94	158	127.5
	d	A	2.04	2.60	44	28.2
2	a	A	1.31	1.50	15	3.9
	b	A	0.93	1.50	15	1.9
	c	A	1.38	2.05	27	8.0
	d	A	1.23	1.60	17	3.9
3	a	A	0.49	1.08	8	0.3
	b	B	0.41	1.20	9	0.2
	c	A	0.76	1.37	12	1.1
	d	A	0.47	1.14	8	0.3
4	a	B	1.02	1.31	11	1.8
	c	B	0.65	1.31	11	0.7
	d	B	0.96	1.38	12	1.8

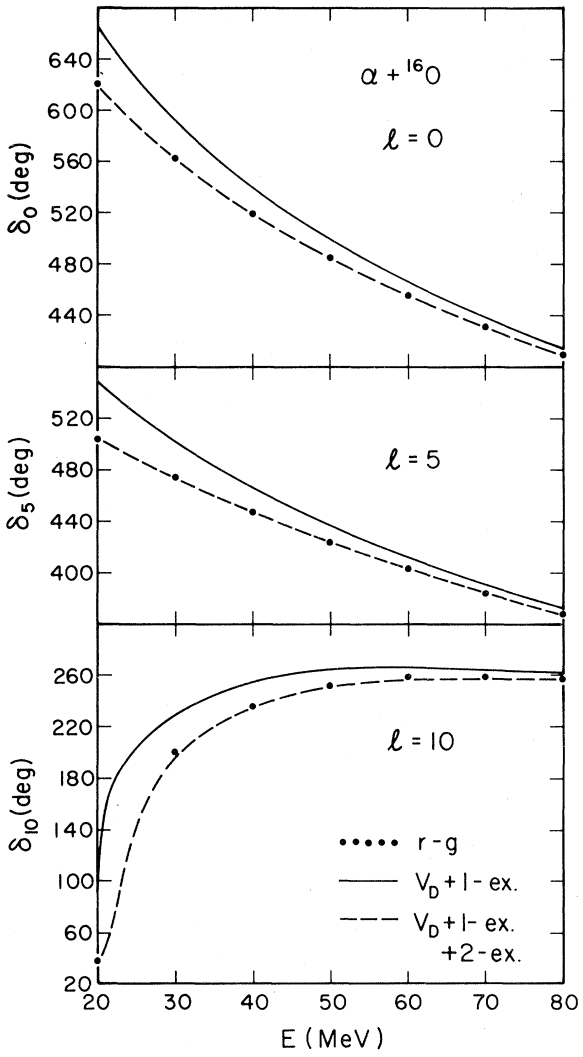

FIG. 5. Comparison of $\alpha+^{16}O$, $l=0$, 5, and 10 phase shifts obtained with the r-g, V_D+1, and V_D+1+2 calculations. Absorptive effects are not included.

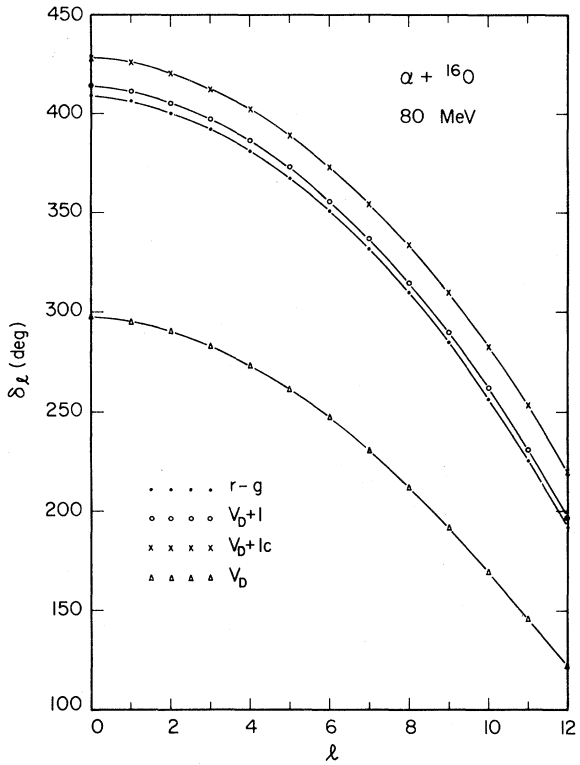

FIG. 6. $\alpha+^{16}O$ phase shifts as a function of l, obtained with the r-g, V_D+1, and V_D+1c calculations. The result obtained with taking only the direct term into consideration is also shown.

sively better as the energy becomes higher.

The usefulness of the V_D+1 calculation at a relatively high energy of, say, 80 MeV is further demonstrated in Fig. 6. From this figure, it is noted that the phase shifts for l up to 12 obtained with the V_D+1 calculation (open circles) differ by only about 5° from those obtained with the r-g calculation (solid dots). Also, in this figure, we show the result obtained with the V_D+1c approximation (crosses). Here one finds that the calculated phase shifts are too large by about 20°. This is not a small discrepancy; however, in view of the fact that the difference in phase shifts between the r-g calculation and a calculation employing V_D alone (triangles) is more than 100°, one could still regard the V_D+1c approximation as a reasonable one at this energy.

Next, we study the situation where absorptive effects are included. For the consideration of such effects, we again introduce into the formulation an imaginary potential of volume Woods-Saxon form with radius and diffuseness parameters equal to 4.2 and 0.6 fm, respectively. Two different cases are then considered at 18 MeV. In the normal-absorption case, we take the depth parameter to be 2 MeV which yields, in the r-g calculation, a reasonable value of 965 mb for the total reaction cross section.[18] In the strong-absorption case, we arbitrarily choose the depth parameter to have a large value of 14 MeV, resulting in a large reaction cross section of 1253 mb. It should be mentioned that we study the latter case here, not because it is realistic for the $\alpha+^{16}O$ system, but because such a study will yield knowledge concerning the influence of strong absorption on exchange effects. Later on, we may then apply such knowledge to many heavy-ion cases where the condition of strong absorption is indeed

fully met.

In Fig. 7, we show in these two cases the reflection coefficient $\exp(-2\delta_l^I)$ as a function of l, with δ_l^I being the imaginary part of the phase shift. The important point to note here is that, for lower values of l, this coefficient acquires moderate values around 0.5 in the normal-absorption case (solid dots) but very small values around 0.03 in the strong-absorption case (open circles). This means that the behavior of the real part of the effective interaction at small intercluster distances is important when there is only normal absorption, but much less so when the absorption is strong. Since it is seen from Table III that the characteristic ranges of the two-exchange terms are much shorter than the characteristic range (2.73 fm) of the direct potential, one expects from this discussion that, at the low energy of 18 MeV, the V_D+1 calculation may yield a rather poor result in the normal-absorption case, but becomes more satisfactory in the strong-absorption case.

That the above expectation is justified can be seen from Fig. 8, where we compare the normal- and strong-absorption results obtained with the r-g (solid dots) and V_D+1 (solid curves) calculations at 18 MeV.[19] Here one notes that, in the normal-absorption case, it is indeed not sufficient to take just one-exchange terms into account. On the other hand, when the absorption is strong, one does see that the V_D+1 calculation yields all the essential features of the resonating-group calculation.

Finally, it should be mentioned that, even under the condition of strong absorption, the V_D+1c calculation yields a rather unsatisfactory result at 18 MeV. As in the ^{3}He+α case, this is again a consequence of the fact that other one-exchange terms (especially type-1a and 1d terms) cannot be omitted at such a low energy.

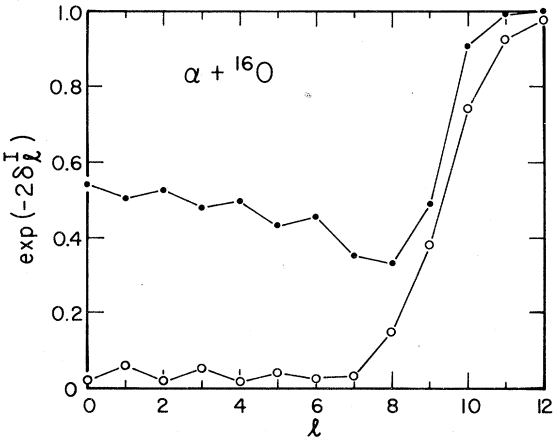

FIG. 7. $\alpha+^{16}$O reflection coefficients as a function of l in the normal-absorption (solid dots) and strong-absorption (open circles) cases.

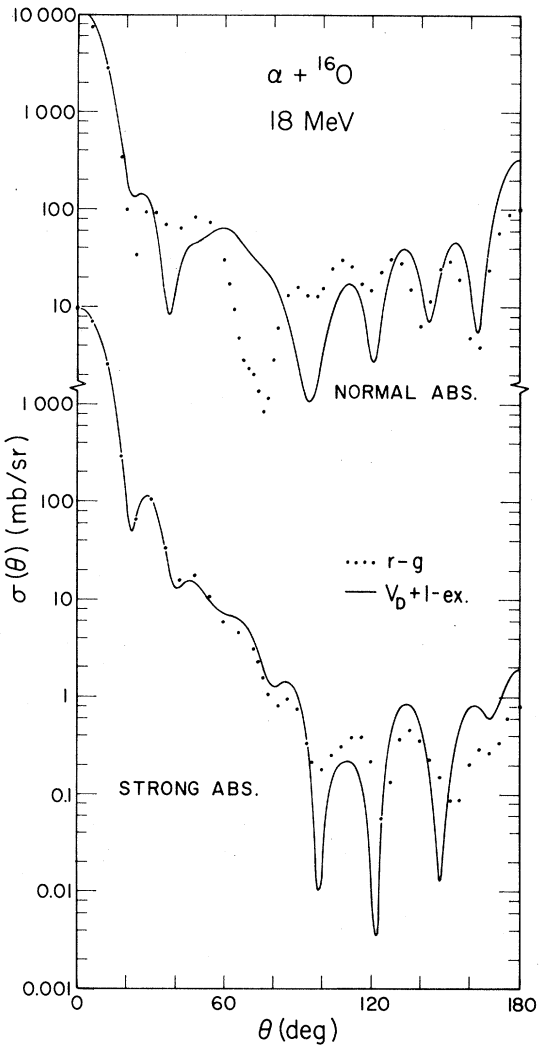

FIG. 8. Comparison of $\alpha+^{16}$O differential cross sections at 18 MeV in the normal-absorption and strong-absorption cases, obtained with the r-g and V_D+1 calculations.

V. CONCLUSION

The most important conclusion reached from this investigation is that the characteristic quantities can be reliably and semiquantitatively employed to determine the relative importance of the various nucleon-exchange terms. This is a very useful finding since, for any two-cluster system, these charac-

teristic quantities can be simply computed by using the general expressions given in previous references.[2,3,5]

Exchange effects are best studied at relatively high energies between about 25 and 50 MeV/nucleon.[20] In this energy region, these effects are significant, but the analysis may be simplified because of the following reasons: (i) The cross-section angular-distribution curve has a distinct V shape and, hence, the classification into class-A and class-B exchange terms becomes particularly useful, and (ii) many nucleon-exchange terms make only minor contributions and, hence, may be omitted from consideration.

Based on the results of this and previous[2] studies, we can make the following general statements[21]:

(i) Among all class-A terms, the one-exchange terms are the most important. At relatively high energies, the type-1c or knockon exchange term has, in particular, a dominant influence.

(ii) Among all class-B terms, the core-exchange terms make the largest contribution. For systems in which absorption is strong, the core-exchange type-a and type-d terms are particularly significant.

(iii) The one-exchange terms are generally important in all scattering systems and over a wide energy range, but the core-exchange terms are generally important only when the nucleon-number difference of the interacting nuclei is rather small.

(iv) At energies higher than about 25 MeV/nucleon, the contribution to the cross section in the decreasing part of the V-shaped angular-distribution curve comes essentially from the direct term and the one-exchange terms (mainly the type-1c term), while the contribution to the cross section in the increasing part of this curve comes essentially from the core-exchange terms.

With the completion of this investigation, we believe that we now understand quite well the general features of antisymmetrization. To proceed further, one must undertake the difficult task of examining the influence of blocking and target clustering on exchange effects.[22] For an understanding of the blocking influence, we intend to carry out a detailed examination of the $\alpha + {}^6$Li system for which a recent resonating-group calculation[9] has yielded reasonable results in comparison with experiment. As for the influence of target clustering, a preliminary attempt[3] based on the properties of the exponential factors occurring in the three-cluster normalization kernel has already been made and an interesting qualitative conclusion has been reached. However, this study is far from complete and one must still perform a careful analysis of the three-cluster Hamiltonian or energy kernel function in order to achieve a semiquantitative and more reliable understanding.

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[19]For both of these comparisons, the absorptive strengths in the V_D+1 calculations are adjusted to yield the same reaction cross sections as those obtained in the corresponding resonating-group calculations.

[20]At energies appreciably higher than 50 MeV/nucleon, the analysis will become more involved due to the necessity of dealing with complicated effects associated with the presence of a short-range repulsive component in the nucleon-nucleon potential.

[21]Based on the WKB approximation and an analysis of the $\alpha+\alpha$ system, Aoki and Horiuchi (Ref. 14) have reached qualitatively the same conclusion.

[22]Very little information on blocking and target-clustering effects can be extracted from this investigation, since the nuclei involved here have either an s-shell configuration or a doubly-closed-shell configuration.