PHYSICAL REVIEW C VOLUME 26, NUMBER 1 JULY 1982

Energy loss, nucleon exchange, and neutron-proton correlation in heavy-ion reactions

S. S. Kapoor and J. N. De
Nuclear Physics Division, Bhabha Atomic Research Centre, Bombay-400 085, India
(Received 20 August 1981)

Transport model description of the correlation between the energy loss and the variance of the charge or mass distributions in the heavy-ion deep inelastic reactions has been modified to include in the theory the effect of the degree of neutron-proton correlation in the particle exchanges. With the use of the revised theoretical expressions and considering the role played by the presence of correlations, the contradictions and discrepancies seen in the earlier analysis are removed. A better quantitative agreement with the experimental results is obtained by the present formalism. This brings out in an unambiguous way the dominant role of the nucleon exchange process in the energy loss mechanism.

[NUCLEAR REACTIONS Heavy-ion reactions, energy loss, nucleon exchanges, neutron-proton correlations.]

Experimentally well established correlation between the dissipated energy and the variance of the fragment charge or mass distribution in the deep inelastic heavy-ion collisions has lent crucial support to the assumption that the nucleon exchange mechanism is an important source of energy loss in these reactions.[1] In recent studies,[2–6] experimental results for a variety of target-projectile combinations and bombarding energies have been compared with the predictions of transport theories based on stochastic transfer of nucleons between the two colliding nuclei in relative motion. It is now recognized[7,4,5] that the fermion nature of the exchanged particles and the associated Pauli blocking effect should be included in a theoretical description of the correlation between energy dissipation and mass dispersion. In the above studies, the effect of the neutron-proton correlation in the exchange process has also been considered in deducing the fragment mass variance σ_A^2 from the experimentally measured fragment charge variance σ_Z^2 for confronting the experimental results with the theoretical transport model description of Randrup.[7] Although the theoretical description can qualitatively account for the experimental results on σ_A^2 versus the energy loss, there still remain important discrepancies, and in particular, conflicting conclusions seem to have been reached in Refs. 4 and 5 with regard to the degree of neutron-proton correlations on the basis of fits with the theory.

In the present work, it is suggested that the above mentioned discrepancies result due to the fact that comparisons with theory were not properly made, since although the effects of correlations were considered in deducing "experimental" σ_A^2 from measured σ_Z^2, the effect of correlated transfers on theoretical results had not been considered. In this paper, modified theoretical expressions with the inclusion of the effect of correlations have been deduced which resolve the above discrepancies, and a better agreement with the experimental results is obtained.

We first consider the discrepancies which can be seen in a comparison of the experimental results with the Randrup model,[7] as has been done in Refs. 4 and 5. The relation between the mass variance σ_A^2, charge variance σ_Z^2, and the neutron number variance σ_N^2 is given by

$$\sigma_A^2=\sigma_Z^2+\sigma_N^2+2\rho\sigma_Z\sigma_N \ .$$

Here, ρ measures the degree of correlation between neutrons and protons during transfer and varies between -1 to $+1$. For uncorrelated motion ($\rho=0$) it can be easily seen that the variances scale according to the corresponding densities,

$$\sigma_N^2=\frac{N}{Z}\sigma_Z^2 \ , \tag{1a}$$

$$\sigma_A{}^2=\sigma_N{}^2+\sigma_Z{}^2=\frac{A}{Z}\cdot\sigma_Z{}^2 . \tag{1b}$$

For fully correlated neutron and proton exchanges ($\rho=1$) such that the equilibrated A/Z ratio is always maintained, it has been shown[8] that

$$\sigma_N{}^2=\left[\frac{N}{Z}\right]^2\sigma_Z{}^2 \tag{2a}$$

and

$$\sigma_A{}^2=(\sigma_N+\sigma_Z)^2=\left[\frac{A}{Z}\right]^2\cdot\sigma_Z{}^2 . \tag{2b}$$

These limiting relationships for uncorrelated and correlated motion have also found support from experiments[9,10] in which variances of both charge and mass distributions are measured.

If one does not consider Fermi motion and Pauli blocking in the nucleon-exchange process and the exchanged particles between the two nuclei in relative motion are taken as classical objects, the following relation holds,[11,1,2] neglecting a small term arising from recoil correction,

$$-\frac{dE}{d\sigma_A{}^2}=\frac{m}{\mu}\cdot E . \tag{3}$$

Here E is the available relative kinetic energy above the Coulomb barrier, m the nucleon mass, and μ the reduced mass of the system. Within the framework of the transport theory, the following results have been deduced by Randrup,[7] with the inclusion of Fermi motion and Pauli blocking,

$$\frac{d\sigma_A{}^2}{dt}=2N_A'T^* , \tag{4}$$

where N_A' is the differential total particle current and T^* is given by

$$T^*=\tfrac{1}{2}\langle\omega\coth(\omega/2T)\rangle_F$$
$$\approx\tfrac{1}{2}\langle\omega^2\rangle_F^{1/2}\coth(\langle\omega^2\rangle_F^{1/2}/2T) . \tag{5}$$

Here ω is the change in excitation energy associated with the transfer of a nucleon, T is the temperature of the system, and the average is taken around the mean value of the Fermi energies. The rate of energy dissipation neglecting the recoil correction term is given by

$$-\frac{dE}{dt}\simeq N_A'\langle\omega^2\rangle_F . \tag{6}$$

It is also a very good approximation to take $\coth\langle\omega^2\rangle_F^{1/2}/2T\approx1$ in Eq. (5) for the cases under consideration and, therefore,

$$T^*\approx\tfrac{1}{2}\langle\omega^2\rangle_F{}^{1/2} .$$

From Eqs. (4) and (6), it then follows that

$$-\frac{dE}{d\sigma_A{}^2}\simeq\langle\omega^2\rangle_F^{1/2} . \tag{7}$$

For peripheral collisions, and for nuclei having nearly equal Fermi energies, Eq. (7) reduces to[4,5]

$$-\frac{dE}{d\sigma_A{}^2}\simeq\left[\frac{m}{\mu}EE_F\right]^{1/2} , \tag{8}$$

where E_F is the Fermi energy. On integrating Eq. (8), one obtains

$$E^{1/2}=E_0^{1/2}-\tfrac{1}{2}\left[\frac{m}{\mu}E_F\right]^{1/2}\sigma_A{}^2 , \tag{9}$$

where E_0 is the initial available kinetic energy. The correlation between the energy loss and the mass dispersion can also be expressed in another form as follows,

$$-\frac{dE}{d\sigma_A{}^2}=\alpha\cdot\frac{m}{\mu}E . \tag{10}$$

In this representation, a value of α larger than unity becomes a reflection of the quantum nature of the particle transfers, since for the classical case the value of α is unity. For peripheral collisions, the quantum nature of particle transfers yield the following value of α on the basis of Eqs. (8) and (10)

$$\alpha=\left[\frac{E_F}{m}\cdot\frac{\mu}{E}\right]^{1/2} . \tag{11}$$

In earlier studies, experimental results have been compared with theory in different ways on the basis of Eqs. (8)–(11). In the experiments[5] with 610 and 710 MeV ^{86}Kr beams on ^{139}La, the charge distributions were measured as a function of available relative kinetic energy. The relationship of mass dispersion $\sigma_A{}^2$ with $\sigma_Z{}^2$ is not unambiguously known, but it has been found that for bombarding energies close to the Coulomb barrier there is good evidence for the relationship

$$\sigma_A{}^2=\left[\frac{A}{Z}\right]^2\sigma_Z{}^2 .$$

At higher bombarding energies,[10] although $\sigma_A{}^2$ starts as $A/Z\cdot\sigma_Z{}^2$ for small energy losses, it approaches $(A/Z)^2\cdot\sigma_Z{}^2$ for large energy losses. The authors of Ref. 5 have deduced experimental $\sigma_A{}^2$ from $\sigma_Z{}^2$ under the two extreme assumptions with regard to correlations, and these results are shown in Fig. 1 in the form of a plot of $-dE/d\sigma_A{}^2$ as a function of $[(m/\mu)EE_F]^{1/2}$ to make comparisons with the prediction of Eq. (8). Similar results from the experiments[12,3,13] with ^{86}Kr+^{197}Au at 620 MeV, ^{86}Kr+^{166}Er at 710 MeV, and ^{84}Kr+^{165}Ho at 714 MeV, again taken from Ref. 5, are shown in Fig. 2. In the present representation of the experimental results, the scales on the left and right have been altered by a factor A/Z so that the left scale corresponds to the experimental values under the assumption of uncorrelated motion, while the right scale is for correlated motion. It may be noted that both the theoretical lines represented by solid and dashed lines have the slope of unity [Eq. (8)] on their respective right and left scales. On the basis of these results of Figs. 1 and 2, the authors of Ref. 5 seem to infer that there is evidence in the data for uncorrelated motion, as only this assumption provides good fit with the theory. This suggestion of completely uncorrelated transfer at all energies runs counter to the experimental findings.[9,10] On the other hand, in accordance with the experimental findings,[9,10] if one assumes that $\sigma_A{}^2=(A/Z)^2\sigma_Z{}^2$, it is seen in Figs. 1 and 2 that the theory predicts significantly larger energy loss per variance $-dE/d\sigma_A{}^2$, as compared to the experimental values. This is difficult to understand. If there are coacting mechanisms like the excitation to giant resonances[14] or uncorrelated particle-hole excitations,[15] the experimental energy loss per variance should be greater than the theoretical expectations.

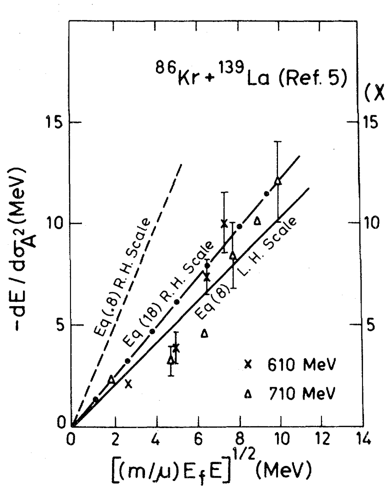

FIG. 1. Plot of $-dE/d\sigma_A{}^2$ as a function of $[(m/\mu)EE_F]^{1/2}$. The left hand scale corresponds to the case of uncorrelated motion and the right hand scale corresponds to the case of correlated motion. The experimental points obtained from $\sigma_Z{}^2$ under the two extreme assumptions have to be read out accordingly. The full and dashed lines obtained from Eq. (8) have the slope of unity on the left hand and right hand scales, respectively. The dashed-dotted line refers to the case when the theory is modified for correlated transfer. The value of E_F is taken to be 37 MeV.

As an example of the results of correlations between the available energy and directly measured $\sigma_A{}^2$, we have picked up the system 710 MeV ^{86}Kr+^{166}Er for which measurements of both $\sigma_A{}^2$ and $\sigma_Z{}^2$ have been made in a separate study and the data were in agreement with the relationship $\sigma_A{}^2=(A/Z)^2\sigma_Z{}^2$. These results are plotted in a different form, as a plot of $E^{1/2}$ vs $\sigma_A{}^2$ to make comparison with the form of Eq. (9). It is clearly seen that Eq. (9) based on the Randrup model is inoperative, as it predicts an even larger energy loss per variance than observed.

A similar anomaly is again seen in Fig. 4, in the plot of the experimental values of α versus the available energy per nucleon, as deduced for a number of cases in Ref. 4 along with the theoretical prediction of α based on Eq. (11), shown as a dashed curve. The results corresponding to directly measured $\sigma_A{}^2$ are shown in Fig. 4(b), while those in Fig. 4(a) correspond to those deduced from directly measured $\sigma_Z{}^2$ under the assumption $\sigma_A{}^2=(A/Z)^2\sigma_Z{}^2$. For the moment, confining ourselves to Fig. 4(a), it is again seen that the theoretical energy loss predicted by Eq. (11) is much larger than observed. In what follows it is shown that the above mentioned anomalies are removed if the theoretical expressions are also suitably modified to take into account the degree of neutron-proton correlations in the nucleon exchange process. Tak-

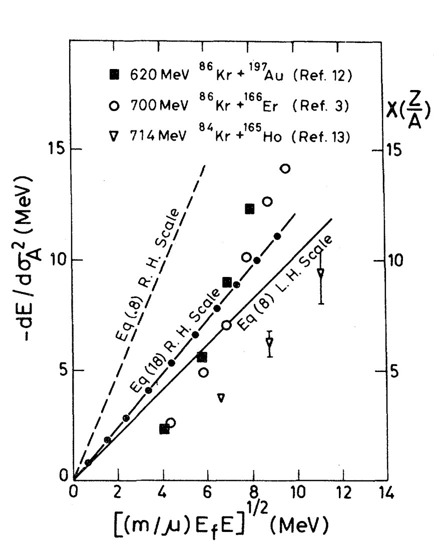

FIG. 2. Same as Fig. 1.

ing into consideration the two component nature of the exchanged particles, one can write individual expressions for the time rate of change of the proton and neutron variances. For independent (uncorrelated) proton and neutron exchanges, we have

$$\frac{d}{dt}\sigma_Z{}^2 = N_Z' T^* \,, \tag{12a}$$

$$\frac{d}{dt}\sigma_N{}^2 = N_N' T^* \,, \tag{12b}$$

where N_Z' and N_N' are the differential proton and neutron currents, proportional to their densities. However, for correlated transfers ($\rho = 1$), the proton and neutron variances may not be as simply related as in Eq. (12), since this will not yield in general the expected relation of Eq. (2). For the simple problem of a one-dimensional random walk, the variance of the mass distribution is given by[16] $\sigma_A{}^2 = nm^2$, if there are n exchanges each involving a unit of mass m. In terms of the total mass exchange $M = nm$, the variance can be written as $\sigma_A{}^2 = Mm$. In the case of correlated motion for a system with $Z = N$, such that a neutron transfer is necessarily accompanied by a proton transfer in the same direction to preserve the value of A/Z, the result for $\sigma_A{}^2$ changes to $\sigma_A{}^2 = (n/2)(2m)^2 = M(2m)$, because now there are $n/2$ exchanges, each involving a unit of mass $2m$.

Thus in the case of a system with $Z = N$, the value of $\sigma_A{}^2$ for correlated transfers should be twice that for uncorrelated motion, for a given value of total mass exchange and hence energy dissipation. For the general case of $Z \neq N$, the following relations are deduced: Assuming for a moment that the protons are transferred independently and the neutrons follow the protons, Eq. (12a) remains unchanged, but Eq. (12b) should change to

$$\frac{d}{dt}\sigma_N{}^2 = \left[\frac{N}{Z}\right] N_N' T^* = \left[\frac{N}{Z}\right]^2 N_Z' \cdot T^* \tag{13}$$

in order to satisfy Eq. (2). Similarly, if it is assumed that a proton transfer is followed by an independent neutron transfer, then Eq. (12b) remains unchanged, but Eq. (12a) changes to

$$\frac{d}{dt}\sigma_Z{}^2 = \frac{Z}{N} \cdot N_Z' T^* \,. \tag{14}$$

Assuming that the protons and neutrons have each one-half probability of independent transfers followed by correlated transfers of neutrons and protons, one obtains the following relations for correlated transfers.

$$\frac{d}{dt}\sigma_Z{}^2 = \tfrac{1}{2} N_Z' T^* + \tfrac{1}{2}\left[\frac{Z}{N}\right] N_Z' T^* = \frac{A}{2N} N_Z' T^* \,, \tag{15a}$$

$$\frac{d}{dt}\sigma_N{}^2 = \tfrac{1}{2}\left[\frac{N}{Z}\right]^2 N_Z' T^* + \tfrac{1}{2}\left[\frac{N}{Z}\right] N_Z' T^* = \frac{A}{2}\frac{N}{Z^2} N_Z' T^* \,. \tag{15b}$$

For $\rho = 1$, then

$$\left[\frac{d}{dt}\sigma_A{}^2\right]_{\text{correlated}} = \frac{A^2}{2ZN}\left[\frac{d\sigma_A{}^2}{dt}\right]_{\text{uncorrelated}} . \tag{16}$$

Thus, association of $\sigma_A{}^2$ with the total number of particle exchanges is valid only for the uncorrelated motion. For fully correlated motion, the value of $\sigma_A{}^2$ is larger than the number of particle exchanges by a factor $A^2/2ZN$, as seen in Eq. (16). As we are considering the correlated and uncorrelated motion for a given number of particle exchanges, the rate of energy dissipation [Eq. (6)] remains unaltered. It then follows that the theoretical value of $dE/d\sigma_A{}^2$ for correlated transfers is related to the expression for uncorrelated motion given by Randrup[7] as follows:

$$\left[\frac{dE}{d\sigma_A{}^2}\right]_{\text{correlated}} = \frac{2ZN}{A^2}\left[\frac{dE}{d\sigma_A{}^2}\right]_{\text{uncorrelated}} . \tag{17}$$

For peripheral collisons, Eqs. (8), (9), and (11) then change to

$$-\frac{dE}{d\sigma_A{}^2} = \frac{2ZN}{A^2}\left[\frac{m}{\mu} E E_F\right]^{1/2} \tag{18}$$

and

$$E^{1/2} = E_0{}^{1/2} - \frac{ZN}{A^2}\left[\frac{m}{\mu} E_F\right]^{1/2} \cdot \sigma_A{}^2 \,. \tag{19}$$

Thus in order to make proper comparisons with the theory, one should compare in Figs. 1 and 2 the experimental results with Eq. (18) if neutron-proton motion is assumed to be correlated, and with Eq. (8) if it is assumed to be uncorrelated. Predictions of Eq. (18) are shown as a dashed-dotted line (right hand scale) in Figs. 1 and 2, from where it is seen that theory is in good agreement with the experimental results irrespective of the assumption made with regard to the degree of correlations. This is so, because the assumption of correlation not only changes the experimental values of $\sigma_A{}^2$ as deduced from measured $\sigma_Z{}^2$, but also changes in the same way the theoretical expression to be expected for $-dE/d\sigma_A{}^2$. On the other hand, if experimental values of $\sigma_A{}^2$ are obtained from direct measurements, the assumption of correlation will alter only the theoretical expression, as can be seen in Fig. 3. For this case there is experimental evidence[9] for fully correlated neutron-proton exchanges and therefore the experimental results should be compared with the theoretical prediction of Eq. (18). It is heartening to note in Fig. 3 that if the theory is modified to include the effect of correlated motion, the experimental results are in good agreement with the theory, removing the discrepancy mentioned earlier. In Fig. 4(a), the theoretical values of α for fully correlated motion, based on Eq. (18), and shown as a solid line, are again seen to be in better agreement with the experimental $\sigma_A{}^2$ deduced from $\sigma_Z{}^2$ assuming correlated motion. It may be noted here that an equally good agreement of experiment with theory would have been achieved by assuming uncorrelated motion both in experimental deduction of $\sigma_A{}^2$ and in theory. In Fig. 4(b) based on direct measurements of $\sigma_A{}^2$, the experimental points are insensitive to assumption on correlations, and only theoretical results change. In this case, it is therefore possible to comment on the degree of correlation on the basis of comparison between experiments and theory. It is interesting to note that the degree of correlations reached in these systems as interpreted on the basis of quality of fit with theory [Fig. 4(b)] is consistent with the results of direct measurements.[10,9] From the earlier discussions, it thus emerges that if the experimental results are compared with the proper theory which includes the effects of neutron-proton correlations, measure-

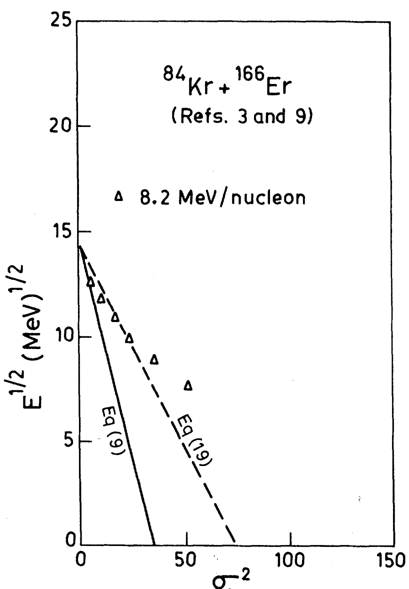

FIG. 3. Plot of $E^{1/2}$ as a function of $\sigma_A{}^2$ for the system $^{86}Kr+^{166}Er$. The experimental data for $\sigma_A{}^2$ are measured directly. The full line refers to the theoretical prediction for the case of uncorrelated neutron-proton transfer and the dashed line results if the theory is modified to include correlated motion.

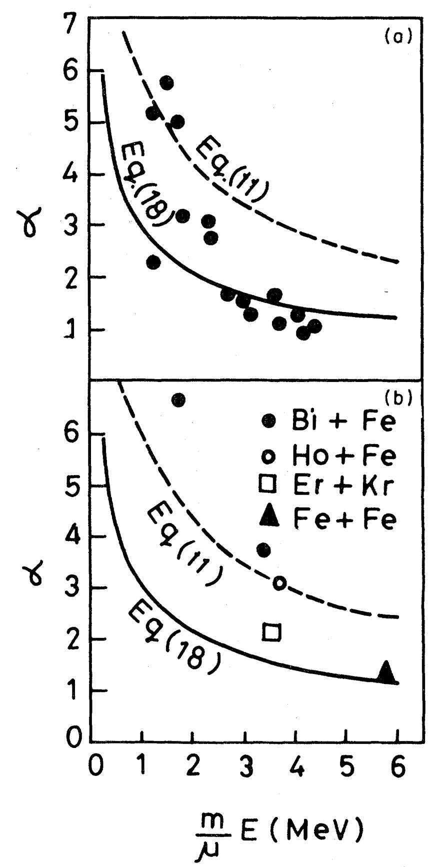

FIG. 4. (a) The blocking parameter α plotted as a function of $(m/\mu)/E$. The dashed curve is obtained when $\sigma_A{}^2$ is evaluated from $\sigma_Z{}^2$ with the assumption of correlated transfer, but without any corresponding modification in Eq. (11). For further details, see text. (b) The values of α have been plotted here for the cases of direct measurements of $\sigma_A{}^2$. The dashed and full line curves are theoretical predictions for the cases of uncorrelated and correlated neutron-proton exchanges, respectively.

ments of σ_Z^2 versus energy loss are better suited to confront the transport theory and to infer as to what extent the observed energy loss can be accounted by the nucleon exchange mechanism alone. In the case of comparison of theory with the experimental correlation of energy loss versus directly measured σ_A^2, the question of the degree of proton-neutron correlation also sensitively enters into comparison. This conclusion is contrary to the earlier belief that the question of correlation is important in the understanding of energy loss versus σ_Z^2, but not versus σ_A^2.

To summarize, it has been shown that the discrepancies observed in earlier analysis of the data on correlation of energy loss with charge or mass dispersion in a number of heavy-ion systems, are removed by a modification of the theoretical transport model description to include the degree of neutron-proton correlation in the exchange process. Comparison of the data for a number of systems with this modified theory shows good quantitative agreement, without much ambiguity arising from the degree of correlations present, as the assumption of correlation modifies both the experimental deduction of σ_A^2, and the theory. The dominant role played by this mechanism in the energy dissipation process in deep inelastic heavy-ion collisions, is strongly supported by the present analysis.

[1]W. U. Schroder and J. R. Huizenga, Annu. Rev. Nucl. Sci. 27, 465 (1977).
[2]W. U. Schroder *et al.*, Phys. Lett 71B, 283 (1977).
[3]G. Rudolf *et al.*, Nucl. Phys. A330, 243 (1979).
[4]W. U. Schroder *et al.*, Phys. Lett. 44, 308 (1980).
[5]P. Dyer *et al.*, Phys. Rev. C 22, 1509 (1980).
[6]M. Dakoswki *et al.*, Phys. Lett. 90B, 379 (1980).
[7]J. Randrup, Nucl. Phys. A327, 490 (1979).
[8]F. Beck *et al.*, Z. Phys. A 289, 113 (1978).
[9]Y. Eyal *et al.*, Phys. Rev. Lett. 42, 826 (1979).
[10]H. Breuer *et al.*, Phys. Rev. Lett. 43, 191 (1979).
[11]W. J. Swiatecki, J. Phys. (Paris) C 5, 45 (1972).
[12]L. Moretto and J. S. Sventek, Proceedings of the Symposium on Macroscopic Features of Heavy-Ion Collisions, Argonne, 1976, Argonne National Laboratory Report ANL/PHY-76-2, 1976, p. 235.
[13]J. R. Huizenga *et al.*, Phys. Rev. Lett. 37, 885 (1976).
[14]R. A. Broglia *et al.*, Phys. Lett. 730B, 405 (1978).
[15]R. Beck and D. H. E. Gross, Phys. Lett. 47B, 143 (1973).
[16]S. Chandrasekhar, Rev. Mod. Phys. 15, 1 (1943).