## Possible existence of backbending in actinide nuclei

J. Dudek, W. Nazarewicz,\* and Z. Szymański Institute for Theoretical Physics, Warsaw University, Hoza 69, PL-00-681 Warsaw, Poland (Received 16 October 1981)

The possibilities for the backbending effect to occur in actinide nuclei are studied using the pairing-self-consistent independent quasiparticle method. The Hamiltonian used is that of the deformed Woods-Saxon potential plus monopole pairing term. The results of the calculations explain why there is no backbending in most actinide nuclei and simultaneously suggest that in some light neutron deficient nuclei around <sup>224</sup>Th and <sup>22</sup>Ra a backbending effect may occur.

NUCLEAR STRUCTURE Collective structure in levels (including rotational bands); collective models.

Considerable progress in understanding the behavior of the high-spin excitations in atomic nuclei was achieved during the last decade. In particular, the mechanism of nucleonic angular momentum alignment<sup>1</sup> has been rather well understood in terms of the independent quasiparticle  $model^{2-6}$ [see also the early Hartree-Fock-Bogoliubov (HFB) cranking model calculations $^{7-10}$ ]. In a nucleus that rotates very fast a substantial rearrangement in structure may occur which may lead to a crossing of the ground-state band with another one of a different alignment of angular momentum. A sharp crossing between bands of two different angular momentum alignments leads to the appearance of the multivalued behavior in nuclear moment of inertia J and other quantities versus angular velocity  $\omega$ . This effect is referred to as backbending. It is known to occur in the yrast spectra of many nuclei in the rare earth region. Here, the high-*j* and low- $\Omega$ orbitals, such as the lowest members of the  $1i_{13/2}$ neutron, or  $1h_{11/2}$  proton orbitals play an essential role owing to the strong Coriolis interaction [cf. Eq. (1)]. In the actinide nuclei, however, no backbending effect has been reported until now. Several experiments were performed recently in this region; they were based mainly on the multiple Coulomb excitation. Rotational states with angular momentum ranging up to  $1 \approx 30\%$  have been observed.<sup>11-15</sup> In some of these nuclei an angular momentum alignment effect has been analyzed.<sup>5,6,14,16</sup> However, no pronounced backbending has ever been seen in this region. At first this may seem surprising since there exist 1 higher-*j* orbitals in the single

particle spectra of these nuclei: the  $1j_{15/2}$  and  $1i_{13/2}$  multiplets for neutrons and protons, respectively. The lack of backbending in any of the actinide nuclei studied so far may, however, be explained by a strong interaction between the ground state band and the excited bands, as discussed in more detail below.

The aim of the present paper is to demonstrate that in a few actinide nuclei a sharp band crossing may nevertheless lead to a pronounced backbending effect. At the same time our calculations show no backbending for most of the actinide nuclei, in agreement with existing experimental data. An explanation is attempted for the *nonexistence* of backbending as the most common feature in this region.

We first calculated the nuclear equilibrium deformations using the Strutinsky method.<sup>17</sup> We adopted the realization of the method proposed by Bolsterli et al.<sup>18</sup> The single particle energies were generated using the optimized parametrization<sup>19</sup> of the Woods-Saxon potential; the corresponding parametrization of pairing was taken from Ref. 20. The results of the calculations are illustrated in Fig. 1 which represents the quadrupole and hexadecapole equilibrium deformations. It is a very important element of the analysis to use the proper estimates for the equilibrium deformations, since the relative positions of the high-*i* orbitals may vary significantly with the deformation. This is illustrated in Figs. 2 and 3 where the Woods-Saxon single particle levels are plotted as functions of the quadrupole  $\beta_2$ and hexadecapole  $\beta_4$  deformations; in fact, both  $\beta_2$ and  $\beta_4$  are varied in such a way as to follow, on the

26

1708



FIG. 1. Quadrupole and hexadecapole deformations of actinide nuclei calculated by making use of the Strutinsky method.

average, the arc in the  $(\beta_2, \beta_4)$  plane of Fig. 1.

Comparing the results given in Figs. 1–3 one learns that most of the calculated equilibrium deformations correspond to  $\beta_2 \approx (0.20-0.25)$  and to positive  $\beta_4$  values. One can see from Fig. 2 that near their Fermi energies nuclei with  $N \ge 142$  do



FIG. 2. Woods-Saxon neutron single particle levels calculated for  $\beta_2$  and  $\beta_4$  deformations varying along the average path of Fig. 1. The symbols near the levels specify nuclei whose valence neutrons reside in the indicated orbitals.



FIG. 3. The same as in Fig. 2 but for protons.

not have any high-*j* orbitals other than those with  $\Omega \gtrsim \frac{7}{2}$ .

Let us now recapitulate the conditions for the occurrence of the pronounced backbending effect. We first need a large difference in angular momentum alignment between two crossing bands, the Stockholm band and the ground band. This is certainly the case if we deal with the low- $\Omega$  members of the high-*j* orbitals as follows from the explicit expression for the matrix element of the Coriolis force

$$\langle j\Omega | j\pm | j\Omega\pm 1 \rangle = [j(j+1) - \Omega(\Omega\pm 1)]^{1/2} .$$
<sup>(1)</sup>

However, there exists a second necessary condition requiring that the interaction V between the two bands is weak enough so that a sharp band crossing results. This interaction is known to be an oscillating function of the shell filling.<sup>21</sup> We are now ready to present explicitly the differences in the backbending behavior between the two regions (rare earth and actinide). According to the analysis by Bengtsson and Frauendorf<sup>2</sup> (see also Refs. 5 and 6), the backbending takes place only if the interaction |V| is weak enough. More precisely, it should not exceed roughly  $j^2/(4J)$  if the functions *i*, *J*, etc., are to remain multivalued in the model of Ref. 2. Now, although the angular momentum contribution *j* is

of the same order of magnitude in both the regions, the moments of inertial J for the actinides are approximately twice as large as those in the rare earth nuclei. This makes the backbending much less probable for these heavy nuclei, especially at intermediate values of  $\Omega$  where the minima in the curve of |V| versus shell filling (see Ref. 21) occur as the very narrow dips. It seems, therefore, quite likely that in the vicinity of the  $j = \frac{13}{2}$  and  $\Omega = \frac{7}{2}$  orbit the backbending effect may be absent in the actinide region although it is still possible in the rare earth nuclei. A similar conclusion has also been drawn by Bengtsson.<sup>5</sup> Thus a pronounced backbending can only occur in nuclei with the Fermi energy close to the energy of the high-j orbital and the smallest possible  $\Omega$ , i.e., with  $\Omega = \frac{1}{2}$  or at most,  $\frac{3}{2}$ . Figure 1 suggests that this may be the case for a few light neutron-deficient nuclei with relatively small quadrupole equilibrium deformation and thus also relatively small moments of inertia. The results illustrated in Figs. 2 and 3 indicate that the best candidates in this respect are <sup>222,224</sup>Th and <sup>222</sup>Ra which, in addition to having small equilibrium deformations, have also particularly active high-j orbitals  $[1j_{15/2(\Omega=1/2)} \text{ and } 1j_{15/2(\Omega=3/2)} \text{ neu-}$ tron and also  $1i_{13/2(\Omega=1/2)}$  proton levels] close to the Fermi energy.

The above simple qualitative arguments are confirmed by the quantitative HFB cranking model calculation results. In Figure 4 a comparison is given between moments of inertia of <sup>237</sup>Np, which is representative for well-deformed actinide nuclei, and those of the "exotic" <sup>224</sup>Th, for which a particularly strong double backbending is predicted. It is worth emphasizing that the method applied here is exactly the same as the one successfully applied to the description of the band crossing in the rare earth nuclei (e.g., in <sup>160</sup>Yb, Ref. 22, in <sup>166,168,170</sup>Yb, Ref. 23, or in <sup>156</sup>Er, Ref. 24). In fact, our calculation extends over a wider region of nuclei including, e.g., some isotopes of Th and U. The results which fully confirm our general conclusion are not given here and will be a subject of the forthcoming publication.

In summary, the presented calculation based on the HFB cranking method gives for the first time a direct explanation for the lack of the backbending effect in the yrast spectra for most of the nuclei in the actinide region. At the same time we set the validity limits for this general conclusion by providing the important exceptional cases. The results do predict a rather pronounced backbending effect for



FIG. 4. The relatively-smooth J vs  $\omega^2$  dependence in <sup>237</sup>Np which is characteristic for well-deformed nuclei in the actinide region (left-hand side) is compared to the pronounced double S-shape dependence of J vs  $\omega^2$  of <sup>224</sup>Th [ $J = I/\omega(I)$ ]. Note that for spin values which are not too high J (I) of <sup>224</sup>Th is significantly smaller than the corresponding values for <sup>237</sup>Np. For comparison the experimental data taken from Ref. 13 is also given.

a few light neutron-deficient nuclei around <sup>224</sup>Th and <sup>222</sup>Ra, and possibly for only very few of the neighboring nuclei (for which, however, no detailed calculations have yet been performed). It may be difficult to verify these predictions experimentally owing to the relatively low fission barriers (the calculated value of the barrier height is ~5 MeV at I = 0 for <sup>224</sup>Th) and a small interaction between the crossing bands, but such an endeavor would be of particular importance for a deeper understanding of the alignment effect in nuclei.

After this paper was completed we learned about the work of Diebel and Mosel<sup>25</sup> who apply the HFB cranking method based on the Nilsson model to several nuclei in the actinide region. These authors obtain a much weaker interaction |V| and, consequently, a stronger and more rapid alignment as compared to our calculations which are based on the Woods-Saxon potential. This effect can be related to the systematic differences between the slopes of the single particle Routhians generated by the Nilsson and the Woods-Saxon potentials<sup>26</sup> and can be traced back to the presence of the  $\vec{L}^2$  term in the Nilsson potential.

The authors are indebted to their colleagues A. Majhofer and J. Skalski for their help in calculating the equilibrium deformations. This work was supported in part by the Polish-American Maria Skłodowska Fund, Grant No. P-F7F037P.

- \*Address: Institute of Physics, Technical University of Warsaw, Koszykowa 75, PL-00-662 Warsaw, Poland.
- <sup>1</sup>F. S. Stephens and R. S. Simon, Nucl. Phys. <u>A183</u>, 257 (1972).
- <sup>2</sup>R. Bengtsson and S. Frauendorf, Nucl. Phys. <u>A314</u>, 27 (1979).
- <sup>3</sup>R. Bengtsson and S. Frauendorf, Nucl. Phys. <u>A327</u>, 39 (1979).
- <sup>4</sup>I. Hamamoto, Nucl. Phys. <u>A271</u>, 15 (1976).
- <sup>5</sup>R. Bengtsson, J. Phys. (Paris) Colloq. <u>C10</u>, 84 (1980).
- <sup>6</sup>S. Frauendorf, Phys. Scr. <u>24</u>, 349 (1981).
- <sup>7</sup>P. Ring, R. Beck, and H. J. Mang, Z. Phys. <u>231</u>, 10 (1970); <u>231</u>, 26 (1970); H. J. Mang, B. Samadi, and P. Ring, Z. Phys. A 277, 325 (1976).
- <sup>8</sup>P. C. Bhargava, Nucl. Phys. <u>A207</u>, 258 (1973).
- <sup>9</sup>A. Goodman, Nucl. Phys. <u>A256</u>, 113 (1976).
- <sup>10</sup>A. Faessler, K. R. Sandhya-Devi, F. Grümmer, K. W. Schmid, and R. R. Hilton, Nucl. Phys. <u>A256</u>, 106 (1976).
- <sup>11</sup>H. Ower, Th.W. Elze, J. Idzko, K. Stelzer, H. Emling, P. Fuchs, E. Grosse, D. Schwalm, H. J. Wollersheim, N. Krafrell, and N. Trautmann, J. Phys. (Paris) Colloq. <u>C10</u>, 102 (1980).
- <sup>12</sup>E. Grosse, A. Balanda, H. Emling, F. Folkmann, P. Fuchs, R. B. Piercey, D. Schwalm, R. S. Simon, H. J. Wollersheim, D. Ewers, and H. Ower, Phys. Scr. <u>24</u>, 337 (1981).
- <sup>13</sup>R. S. Simon, F. Folkmann, C. Briancon, J. Libert, J. P. Thibaud, R. J. Walen, and S. Frauendorf, Z. Phys. <u>298</u>, 121 (1980).
- <sup>14</sup>R. B. Piercey, J. H. Hamilton, A. V. Ramayya, H.

Emling, P. Fuchs, E. Grosse, D. Schwalm, H. J. Wollersheim, N. Trautmann, A. Faessler, and M. Ploszajczak, Phys. Rev. Lett. <u>46</u>, 415 (1981); see also A. Faessler, invited talk given at the European Physical Society meeting on Heavy Ion Physics in Nuclei and Atoms, Bucharest, 1981, Institute of Physics and Nuclear Engineering, Bucharest (to be published).

- <sup>15</sup>O. Häusser, H. Gräf, L. Grodzins, E. Jaensche, V. Metag, D. Habs, D. Pelte, H. Emling, E. Grosse, R. Kulessa, D. Schwalm, R. S. Simon, and J. Keionen, Phys. Rev. Lett. <u>48</u>, 383 (1982).
- <sup>16</sup>L. K. Peker, S. Pearlstein, and J. H. Hamilton, Phys. Lett. <u>100B</u>, 281 (1981).
- <sup>17</sup>V. M. Strutinsky, Nucl. Phys. <u>A95</u>, 420 (1967).
- <sup>18</sup>M. Bolsterli, E. O. Fisset, J. R. Nix, and J. L. Norton, Phys. Rev. C <u>5</u>, 1050 (1972).
- <sup>19</sup>J. Dudek, A. Majhofer, J. Skalski, T. Werner, S. Cwiok, and W. Nazarewicz, J. Phys. G <u>5</u>, 1359 (1979).
- <sup>20</sup>J. Dudek, A. Majhofer, and J. Skalski, J. Phys. G <u>6</u>, 447 (1980).
- <sup>21</sup>R. Bengtsson, I. Hamamoto, and B. R. Mottelson, Phys. Lett. <u>73B</u>, 259 (1978).
- <sup>22</sup>S. Cwiok, W. Nazarewicz, J. Dudek, J. Skalski, and Z. Szymanski, Nucl. Phys. <u>A333</u>, 139 (1980).
- <sup>23</sup>S. Cwiok, J. Dudek, W. Nazarewicz, and Z. Szymanski, Phys. Rev. C <u>21</u>, 448 (1980).
- <sup>24</sup>J. Dudek, W. Nazarewicz, and Z. Szymanski, Phys. Scr. <u>24</u>, 309 (1981).
- <sup>25</sup>M. Diebel and U. Mosel, Z. Phys. A <u>303</u>, 131 (1981).
- <sup>26</sup>J. Dudek, A. Majhofer, W. Nazarewicz, and Z. Szymanski, Phys. Lett. <u>112B</u>, 1 (1982).