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Three-body kinematics for the binding effect in pion-nucleus elastic scattering

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A simple and effective method to include the nucleon binding effect in pion-nucleus elastic scattering by using the three-body kinematics is proposed. It will be shown that the binding effect can be incorporated into the usual form of the first order optical potential calculation by making the two-body energy argument a variable parameter depending not only on the energy, but also on the nucleon Fermi momentum. Numerical examinations of the binding effect are carried out by π-^{12}C elastic scattering and the binding effect will be shown to have significant implications over the entire energy range considered, from low through resonance to above resonance regions.

[NUCLEAR REACTIONS Calculated $\sigma(\theta)$ for $^{12}C(\pi,\pi)$, $E=50-250$ MeV. Binding effect. Three-body kinematics. Importance of the Fermi motion.]

I. INTRODUCTION

For microscopic calculations in multiple scattering theory, the use of the impulse approximation[1,2] has been almost imperative in the past. Although there exist many papers[3–16] which have purportedly employed a nonimpulse approximation, there exist rather larger differences among these calculations and hence a realistic alternative to the impulse approximation has not yet been satisfactorily developed.

The importance of the binding effect has been rather elusive especially as to where it should appear. It may show up at low energy,[1,2] or around the P_{33} resonance peak[1,12] in pion-nucleus elastic scattering, or at the high energy side of the resonance tail.[8] The past attempts to dispense with or estimate the validity of the impulse approximation may be grouped into two categories within the context of multiple scattering theory. One is to simultate the binding interaction by a simple model like a nucleon bound inside the harmonic oscillator potential well,[3,4,12] and the pion-bound nucleon scattering matrix so obtained is substituted into multiple scattering theory. Another is to set up a three-body system and replace the real scattering dynamics by the three-body kinematics (Refs. 5–8, 13, 17, and 19). Unfortunately the conventional three-body problem can be solved only approximately,[5,8,9,13] although for separable pairwise interactions[20] and for the deuteron target[16,20,21] it can be solved exactly. Recently a new three-body method has been put forward[17–19] in which the usual cyclic symmetry of the standard three-body formalism[20] is missing. However, a practical demonstration of its usefulness has yet to be seen. We refer to Ref. 22 for more extensive references.

In this paper we take a slightly different path to this second approach. We will make use of the three-body kinetics, but not as the three-body equations *per se*, but rather to take advantage of the difference between the free and bound state Green's functions. A similar step was taken previously by Schmit[4] and Gurvitz *et al.*[15] The common idea is to devise kinematics by which the series expansion of the projectile-bound nucleon t matrix in terms of the projectile-free nucleon t matrix may be truncated after the first term. Here we employ the three-body kinematics to achieve this goal. Because of this special kinematics it will be shown later that the nucleon Fermi motion effect must necessarily be included in the nuclear matrix element calculation, that is, in the three-body kinematics one cannot separate the nucleon Fermi motion from the binding effect.

In Sec. II we develop the basic formalism for the inclusion of the binding effect. Section III deals with the off-shell kinematics and approximation schemes. Final results and some comments are given in Sec. IV.

II. THEORY

In general, the many-body operator τ for the two-body process in the integral equation formalism

of Kerman, McManus, and Thaler (KMT) (Ref. 2) can be expanded in terms of a free two-body operator in many-body space t as [13,18,19]

$$\tau(E)=t(\epsilon)+t(\epsilon)G_0(\epsilon)\times\{\epsilon-E-H_0+H_A\}G(E)\tau(E)\,, \quad (1)$$

where

$$G_0(\epsilon)=(\epsilon^+-K_0-H_0)^{-1}\,,$$
$$G(E)=(E^+-K_0-H_A)^{-1}=(E^+-K_0-H_0-U)^{-1}\,, \quad (2)$$

with K_0, H_A, and U being the kinetic energy operator of projectile, nuclear Hamiltonian, and the binding potential. The energy parameter E is uniquely determined by the initial scattering configuration; however ϵ, being a parameter of the two-body interaction in many-body space, is a completely arbitrary constant, although once chosen it has to be kept in all subsequent calculations. The impulse approximation consists of truncating Eq. (1) after the first term and setting $\epsilon=E$. The idea to dispense with or to minimize the error in using the impulse approximation[4,15] is to make the second term of Eq. (1) vanish, namely the term enclosed by the brace. Since ϵ is at our disposal, if the eigenvalues of Hamiltonians take tangible forms, we can *choose* ϵ so as to make this term disappear. In order to see if this is possible, Eq. (1) is rewritten in detail

$$\langle\vec{K}';B\,|\,\tau(E)\,|\,\vec{K}_0;B\rangle=\langle\vec{K}';B\,|\,t(\epsilon)\,|\,\vec{K}_0;B\rangle+\int d^3K\int\prod_{j=1}^{A}d^3p_j\,\delta^{(3)}\left[\sum_{i=1}^{A}\vec{p}_i-\vec{K}\right]\times\frac{\langle\vec{K}';B\,|\,t(\epsilon)\,|\,\vec{K};\vec{p}_1,\ldots,\vec{p}_A\rangle\{\epsilon-E-\epsilon(\vec{K};\vec{p}_1,\ldots,\vec{p}_A)+E(K)\}}{\epsilon-\epsilon(\vec{K};\vec{p}_1,\ldots,\vec{p}_A)+i\eta'}\times\frac{\langle\vec{K};\vec{p}_1,\ldots,\vec{p}_A\,|\,\vec{K};B\rangle\langle\vec{K};B\,|\,\tau(E)\,|\,\vec{K}_0;B\rangle}{E-E(K)+i\eta}\,, \quad (3)$$

where $\vec{K}$, $\vec{p}_i$, $E(K)$, and B denote the momenta of pion and ith nucleon and the total energy in the πA c.m.s. (center of mass system), and the nuclear ground state. Furthermore, $|\vec{K};\vec{p}_1,\ldots,\vec{p}_A\rangle$ is an aggregate of plane wave states with respective momenta for pion and nucleons and $\epsilon(\vec{K};\vec{p}_1,\ldots,\vec{p}_A)$ its energy. Primed and unprimed momenta refer to after and before scattering, respectively. In deriving we have used the fact that the states $|\vec{K};\vec{p}_1,\ldots,\vec{p}_A\rangle$ and $|\vec{K};B\rangle$ are the eigenstates of H_0 and H_A, respectively, and that both H_0 and H_A do not interact with pions. In order to carry out integrations on nucleon momenta, we introduce two approximations, the independent particle model for a nucleus which is consistent with the lowest order approximation of the optical potential and the three-body kinematics for the evaluation of the $\epsilon(\vec{K};\vec{p}_1,\ldots,\vec{p}_A)$ term. Then the multidimensional integrals on intermediate nucleon momenta can effectively be reduced to a single integral.

In order to choose the value of ϵ, first we employ nonrelativistic expressions[19] of energy for the nucleon and core nucleus, and second the average nucleon mass $m=938.906$ MeV is distinguished from the atomic mass unit $m^*=931.481$ MeV in order to take the binding effect into account from the many-body point of view. The use of m^* is particularly appropriate for the discussion of the binding effect in the context of the multiple scattering theory which treats all nucleons as equivalent,[2] for the binding effect is operative among core nucleons at the same time when the binding effect of the valence nucleon is examined and the bulk effect of the former can be incorporated through the use of m^*.

Thus when a pion of mass μ and the laboratory kinetic energy of T_π (lab) collides with a static nucleus of A nucleons, the total energy of the system in the center of mass is given by

$$E=E(K_0)=\{\mu^2+(Am^*)^2+2Am^*[\mu+T_\pi(\text{lab})]\}^{1/2}=E_\pi(K_0)+Am^*+\frac{K_0^2}{2Am^*}\,, \quad (4)$$

where the pion energy $E_\pi(K_0)=\sqrt{\mu^2+K_0^2}$ and the on-shell momentum $\vec{K}_0$ in the πA c.m.s. are defined by the last line. Similarly the energy $E(K)$ for off-shell momentum $\vec{K}$ in Eq. (3) is given by

$$E(K)=E_\pi(K)+Am^*+\frac{K^2}{2Am^*}\,. \quad (5)$$

Then the three-body kinematics replace $\epsilon(K;,p_1,\ldots,p_A)$ by

$$\epsilon(\vec{K};\vec{p}_1,\vec{p}_{A-1})=E_\pi(K)+m+\frac{1}{2m}\left[\vec{q}-\frac{\vec{K}}{A}\right]^2+(A-1)m^*+\frac{1}{2(A-1)m^*}\left[\vec{q}+\frac{A-1}{A}\vec{K}\right]^2$$

$$=E_\pi(K)+Am^*+\frac{K^2}{2Am^*}+\frac{A}{2(A-1)m}q^2+m-m^*$$

$$-(m-m^*)\left[\frac{q^2}{2(A-1)mm^*}-\frac{\vec{q}\cdot\vec{K}}{Amm^*}+\frac{K^2}{2Amm^*}\right], \tag{6}$$

where $\vec{q}$ is the nucleon Fermi momentum. Since the last term in the parenthesis is very small, this may be neglected. The additional term

$$[A/2(A-1)m]q^2+m-m^*,$$

when compared with Eq. (5), can be easily seen to be the negative of the binding potential. With this approximation, the term in the brace of Eq. (3) can be seen to vanish, if we take

$$\epsilon=E+\frac{A}{2(A-1)m}q^2+m-m^*. \tag{7}$$

Thus in the present three-body framework, we can abolish the impulse approximation simply by taking the nucleon Fermi motion fully into account and making the two-body energy parameter in the three-body space Fermi momentum dependent in our calculations of the nuclear matrix element. The importance of the nucleon Fermi motion has been known (Refs. 6, 9, 10, 19, and 23–25) not only by virtue of itself, but also as a significant ingredient in deriving πN c.m.s. energy and momentum from the corresponding πA c.m.s. counterparts.[9,11,19,25] But the relationship between the $\vec{q}$ integral and impulse approximation has never been fully examined and these two subjects were treated independently, and since the essential change in t matrix was not expected arising from the Fermi motion, most calculations resorted to the so-called factorizing approximation within the impulse approximation (Refs. 4, 5, 8–13, 19, 22, 24, and 25). However, here we now have a new situation in which the inclusion of the $\vec{q}$ integral is essential to dispense with the impulse approximation. This is very natural, since in the three-body kinematics the nucleon Fermi motion must be explicitly taken into consideration and since the binding interaction is inseparably related to the Fermi motion, the three-body kinematics automatically link up the Fermi motion with the binding interaction. We cannot talk about one without another in the three-body kinematics, as in the past. Thus, although ϵ and hence the πN c.m.s. energy become functions of $\vec{q}$, by including the $\vec{q}$ integral we can achieve both elimination of the impulse approximation and improvement of the nuclear matrix element at the same time.

Once the two-body energy in the three-body space ϵ is determined, the πN c.m.s. energy S can be obtained from the invariant πN mass relationship[25]

$$S^2=W^2-Q_c^{\,2}=(\epsilon-E_c)^2-Q_c^{\,2}, \tag{8}$$

where W, E_c, and $\vec{Q}_c$ denote the total energy of the πN system and energy momentum of the core nucleus in the πA c.m.s. at the incident energy. The expressions for the latter should be clear from Eq. (6). As the $\vec{q}$ integral is performed, we have the well-known complexity that the S may decrease below the threshold $(m+\mu)$ (Refs. 9, 11, 19, 20, and 25). Since we use a separable πN $\bar{t}$ matrix,[26,27] the extrapolation into the unphysical region is conveniently taken care of by the Fredholm determinant.[19]

III. CALCULATION

A. Off-shell scattering kinematics

We employ the method of Heller *et al.*[28] to relate a πN scattering matrix element in the πN c.m.s. with the corresponding matrix element in the πA c.m.s. In their formalism, a half-shell matrix element in the πA c.m.s. is given by

$$t[E(K,p);\vec{K}',\vec{p}',\vec{K},\vec{p}]=N(K',p',K,p)F(P;\kappa',\kappa)\bar{t}[\omega(\kappa);\vec{\kappa}',\vec{\kappa}], \tag{9}$$

where $\bar{t}$ is the two-body matrix element in the πN c.m.s., and we use the following notations:

$$
\begin{aligned}
&E(K,p)=E_1(K)+E_2(p)\ ,\\
&\omega(\kappa)=\omega_1(\kappa)+\omega_2(\kappa)=[E(K,p)^2-P^2]^{1/2}\ ,\\
&E_i(K)=\sqrt{m_i^2+K^2},\quad \omega_i(\kappa)=\sqrt{m_i^2+\kappa^2}\ ,\\
&N(K',p',K,p)=\left[\frac{E(K',p')\omega_1(\kappa')\omega_2(\kappa')E(K,p)\omega_1(\kappa)\omega_2(\kappa)}{E_1(K')E_2(p')\omega(\kappa')E_1(K)E_2(p)\omega(\kappa)}\right]^{1/2}\ ,\\
&F(P;\kappa',\kappa)=\frac{\omega(\kappa')+\omega(\kappa)}{E(K',p')+E(K,p)}\ ,\\
&\vec{\mathrm{P}}=\vec{\mathrm{K}}+\vec{\mathrm{p}}=\vec{\mathrm{K}}'+\vec{\mathrm{p}}'\ .
\end{aligned}
\tag{10}
$$

The πN c.m.s. momentum $\vec{\kappa}$ is obtained from $\vec{\mathrm{K}}$ and $\vec{\mathrm{p}}$ by the proper Lorentz transformation of the relative momentum.

$$
\vec{\kappa}=\frac{1}{2\omega(\kappa)\{E(K,p)+\omega(\kappa)\}}\{[\omega^2(\kappa)+2\omega(\kappa)E_2(p)+m_2{}^2-m_1{}^2]\vec{\mathrm{K}}-[\omega^2(\kappa)+2\omega(\kappa)E_1(K)+m_1{}^2-m_2{}^2]\vec{\mathrm{P}}\}\ .
\tag{11}
$$

This was called a canonical transformation in Ref. 28. The special transformation of Aaron *et al.*,[29] when applied to the relative momentum, will also reduce to this form. After calculating a similar expression for κ', the scattering angle in the πN c.m.s. is given by[19]

$$
\cos\theta=\frac{\vec{\kappa}\cdot\vec{\kappa}'}{\kappa\kappa'}\ .
\tag{12}
$$

The product NF reduces to the conventional transformation factor γ^1 in the on-shell limit.

On the other hand, a fully off-shell matrix element in the πA c.m.s. is expressed by the integral of the form

$$
\begin{aligned}
\frac{t(E;\vec{\mathrm{K}}',\vec{\mathrm{p}}',\vec{\mathrm{K}},\vec{\mathrm{p}})}{N(K',p',K,p)}&=F(P;\kappa',\kappa)\bar{t}[\omega(\kappa);\vec{\kappa}',\vec{\kappa}]\\
&\quad+\int d^3k\,F(P;\kappa',k)F(P;\kappa,k)\bar{t}[\omega(k);\vec{\kappa}',\vec{\mathrm{k}}]\bar{t}^*[\omega(k);\vec{\kappa},\vec{\mathrm{k}}]\\
&\quad\times\left[\frac{F^{-1}(P;k_0,k)}{\omega(k_0)+i\eta-\omega(k)}-\frac{F^{-1}(P;\kappa,k)}{\omega(\kappa)+i\eta-\omega(k)}\right]\ ,
\end{aligned}
\tag{13}
$$

$$
\omega(k_0)=\sqrt{E^2-P_0{}^2}\ .
$$

Here the index zero implies the incident scattering configuration. Since the inclusion of the above integral in calculating the nuclear matrix element is extremely time consuming, it has never been done thus far. Hence we introduce the following accurate approximation. First, the above equation can be rewritten as follows, after factoring out the common angular dependent part,

$$
\begin{aligned}
\frac{t(E;K',p',K,p)}{N(K',p',K,p)}&=F(P;\kappa',\kappa)\bar{t}(\omega(k_0);\kappa',\kappa)\\
&\quad-\left\{F(P;\kappa',\kappa)\bar{t}[\omega(k_0);\kappa',\kappa]-\int k^2dk\frac{F(P;\kappa',k)F(P;\kappa,k)F^{-1}(P;k_0,k)}{\omega(k_0)+i\eta-\omega(k)}\right.\\
&\qquad\left.\times\bar{t}[\omega(k);\kappa',k]\bar{t}^*[\omega(k);\kappa,k]\right\}\\
&\quad+\left\{F(P;\kappa',\kappa)\bar{t}[\omega(\kappa);\kappa',\kappa]-\int k^2dk\,\frac{F(P;\kappa',k)\bar{t}[\omega(k);\kappa',k]\bar{t}^*[\omega(k);\kappa,k]}{\omega(\kappa)+i\eta-\omega(k)}\right\}\ .
\end{aligned}
\tag{14}
$$

Then we drop the second and third terms on the right hand side altogether as an approximation. This is because the differences in each brace can be shown to be roughly the same constant below the inelastic threshold in the separable model of πN $\bar{t}$ matrix elements and thus the combined terms vanish. In order to show this assertion, we write the terms in the first brace by the separable model. After omitting the common factor it takes the form[27]

$$F(P;\kappa',\kappa)\frac{\bar{t}[\omega(k_0)]}{f^2(k_0)}+\frac{1}{\pi}\int d\omega(k)\frac{F(P;\kappa',k)F(P;\kappa,k)F^{-1}(P;k_0,k)}{\omega(k_0)+i\eta-\omega(k)}\frac{\mathrm{Im}\,\bar{t}[\omega(k)]}{f^2(k)}\ ,$$

where $f(k)$ and $\bar{t}[\omega(k)]$ are the off-shell function and the on-shell matrix element. Since the function $F(P;\kappa',\kappa)$ and the combination of F's in the integrand are roughly the same (in both the nonrelativistic and on-shell limits, they are exactly the same; otherwise they are slowly varying functions), neglecting the difference and separating the whole expression into the real and imaginary parts, we can see the above quantity to be approximately some constant σ.[30] Since the same constant will appear from the second brace of Eq. (14), the combined terms vanish within the above mentioned approximation. The approximate matrix element so obtained bears the correct off-shell character and makes a smooth transition to the half-shell matrix element.

B. Approximation schemes

Although the above equations are mostly exact, in practice the inclusion of the nucleon Fermi motion requires a very long computing time.[22,24] Thus it is always desirable to seek some approximation schemes to circumvent the integration on the Fermi momentum, without altering the content of the exact matrix element too much. This is usually called the factorizing approximation (Refs. 4, 5, 8–13, 22, 24, and 25). Theoretically this approximation is not expected to be a reliable one[22] when the πN t matrix undergoes rapid variation as around resonances, but in practice it works well owing mainly to the greater insensitivity of cross sections to the details of input.[8,12,22] Here we propose two approximation schemes which do cut down computing time, but without resorting to the factorizing approximation.

The nuclear matrix element becomes in the symmetric form[24] and in the separable model

$$4\left[\frac{b}{\sqrt{\pi}}\right]^3\exp[-(Qb)^2]\int_0^\infty k^2dk\left\{1+\frac{A-4}{6}(k^2-Q^2)b^2\right\}\exp[-(kb)^2]\int d\Omega(\hat{k})g(\vec{p}\,'[\vec{k}])g_0(E,\vec{k})g(\vec{p}[\vec{k}])\ , \tag{15}$$

where the harmonic oscillator wave functions with the oscillator parameters "b" are used. g and g_0 are the off- and on-shell factors including the FN term in the previous subsection.

$\vec{Q}=(A-1)(\vec{K}-\vec{K}')/(2A)$, and $\vec{k}=\vec{q}+\vec{Q}$. First the $\vec{k}$ integral becomes symmetric if the $\hat{k}$ direction is chosen along $\vec{Q}$ or $\vec{K}+\vec{K}'$, which will roughly halve the computing time. Then, since for a spherical nucleus the integrand arising from the dominant nuclear part is angle independent, the first approximation is obtained by neglecting the azimuthal angle dependence out of the off-shell factors, i.e., by reducing two-dimensional angle integrations to one-dimensional ones, and the second approximation is obtained by completely neglecting the angle dependence out of the off-shell factors. We remark that the full angular integral is still to be carried out over the on-shell part of the t matrix element. If one further neglects even the k integral itself over the off-shell factors, then one obtains an improved Fermi averaged matrix element in which the nuclear overlap functions are exact.

In order to elucidate the role played by the binding effect, we have also calculated cross sections by using the conventional kinematics, i.e., fully relativistic, asymmetric, and

$$S^2=[\sqrt{\mu^2+K_0^2}+\sqrt{m^2+(\vec{p}-\vec{K}_0/A)^2}]^2-Q_c^2\ .$$

To separate the effect of Fermi motion out, an old program was improved by including the q integral and the full $\hat{q}$ integral over the on-shell part of t matrix element. This approximation is called the third approximation in the following.

We are now ready to calculate the cross sections. The basic formalism is a momentum space integral equation.[31] The harmonic oscillator constant $r_0=1.687$ fm and the size parameter $(A-4)/6=1.067$ were taken from the table by Jager *et al.*[32] which includes the proton size correction.

IV. RESULTS AND DISCUSSIONS

Figure 1 displays differential cross sections from 50 to 250 MeV pion kinetic energy in the laboratory system T_π (lab), and Fig. 2 shows the total and elastic total cross sections and the real forward scattering amplitudes.

First of all, we notice that the binding effect increases the differential cross sections above the impulse results over the entire angle domain, save for the extremely forward direction, and second it flattens out both the total cross section and real forward scattering amplitude by reducing the peaks and by filling in the valleys; however, it does not shift the peak positions. On the other hand, the elastic total cross sections are somewhat increased by the binding effect, which is related with the increased differential cross sections.

As far as the total cross section is concerned, the binding effect shows up at all three energy domains mentioned in the Introduction. The increase of σ_t at the high energy tail of resonance[8] was also observed in Refs. 12 and 13, however, in Ref. 12 it was obtained by lifting up the σ_t over the entire energy region, while in Ref. 13 it was obtained by shifting the overall energy dependence of σ_t upward. In our case the increase was obtained by neither cause. Thus the binding effect in terms of the increased σ_t at high energy in the present calculation is more pronounced than the previous calculations. Although this finding helps improve the agreement with the experimental data at high energy, it is detrimental at low energy, since it raises σ_t above the impulse results and away from the experimental results.

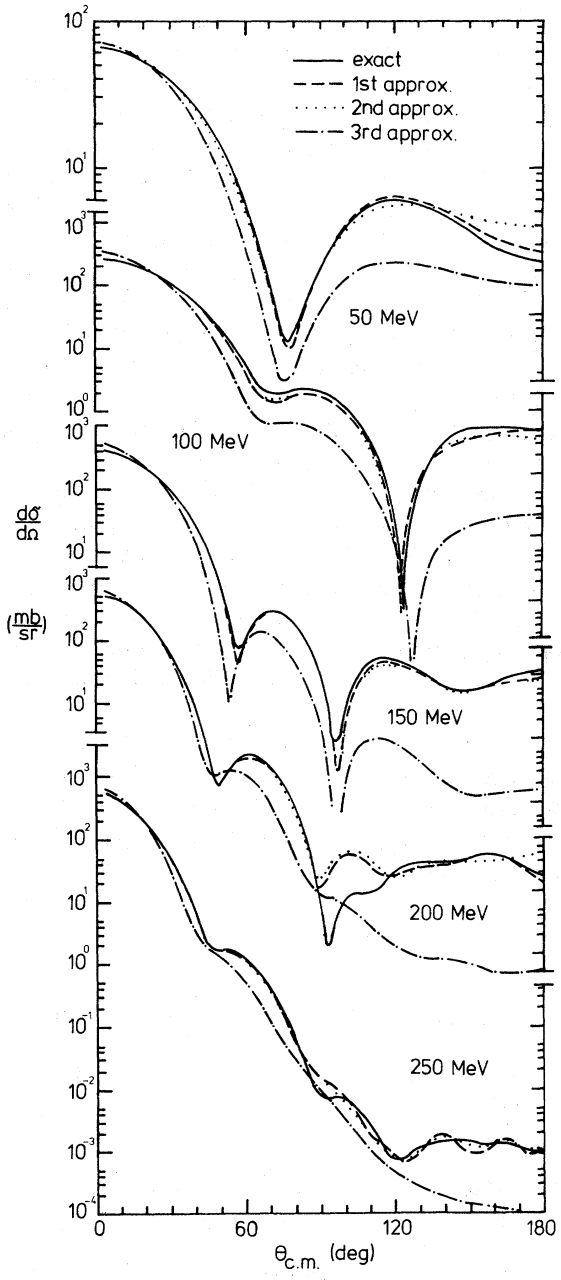

FIG. 1. Elastic scattering differential cross sections for π-^{12}C collision. Explanation on the approximations is in the main text.

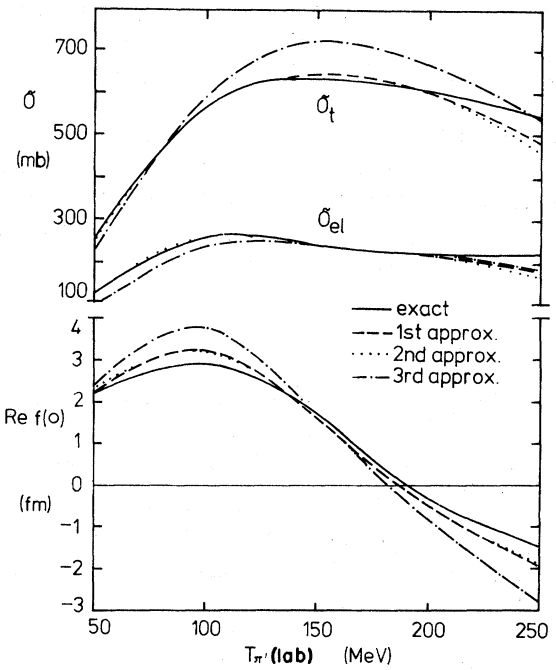

FIG. 2. Total and elastic total cross sections and real forward scattering amplitudes. See caption of Fig. 1.

The reduction of the peak value has a somewhat different bearing. In Ref. 13 it was reduced as in our case, but in Ref. 12 it was markedly increased. It is very likely that this difference was caused by the special arrangement for the intermediate states in Ref. 12. In this connection, we remember that the reduction of peak value can be obtained even by the effect of the Fermi motion alone.[9,23,24]

A rather large difference between the impulse and binding differential cross sections was caused not only by the binding effect, but also by the difference due to the use of asymmetric and symmetric kinematics. Since the asymmetric kinematics places too much emphasis on the forward direction, it tends to overestimate differential cross sections near forward direction by supplanting the contributions from large angles. This defect is amended by the symmetric kinematics. The increased binding differential cross section has also its cause in the use of m^* as an embodiment of the binding effect. There are many calculations (Refs. 4, 5, 15, 19, and 25) in which the binding energy E_B is assumed to be the separation energy E_S. The use of E_B and the subsequent replacement by E_S gives distinctive differences from our kinematics. First of all, the use of E_B amounts to some averaging of the major part of the nucleon Fermi motion effect, which may be perhaps an oversimplification of the three-body kinematics, and thus it is better to treat the binding interaction as a dynamical effect through the Fermi motion rather than some averaged and static treatment. Second, although E_S was introduced as the mass difference between the nucleus and the core-nucleon state,[25] in the subsequent calculations the nuclear and core masses were treated[19,25] as Am and $(A-1)m$, respectively, which is inconsistent. The introduction of m^* is consistent with the picture that an approaching pion sees the nucleus as a target of mass Am^*, and not Am.

Turning to the difference between the exact solution and its two approximation methods, we observe that both the first and second approximations follow the exact calculations rather faithfully, except for $\mathrm{Re}f(0)$. Furthermore, the difference between two approximation methods is almost negligible. This off-shell insensitivity has been pointed out previously.[22] Hence, in practice, the second approximation will give a very useful calculational tool which can closely reproduce most of the binding effects with minimum computing time.

Since there has not been a thorough and reliable calculation of the binding effect, it is difficult to give a quantitative estimate of various approaches to the binding effect, which can be tested only by comparing with the most exact theory if it exists. However, elimination of a free parameter E_S, internal consistency between m^* and m, and the proper perspective on the Fermi motion in the present method of nonimpulse calculation should help deepen our understanding of the binding effects.

We have proposed another method to include the binding effects in particle-nucleus elastic scattering. It is designed for a momentum space calculation in multiple scattering theory, based on a simple yet realistic three-body kinematics. It was found that the binding effects have significant implications on results of the first order optical model calculation. It was also found that the nucleon Fermi motion is an integral part of binding corrections in the three-body kinematics, both in principle and in practice.

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