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One-step and two-step contributions to two-nucleon transfer reactions

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Qualitative arguments are given to show that although first-order distorted-wave Born approximation calculations are adequate for the treatment of single-nucleon transfer reactions, there is little reason to trust them for two-nucleon or multinucleon transfer reactions. A first-plus-second Born approximation calculation is presented for the ^{208}Pb(^{16}O,^{18}O)^{206}Pb ground state transition, at bombarding energies below and slightly above the Coulomb barrier. It is found that successive transfer (two-step) processes account for almost all the observed cross section.

[NUCLEAR REACTIONS Theory of one- and two-step contributions to ^{208}Pb(^{16}O,^{18}O)^{206}Pb.]

I. INTRODUCTION

Early analyses[1-3] of two-nucleon transfer data with light ions involved simple generalizations of the one-particle transfer first-order DWBA codes then available. Because these generalizations did not use a normalizable interaction, it was impossible to use them to calculate absolute cross sections. They were therefore limited to the calculation of the shapes of angular distributions, or the calculation of the relative cross section for the population of different final states in the residual nucleus. The first attempts[4,5] to calculate two-nucleon transfer cross sections using a realistic, normalized interaction in a DWBA calculation underpredicted the observed cross section by factors of 3 to 10. Similarly, first-order DWBA calculations of two-nucleon transfer between heavy ions,[6-9] performed with careful treatment of recoil effects, also underpredicted the cross section. Was this due to remaining uncertainties in the one-step DWBA calculation (form factors, configuration mixing, optical potentials), or was it due to the fact that the physical transfer process involved multistep routes that were not included in the one-step calculation? This is still a controversial question.

One type of multistep route involves inelastic excitation of the target, projectile, ejectile, or residual nucleus. If such inelastic excitations are strong, it is clearly important to include them, and this can readily be done by replacing the optical model calculations of DWBA by coupled-channels calculations. When we consider ground state transitions for a spherical even-even target and projectile, such inelastic multistep routes should be of minor importance. However, for two-nucleon transfer we also have the possibility of a multistep route involving two successive single-nucleon transfers. This is the multistep process that will be investigated in this paper.

Some workers have found that if they take sufficient care in the treatment of the interaction and bound states, they can account for the observed cross section with a one-step calculation.[10] However, those who have attempted to calculate the contribution of successive transfer processes[11-18] have usually found them to make comparable or greater contributions to the reaction amplitude than does the one-step process.

The uncertainties in these calculations associated with the optical potentials can be minimized by considering bombarding energies below or at the Coulomb barrier. Franey *et al.*[19] studied the ^{208}Pb(^{16}O,^{18}O)^{206}Pb ground state transition below the Coulomb barrier and found that a one-step, finite range, full-recoil calculation underpredicted the observed cross section by a factor of about 10. They then used a semiclassical method[18] to estimate the contribution of a process in which the two neutrons are transferred successively, and found that the inclusion of this process brought the predicted cross section close to the observed one. It was assumed that the oxygen and lead nuclei move past each other on a classical Coulomb trajectory, during which time the two neutrons can make the transition from one well to the other. Of course, it would be more satisfactory to have a fully quantum-mechanical treatment of the successive transfer pro-

cess. This is especially important for bombarding energies at, or slightly above, the Coulomb barrier. Here the classical trajectory brings the colliding nuclei close enough so that they are absorbed by the imaginary part of the potential. It fails to include the quantum mechanical reflection due to the change in potential, even when the bombarding energy is over the Coulomb barrier. This effect can be included by means of the Wentzel-Kramers-Brillouin (WKB) method using complex turning points.[18]

In this paper we have investigated the effect of successive-transfer processes within the framework of the second Born approximation. We first present some qualitative arguments to show that although first-order DWBA is probably adequate for single-nucleon transfer, there is little reason to expect it to be adequate for two-nucleon or multinucleon transfer. We then apply to the ^{208}Pb(^{16}O,^{18}O)^{206}Pb$_{g.s.}$ reaction studied by Franey *et al.*[19] a combined first-plus-second Born approximation analysis, and find that the second Born approximation terms contribute most of the transition amplitude. The resulting cross section is in reasonable agreement with the available experimental data.

II. QUALITATIVE COMPARISON OF ONE-STEP AND TWO-STEP TRANSFER

In this section we will present qualitative arguments showing that one-step DWBA can be expected to be a poorer approximation for two-nucleon transfer (2NT) than for one-nucleon transfer (1NT) reactions.

We label the participants in the reaction in the usual way

$$A+a\rightarrow b+B \tag{1}$$

with

$$a=b+n_1,\quad B=A+n_1\ (\text{1NT}) \tag{2a}$$

or

$$a=b+n_1+n_2,\quad B=A+n_1+n_2\ (\text{2NT})\ . \tag{2b}$$

In the post representation, the "exact" transition amplitudes require the evaluation of the matrix elements

$$T_{\text{1NT}}=\langle\chi_f^{(-)}(\vec{r}_{Bb})\phi_B(\xi_A,\vec{r}_{A1})\phi_b(\xi_b)\,|\,v(r_{b1})\,|\,\Psi_{\text{1NT}}^{(+)}\rangle\ , \tag{3a}$$

$$T_{\text{2NT}}=\langle\chi_f^{(-)}(\vec{r}_{Bb})\phi_B(\xi_A,\vec{r}_{A1},\vec{r}_{A2})\phi_b(\xi_b)\,|\,v(r_{b1})+v(r_{b2})\,|\,\Psi_{\text{2NT}}^{(+)}\rangle$$
$$=2\langle\chi_f^{(-)}(\vec{r}_{Bb})\phi_B(\xi_A,\vec{r}_{A1},\vec{r}_{A2})\phi_b(\xi_b)\,|\,v(r_{b1})\,|\,\Psi_{\text{2NT}}^{(+)}\rangle\ . \tag{3b}$$

In (3), $\Psi_{1,\text{2NT}}^{(+)}$ is the exact wave function, subject to the boundary conditions appropriate to an incoming $A+a$ channel. ξ_A and ξ_b represent internal degrees of freedom of the "core" nuclei A and b, and $\vec{r}_{A1}$, $\vec{r}_{A2}$, $\vec{r}_{b1}$, and $\vec{r}_{b2}$ locate the transferring nucleons relative to the centers of the cores. $\chi_f^{(-)}(\vec{r}_{Bb})$ is the optical wave function of relative motion of B and b in the final channel, and $v(r_{bi})$ is the interaction between transferring nucleon i and the core b. We take v to have a Woods-Saxon shape,

$$v(r)=\frac{-V}{1+\exp\left[\dfrac{r-R}{a}\right]}\ ,$$

with depth and geometry appropriate to a shell-model description of nucleus a. In particular, the diffusivity parameter has a value of $a=0.65$ fm, so that for $r>R$, $v(r)$ falls like $\exp(-r/0.65\ \text{fm})$.

The first order Born approximation replaces the exact wave functions $\Psi^{(+)}$ in (3) by optical model approximations to the elastic channel alone

$$\Psi_{\text{1NT}}^{(+)}(\text{DWBA})\equiv\phi_A(\xi_A)\phi_a(\xi_b,\vec{r}_{b1})\chi_i^{(+)}(\vec{r}_{Aa})\ , \tag{4a}$$

$$\Psi_{\text{2NT}}^{(+)}(\text{DWBA})\equiv\phi_A(\xi_A)\phi_a(\xi_b,\vec{r}_{b1},\vec{r}_{b2})\times\chi_i^{(+)}(\vec{r}_{Aa})\ , \tag{4b}$$

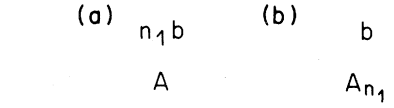

FIG. 1. Schematic representation of two regions of configuration space needed for the evaluation of the one-nucleon transfer transition amplitude, T_{1NT}. The region in which n_1 is far from b makes a smaller contribution to T_{1NT} because of the presence of $v(r_{b1})$ in the integrand (3a).

leading to

$$T_{1NT}^{1\,\text{step}} = \langle \chi_f^{(-)}(\vec{r}_{Bb})\phi_B(\xi_A,\vec{r}_{A1})\phi_b(\xi_b) \mid v(r_{b1}) \mid \phi_A(\xi_A)\phi_a(\xi_b,\vec{r}_{b1})\chi_i^{(+)}(\vec{r}_{Aa})\rangle \ , \tag{5a}$$

$$T_{2NT}^{1\,\text{step}} = 2\langle \chi_f^{(-)}(\vec{r}_{Bb})\phi_B(\xi_A,\vec{r}_{A1},\vec{r}_{A2})\phi_b(\xi_b) \mid v(r_{b1}) \mid \phi_A(\xi_A)\phi_a(\xi_b,\vec{r}_{b1},\vec{r}_{b2})\chi_i^{(+)}(\vec{r}_{Aa})\rangle \ . \tag{5b}$$

The adequacy with which $T^{1\,\text{step}}$ approximates T depends upon how well the elastic channel wave function (4) represents the exact wave function $\Psi^{(+)}$ in the region of configuration space important for the evaluation of the matrix element (3).

Figure 1 shows two regions of configuration space included in the evaluation (3a) of T_{1NT}. The region illustrated by Fig. 1(a) resembles the incoming channel. In this region we have the most reason to expect that $\Psi_{1NT}^{(+)}$ is well represented by $\Psi_{1NT}^{(+)}$(DWBA). On the other hand, the region illustrated in Fig. 1(b) looks quite different from the incoming channel, and here we would expect that $\Psi_{1NT}^{(+)}$(DWBA) is a poor approximation to $\Psi_{1NT}^{(+)}$. Fortunately, the factor $v(r_{b1})$ in (3a) implies that regions with particle 1 far from b play a small role in the evaluation of T_{1NT}. Thus we have reason to expect that $T_{1NT}^{1\,\text{step}}$ of (5a) is a good approximation to T_{1NT} of (3a), since $\Psi_{1NT}^{(+)}$(DWBA) is a good approximation to $\Psi_{1NT}^{(+)}$ in the region of configuration space that dominates the matrix element. In some cases it may be important to improve this approximation by adding to $\Psi_{1NT}^{(+)}$(DWBA) terms corresponding to excited states of a and/or A. This can be done by replacing the optical model calculation of $\chi_i^{(+)}(\vec{r}_{Aa})$ by a coupled-channels calculation including inelastic excitations of the $A+a$ system.

Figure 2 shows two regions of configuration space included in the evaluation (3b) of T_{2NT}. The configuration of the incoming channel is represented by Fig. 2(a), and here we would expect that $\Psi_{2NT}^{(+)}$ and $\Psi_{2NT}^{(+)}$(DWBA) are quite similar. As before, we would not expect $\Psi_{2NT}^{(+)}$ and $\Psi_{2NT}^{(+)}$(DWBA) to be similar in the region of configuration space illustrated by Fig. 2(b), since particle 2 here is far from b. However, there is no factor $v(r_{b2})$ in (3b) to tell us that configurations such as Fig. 2(b) are unimportant in the evaluation of T_{2NT}. On the contrary, since particle 2 is bound to A in the final state, the region illustrated in Fig. 2(b) may be quite important. Since we have little reason to believe that $\Psi_{2NT}^{(+)}$(DWBA) is a good approximation to $\Psi_{2NT}^{(+)}$ throughout the entire region of configuration space important to the evaluation of T_{2NT}, we have little reason to believe that $T_{2NT}^{1\,\text{step}}$ is a good approximation to T_{2NT}.

We attempt to improve the approximation to $\Psi_{2NT}^{(+)}$ by adding a term χ to $\Psi_{2NT}^{(+)}$(DWBA), where χ

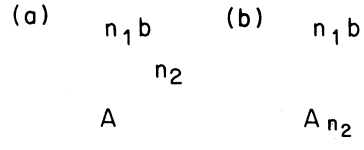

FIG. 2. Schematic representation of two regions of configuration space needed for the evaluation of a two-nucleon transfer matrix element, T_{2NT}. Since the integrand (3b) contains a factor $v(r_{b1})$, but no factor $v(r_{b2})$, there is no reason to neglect the configuration in which r_{b2} is large.

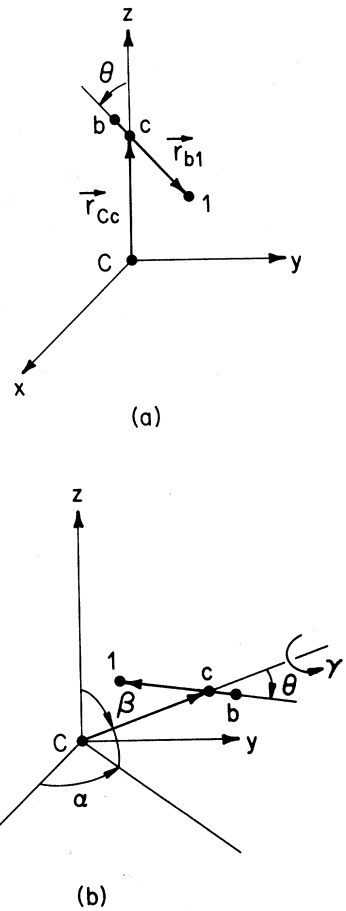

FIG. 3. (a) C, b, and n_1 in "standard" configuration, with $\vec{r}_{Cc}$ along the $\hat{z}$ axis, $\vec{r}_{b1}$ in the x-z plane, and $(\vec{r}_{b1})_x < 0$. (b) The result of applying rigid-body rotation (α, β, γ) to the configuration of (a). Note that changing γ causes $\vec{r}_{b1}$ to rotate about the direction of $\vec{r}_{Cc}$.

gives a better description of the wave function in the region shown in Fig. 2(b). The calculation of χ is similar to finding the amplitude for transferring particle 2 to target A in a one-nucleon transfer reaction

$$A+a\rightarrow c+C,\quad c=b+n_1,\quad C=A+n_2\ .$$

The contribution of χ to (3b) has the structure of a two-step process, with v acting twice. The relative contributions of $\Psi^{(+)}_{2\mathrm{NT}}(\mathrm{DWBA})$ and χ in (3b) depends upon the tightness with which particles 1 and 2 are bound to each other and to the cores b and A.[18] If particles 1 and 2 are bound very tightly to each other compared to their binding to b and A, then the configuration represented by Fig. 3(b) will have a very small amplitude, and $T_{2\mathrm{NT}}$ will be dominated by $T^{1\,\mathrm{step}}_{2\mathrm{NT}}$. If this binding is not so great, the regions of Figs. 2(a) and (b) may make comparable contributions to $T_{2\mathrm{NT}}$, and then the approximation $T_{2\mathrm{NT}}\approx T^{1\,\mathrm{step}}_{2\mathrm{NT}}$ will be poor. It is difficult to predict *a priori* which regime is appropriate to the actual physical situation. Most analyses of two-nucleon transfer data reported in the literature have assumed that $T_{2\mathrm{NT}}=T^{1\,\mathrm{step}}_{2\mathrm{NT}}$. However, in those cases where two-step contributions have been calculated, they have been found to be significant.[11–18] This is also the situation for the analysis of $^{208}\mathrm{Pb}(^{16}\mathrm{O},^{18}\mathrm{O})^{206}\mathrm{Pb}$ reported in this paper.

III. DERIVATION OF FORMULAS

We assume that the a ground state consists of a doubly-closed b core plus a zero-coupled pair of neutrons. Similarly the A ground state consists of two zero-coupled holes in a doubly-closed B core. The low-lying single-hole states of C and single-particle states of c are represented by $\phi_C^{j_f,m_f}(\xi_A,\vec{r}_{A2},\sigma_2)$ and $\phi_c^{j_i,m_i}(\xi_b,\vec{r}_{b1},\sigma_1)$, respectively. The basic assumption of this work is that we achieve a significant improvement to $\psi^{(+)}$ in the configuration of Fig. 2(b) by including only the lowest few single-particle states of c and single-hole states of C:

$$\Psi^{(+)}_{2\mathrm{NT}}\approx\Psi^{(+)}_{2\mathrm{NT}}(\mathrm{DWBA})+\sum_{\substack{j_i,j_f\\K,M}} u_{KM}^{j_f,j_i}(\vec{r}_{Cc})[\phi_C^{j_f}(\xi_A,\vec{r}_{A2},\sigma_2)\phi_c^{j_i}(\xi_b,\vec{r}_{b1},\sigma_1)]^K_M\ . \tag{6}$$

The vector $\vec{r}_{Cc}$ in (6) goes from the mass center of $C(=A+n_2)$ to the mass center of $c(=b+n_1)$. Our main task is to find the functions $u_{KM}(\vec{r}_{Cc})$ which describe the relative motion of C and c during the intermediate stage of the two-step process. To accomplish this, we consider the related quantity $f_{KM}(\vec{r}_{Cc})$ defined by

$$f^{j_1j_2}_{KM}(\vec{r}_{Cc})=\langle[\phi_C^{j_f}\phi_c^{j_i}]^K_M\mid\psi^{(+)}_{2\mathrm{NT}}\rangle_{\vec{r}_{Cc}\,\mathrm{fixed}} \tag{7a}$$

$$=\langle[\phi_C^{j_f}\phi_c^{j_i}]^K_M\mid\chi_i^{(+)}(\vec{r}_{Aa})\phi_A(\xi_A)\phi_a(\xi_b,\vec{r}_{b1}\sigma_1,\vec{r}_{b2}\sigma_2)\rangle_{\vec{r}_{Cc}\,\mathrm{fixed}}+u_{KM}^{j_fj_i}(\vec{r}_{Cc})\ . \tag{7b}$$

We can find a differential equation for f_{KM} by taking the scalar product of the Schrödinger equation

$$(E-H)\Psi^{(+)}_{2\mathrm{NT}}=0=\left[E-H_C-H_c+\frac{\hbar^2}{2\mu_{Cc}}\nabla_{\vec{r}_{Cc}}{}^2-U(\vec{r}_{Cc})-v(r_{c2})\right]\Psi^{(+)}_{2\mathrm{NT}} \tag{8}$$

with $[\phi_C^{j_f}\phi_c^{j_i}]^K_M$. The result is

$$\left[\nabla_{\vec{r}_{Cc}}{}^2+k_{Cc}{}^2-\frac{2\mu_{Cc}}{\hbar^2}U(\vec{r}_{Cc})\right]f^{j_fj_i}_{KM}(\vec{r}_{Cc})=\frac{2\mu_{Cc}}{\hbar^2}\langle[\phi_C^{j_f}\phi_c^{j_i}]^K_M\mid v(r_{c2})\mid\psi^{(+)}_{2\mathrm{NT}}\rangle_{\vec{r}_{Cc}\,\mathrm{fixed}}\ . \tag{9}$$

The use of the optical potential $U(\vec{r}_{Cc})$ to represent the interaction between c and A is consistent with the usual procedures of one-particle transfer theory. We write the solution of (9) in terms of a Green's function, $G(\vec{r}_{Cc},\vec{r}\,'_{Cc})$:

$$f^{j_fj_i}_{KM}(\vec{r}_{Cc})=\frac{2\mu_{Cc}}{\hbar^2}\int d^3r'_{Cc}\,G(\vec{r}_{Cc},\vec{r}\,'_{Cc})\langle[\phi_C^{j_f}\phi_c^{j_i}]^K_M\mid v(r_{c2})\mid\Psi^{(+)}_{2\mathrm{NT}}\rangle_{\vec{r}\,'_{Cc}\,\mathrm{fixed}}\ . \tag{10}$$

To use (10) for the calculation of $f_{KM}(\vec{r}_{Cc})$, we require an approximation for $\Psi^{(+)}_{2\mathrm{NT}}$. Here we use the presence of $v(r'_{C2})$ in the integrand to conclude that $\Psi^{(+)}_{2\mathrm{NT}}\mathrm{DWBA}$ of (4b) will be adequate for this purpose. This is essentially a one-nucleon transfer matrix element, and DWBA should suffice. With this approximation, Eqs. (10), (7b), (6), and (3b) can be combined to give

$$T_{2\mathrm{NT}} \approx \sum_{j_f j_i} C_{j_f}(B,A) C_{j_i}(a,b) T_{2\mathrm{NT}}^{1\,\mathrm{step}}(j_f,j_i) + S_{j_f}(B,C) S_{j_f}(C,A) S_{j_i}(a,c) S_{j_i}(c,b) \times [T_{2\mathrm{NT}}^{VV}(j_f,j_i) - T_{2\mathrm{NT}}^{\mathrm{NO}}(j_f,j_i)] , \tag{11a}$$

where

$$T_{2\mathrm{NT}}^{1\,\mathrm{step}}(j_f,j_i) = 2 \sum_{\sigma_1 \sigma_2} \int d^3 r_{Cc} d^3 r_{b1} d^3 r_{A2} [\psi^{j_f}(\vec{r}_{A1},\sigma_1) \psi^{j_f}(r_{A2},\sigma_2)]_0^0 \, {}^* \chi_f^{(-)}(\vec{r}_{Bb}) {}^* \times v(r_{b1}) [\psi^{j_i}(\vec{r}_{b1},\sigma_1) \psi^{j_i}(\vec{r}_{b2}\sigma_2)]_0^0 , \tag{11b}$$

$$T_{2\mathrm{NT}}^{VV}(j_f,j_i) = 2 \sum_{K,M} \sum_{\substack{\sigma_1,\sigma_2 \\ \sigma_1',\sigma_2'}} \int d^3 r_{Cc} d^3 r_{b1} d^3 r_{A2} \chi_f^{(-)}(\vec{r}_{Bb}) {}^* [\psi^{j_f}(\vec{r}_{A1},\sigma_1) \psi^{j_i}(\vec{r}_{A2},\sigma_2)]_0^0 \, {}^* \times v(r_{b1}) [\psi^{j_f}(\vec{r}_{A2},\sigma_2) \psi^{j_i}(\vec{r}_{b1},\sigma_1)]_M^K \times \int_{\vec{r}_{Cc}\,\mathrm{fixed}} d^3 r'_{Cc} d^3 r'_{A2} d^3 r'_{b1} G(\vec{r}_{Cc},\vec{r}'_{Cc}) \times [\psi^{j_f}(\vec{r}'_{A2},\sigma_2') \psi^{j_i}(\vec{r}'_{A1},\sigma_1')]_M^K \frac{2\mu_{Cc}}{\hbar^2} v(r'_{c2}) \times [\psi^{j_i}(\vec{r}'_{b1},\sigma_1') \psi^{j_i}(\vec{r}'_{b2},\sigma_2')]_0^0 \chi_i^{(+)}(\vec{r}'_{Aa}) , \tag{11c}$$

$$T_{2\mathrm{NT}}^{\mathrm{NO}}(j_f,j_i) = 2 \sum_{K,M} \sum_{\substack{\sigma_1,\sigma_2 \\ \sigma_1',\sigma_2'}} \int d^2 r_{Cc} d^3 r_{b1} d^3 r_{A2} \chi_f^{(-)}(\vec{r}_{Bb}) {}^* [\psi^{j_f}(\vec{r}_{A1},\sigma_1) \psi^{j_f}(\vec{r}_{A2},\sigma_2)]_0^0 \, {}^* \times v(r_{b1}) [\psi^{j_f}(\vec{r}_{A2},\sigma_2) \psi^{j_i}(\vec{r}_{b1},\sigma_1)]_M^K \times \int_{\vec{r}_{Cc}\,\mathrm{fixed}} d^3 r'_{A2} d^3 r'_{b1} [\psi^{j_f}(\vec{r}'_{A2},\sigma_2') \psi^{j_i}(\vec{r}'_{b1},\sigma_1')]_M^K \times [\psi^{j_i}(\vec{r}'_{b1},\sigma_1') \psi^{j_i}(\vec{r}'_{b2},\sigma_2')]_0^0 \chi_i^{(+)}(\vec{r}'_{Aa}) . \tag{11d}$$

Here $C_{j_f}(B,A)$ is the spectroscopic amplitude for the addition of zero-coupled $[\psi^{j_f}\psi^{j_f}]_0^0$ to the ground state of A to yield the ground state of B, and $C_{j_i}(a,b)$ is the corresponding spectroscopic amplitude for the (a,b) pair. $S_{j_i}(c,b)$ is a one-nucleon spectroscopic amplitude for the addition of a j_i nucleon to the ground state of b, leading to an angular momentum j_i state of c, etc.

The nonorthogonality term $T_{2\mathrm{NT}}^{\mathrm{NO}}$ is similar in structure to $T_{2\mathrm{NT}}^{1\,\mathrm{step}}$, in that both (11b) and (11d) contain only one interaction, $v(r_{b1})$, whereas $T_{2\mathrm{NT}}^{VV}$ of (11c) contains two interaction factors $v(r_{b1})v(r'_{c2})$. Götz *et al.*[18] have shown that if a no-recoil approximation is made, and it is assumed that the interaction between the transferred nucleons is negligible, then $T_{2\mathrm{NT}}^{\mathrm{NO}}$ cancels $T_{2\mathrm{NT}}^{1\,\mathrm{step}}$ exactly. This is reasonable, since this limit is governed by a Hamiltonian which is completely separable in the two nucleons, and the amplitude for two-nucleon transfer is

just the product of the amplitudes for their separate transfer. Each amplitude contains an interaction, so the two-nucleon transfer amplitude should contain no terms with only one interaction.

To evaluate (11c) we need a partial-wave expansion of the Green's function

$$G(\vec{r},\vec{r}')=\sum_{l,m}\frac{Y_m^l(\hat{r})Y_m^{l\,*}(\hat{r}')}{-ikrr'}f_l(k,r_<)P_l(k,r_>)\ . \tag{12}$$

Here f_l and P_l are solutions of the homogeneous equation

$$\left[\frac{d^2}{dr^2}-\frac{l(l+1)}{r^2}-\frac{2\mu_{Cc}}{\hbar^2}U(r)+k_{Cc}{}^2\right]\psi(\vec{r})=0\ ,$$

respectively, regular and irregular at the origin. At large r, P_l is purely outgoing. Summation over the spin components and some algebraic simplification yields the following expression:

$$T_{2\mathrm{NT}}^{VV}(j_i,j_f)=\frac{128i(\mu_{Cc}/\hbar^2)^2(-1)^{l_f+l_i}}{k_{Aa}k_{Bb}k_{Cc}[(2j_i+1)(2j_f+1)]^{1/2}}$$

$$\times\sum_K(2K+1)((l_f\tfrac{1}{2})_{j_f}(l_i\tfrac{1}{2})_{j_i}\,|\,(l_fl_i)_K(\tfrac{1}{2}\tfrac{1}{2})_0)_K{}^2$$

$$\times\sum_{l,l_c}\frac{\exp(i[\sigma_l^i+\sigma_l^f])}{(2l+1)^{1/2}}Y_0^l(\hat{k}_{Bb})S_{Kl_cl}\ , \tag{13a}$$

$$S_{Kl_cl}\equiv\int d^3r_{Cc}d^3r_{b1}v(r_{b1})\frac{s_{Kl_cl}(r_{Cc})}{r_{Cc}}\frac{F_l(r_{Bb})}{r_{Bb}}u_{l_f}(r_{C1})u_{l_c}(r_{b1})$$

$$\times[Y^{l_c}(\hat{r}_{Cc})Y^l(\hat{r}_{Bb})]_M^K\,{}^*[Y^{l_f}(\hat{r}_{C1})Y^{l_i}(\hat{r}_{b1})]_M^K\ , \tag{13b}$$

$$s_{Kl_cl}(r_{Cc})\equiv\int_{r_{Cc}\,\mathrm{fixed}}d^3r'_{Cc}d^3r'_{A2}\frac{f_{l_c}(k_{Cc},r_<)P_{l_c}(k_{Cc}r_>)}{r'_{Cc}}\frac{F_l(r'_{Aa})}{r'_{Aa}}v(r'_{c2})$$

$$\times u_{l_f}(r'_{A2})u_{l1}(r'_{c2})[Y^{l_f}(\hat{r}'_{A2})Y^{l_i}(\hat{r}'_{c2})]_M^K\,{}^*[Y^{l_c}(\hat{r}'_{Cc})Y^l(\hat{r}'_{Aa})]_M^K\ . \tag{13c}$$

Here $F_l(r)$ is the radial part of the lth partial wave in the expansion of the optical wave function

$$\chi^{(+)}(\vec{r}_{Aa})=\sum_l i^l e^{i\sigma_l^i}F_l(r_{Aa})Y_0^l(\hat{r}_{Aa})\frac{[4\pi(2l+1)]^{1/2}}{k_{Aa}r_{Aa}}\ , \tag{14a}$$

$$\chi^{(-)}(\vec{k}_{Bb},\vec{r}_{Bb})^*=\sum_{l,m}\frac{4\pi}{k_{Bb}r_{Bb}}i^{-l}e^{i\sigma_l^f}F_l(r_{Bb})Y_m^l(\hat{r}_{Bb})Y_m^L(\hat{k}_{Bb})^*\ , \tag{14b}$$

and the $\sigma_l^{i,f}$ are Coulomb phase shifts. The quantity

$$((l_f\tfrac{1}{2})_{j_f}(l_i\tfrac{1}{2})_{j_i}\,|\,(l_fl_i)_K(\tfrac{1}{2}\tfrac{1}{2})_0)_K$$

is a j-j to L-S transformation amplitude, in an obvious notation. Note that the integrals in (13b) and (13c) are both independent of M, since they are essentially scalar products of eigenstates of the total angular momentum.

One further approximation has gone into the derivation of (13) from (11). The function $\psi_{m_f}^{j_f}(\vec{r}_{A1},\sigma_1)$ in (11c) has been replaced in (13b) by $\psi_{m_f}^{j_f}(\vec{r}_{c1},\sigma_1)$. The justification for this approximation is that $\vec{r}_{A1}$ and $\vec{r}_{c1}$ differ by

$$\frac{1}{M_A+1}\vec{r}_{A2}\ ,$$

and the bound-state wave function $\psi^{j_f}_{m_f}$ changes by a small fraction of itself in this range. We must also make a similar replacement of $\psi^{j_i}_{m_i}(\vec{r}'_{b2},\sigma_2)$ by $\psi^{j_i}_{m_i}(\vec{r}'_{c2},\sigma_2)$. Notice that we do not attempt such "no-recoil" approximations in the optical wave function $\chi^{(+)}_i(\vec{r}_{Aa})$, $\chi^{(-)}_f(\vec{r}_{Bb})$, $f_l(k,r_<)$, or $P_l(k,r_>)$. At the high relative momenta associated with nucleus-nucleus collisions at tandem Van de Graaff energies, these functions are highly oscillatory and no-recoil approximations are of doubtful validity. We have more confidence in the approximations made here, since they involve only the more slowly varying bound-state wave functions $\psi^{j_i}_{m_i}$ and $\psi^{j_f}_{m_f}$.

The integrals in (13b) and (13c) are each six-dimensional. We can reduce them to three-dimensional integrals by using rotational invariance. Consider, for example, S_{Kl_cl} of (13b). Since it is independent of M, we can sum over M and divide by $2K+1$. The resulting M sum yields an integrand

$$\frac{1}{2K+1}\frac{s_{Kl_cl}(r_{Cc})}{r_{Cc}}\frac{F(r_{Bb})}{r_{Bb}}u_{l_f}(r_{c1})u_{l_i}(r_{b1})\sum_{M=-K}^{K}[Y^{l_c}(\hat{r}_{Cc})Y^{l}(\hat{r}_{Bb})]^K_M{}^*[Y^{l_f}(\hat{r}_{c1})Y^{l_i}(\hat{r}_{b1})]^K_M\ , \tag{15}$$

which is spherically symmetric. Now we note that any configuration of C, b, and $\vec{n}_1$ can be reached by applying a rigid-body rotation to a configuration in which $\vec{r}_{Cc}$ lies along the $\hat{z}$ axis and $\vec{r}_{b1}$ lies in the x-z plane with a negative x component (see Fig. 3). We choose the six variables α, β, γ, θ, r_{b1}, and r_{Cc} as new integration variables for (13b). Since the integrand (15) is rotationally invariant, it is independent of α, β, γ, so the α, β, γ integration simply yields a factor of $8\pi^2$. Thus the six-dimensional integral reduces to a three-dimensional integral over θ, r_{b1}, and r_{Cc}. To evaluate the integrand (15) for a given θ, r_{b1}, and r_{Cc} we can use the "standard" configuration of Fig. 3(a), since we know that the integrand has the same value for any orientation. The result is

$$S_{K,l_c,l}=\frac{8\pi^2}{2K+1}\sum_{M=-K}^{K}\left[\frac{2K+1}{4\pi}\right]^{1/2}(l_c l 0M\mid KM)$$
$$\times\int r_{Cc}{}^2dr_{Cc}r_{b1}{}^2dr_{b1}\sin\theta\,d\theta v(r_{b1})\frac{s_{Kl_cl}(r_{Cc})}{r_{Cc}}\frac{F_l(r_{Bb})}{r_{Bb}}u_{l_f}(r_{C1})u_{l_c}(r_{b1})$$
$$\times Y^K_M(\hat{r}_{Bb})^*[Y^{l_f}(\hat{r}_{C1})Y^{l_i}(\pi-\theta,0)]^K_M\ , \tag{16a}$$

where $\vec{r}_{Bb}$ and $\vec{r}_{C1}$ are obtained from r_{Cc}, r_{b1}, and θ by

$$\vec{r}_{Bb}=\left[\frac{m_A+1}{m_A+2}r_{Cc}+\frac{m_A+m_b+2}{(m_b+1)(m_A+2)}r_{b1}\cos\theta\right]\hat{z}+\frac{m_A+m_b+2}{(m_b+1)(m_A+2)}r_{b1}\sin\theta\hat{x}\ , \tag{16b}$$

$$\vec{r}_{C1}=\left[r_{Cc}-\frac{m_b}{m_b+1}r_{b1}\cos\theta\right]\hat{z}-\frac{m_b}{m_b+1}r_{b1}\sin\theta\hat{x}\ . \tag{16c}$$

For computational purposes, the vector-coupled quantity in (16) can be expressed in terms of the generalized spherical harmonic addition theorem described in Appendix 2 of Ref. 5. The remaining three integrals in (16a) were evaluated by Gaussian quadratures. A similar analysis was applied to reduce $s_{Kl_cl}(r_{Cc})$ of (13c) to

$$s_{Kl_cl}(r_{Cc})=\left[\frac{16\pi^3}{2K+1}\right]^{1/2}\sum_{M=-K}^{K}(l_c l 0M\mid KM)$$
$$\times\int_{r_{Cc}\text{ fixed}}r_{Cc}{}^{2'}dr'_{Cc}r_{A2}{}^{2'}dr'_{A2}d\cos\theta'\frac{f_{l_c}(k_{Cc},r_<)P_{l_c}(k_{Cc},r_>)}{r'_{Cc}}\frac{F_l(r'_{Aa})}{r'_{Aa}}$$
$$\times v(r'_{c2})u_{l_f}(r'_{A2})u_{l_i}(r'_{c2})[Y^{l_f}(\hat{r}'_{A2})Y^{l_i}(\hat{r}'_{c2})]^K_M{}^*Y^l_M(\hat{r}'_{Aa})\ , \tag{17a}$$

TABLE I. Single-particle spectroscopic amplitudes. If these are used together with normalized single-neutron radial functions calculated in the Woods-Saxon wells specified below, the resulting asymptotic tails will be consistent with those measured by Franey *et al.* in their study of sub-Coulomb single-neutron transfer. The wells have the form

$$V(r)=-V_0 f(r,R,a)-\left[\frac{\hbar}{m_\pi c}\right]^2 V_{so}\frac{1}{r}\frac{d}{dr}$$
$$\times f(r,R,a)\vec{l}\cdot\vec{\sigma}\ ,$$
$$f(r,R,a)=[\exp((r-R)/a)+1]^{-1}\ .$$

For the oxygen systems, $V_{so}=6$ MeV, $a=0.65$ fm, and $R=1.24A^{1/3}$ fm. For the lead systems, $V_{so}=7$ MeV, $a=0.65$ fm, and $R=1.30A^{1/3}$ fm. In all cases, V_0 was adjusted to yield the observed single-neutron separation energy.

Nuclei	j^π	S_j
(^{18}O, ^{17}O)	$\frac{5}{2}^+$	0.901
(^{18}O, ^{17}O)	$\frac{1}{2}^+$	0.344
(^{17}O, ^{16}O)	$\frac{5}{2}^+$	0.937
(^{17}O, ^{16}O)	$\frac{1}{2}^+$	0.950
(^{208}Pb, ^{207}Pb)	$\frac{1}{2}^-$	0.957
(^{208}Pb, ^{207}Pb)	$\frac{5}{2}^-$	0.861
(^{208}Pb, ^{207}Pb)	$\frac{3}{2}^-$	0.875
(^{207}Pb, ^{206}Pb)	$\frac{1}{2}^-$	0.720
(^{207}Pb, ^{206}Pb)	$\frac{5}{2}^-$	0.368
(^{207}Pb, ^{206}Pb)	$\frac{3}{2}^-$	0.546

where

$$\vec{r}'_{c2}=-\left[r'_{Cc}+\frac{m_A}{m_A+1}\cos\theta'\right]\hat{z}$$
$$-\frac{m_A}{m_A+1}r'_{A2}\sin\theta'\hat{x}\ , \qquad (17b)$$

$$\vec{r}'_{Aa}=\left[\frac{m_a-1}{m_a}r'_{Cc}-\frac{m_A+m_a}{m_a(m_A+1)}r'_{A2}\cos\theta'\right]\hat{z}$$
$$-\left[\frac{m_A+m_a}{m_a(m_A+1)}r'_{A2}\sin\theta'\right]\hat{x}\ . \qquad (17c)$$

To evaluate $T^{NO}_{2NT}(j_f,j_i)$, we note that (11d) can be obtained by replacing

$$G(\vec{r}_{Cc},\vec{r}'_{Cc})\frac{2\mu_{Cc}}{\hbar^2}v(r'_{C2})$$

in (11c) by $\delta(\vec{r}_{Cc}-\vec{r}'_{Cc})$. Then if we compare the partial wave expansion of the δ function with (12), we see that (16) and (17) can be used to obtain $T^{NO}_{2NT}(j_f,j_i)$ if we simply replace

$$v(r'_{C2})f_{l_c}(k_{Cc},r_<)P_{l_c}(k_{Cc},r_>)$$

in (17a) by

$$-ik_{Cc}\frac{h^2}{2m}\delta(r_{Cc}-r'_{Cc})\ .$$

The calculation of the nine-dimensional integral $T^{1\,step}_{2NT}(j_f,j_i)$ of (11b) was performed using the methods of Ref. 6.

DeVries, Satchler, and Cramer[20] have pointed out that it is more correct to use

$$v(r_{b1})+v(r_{b2})+V^{Coul}(\vec{r}_{Ab})-V^{Coul}(\vec{r}_{Bb}) \qquad (18)$$

as the interaction in (3b). In all the calculations reported here, this Coulomb recoil correction has been included. Since our application is to a neutron-transfer reaction, the effect of this correction is small.

IV. APPLICATION TO ^{206}Pb(^{16}O, ^{18}O$_{g.s.}$)^{206}Pb AT $E_{lab}=69$, 73, AND 86 MeV

The potential used for the calculation of the optical parameters was of Woods-Saxon shape, with real well parameters $V=-65$ MeV, $r_0=1.35$ fm, $a=0.34$ fm, $r_{0c}=1.30$ fm, and volume imaginary

TABLE II. Comparison of calculated relative partial cross sections at $\theta=157°$. For each energy, $\sigma_{s_{1/2}p_{3/2}}$ is normalized to 1.0. The first number is $\sigma_{j_ij_f}$ from the one-step calculation. The number following it in parentheses is the corresponding $\sigma_{j_ij_f}$ from the one-step-plus-two-step calculation.

Energy (MeV)	$\sigma_{d_{5/2}p_{1/2}}$	$\sigma_{d_{5/2}p_{3/2}}$	$\sigma_{d_{5/2}f_{5/2}}$	$\sigma_{s_{1/2}p_{1/2}}$	$\sigma_{s_{1/2}p_{3/2}}$	$\sigma_{s_{1/2}f_{5/2}}$
69	0.150 (0.177)	0.094 (0.091)	0.147 (0.185)	0.5 (0.671)	1.0 (1.0)	0.171 (0.207)
73	0.118 (0.172)	0.047 (0.086)	0.170 (0.231)	0.5 (0.674)	1.0 (1.0)	0.194 (0.229)
86	0.090 (0.120)	0.043 (0.045)	0.177 (0.298)	0.5 (0.602)	1.0 (1.0)	0.116 (0.261)

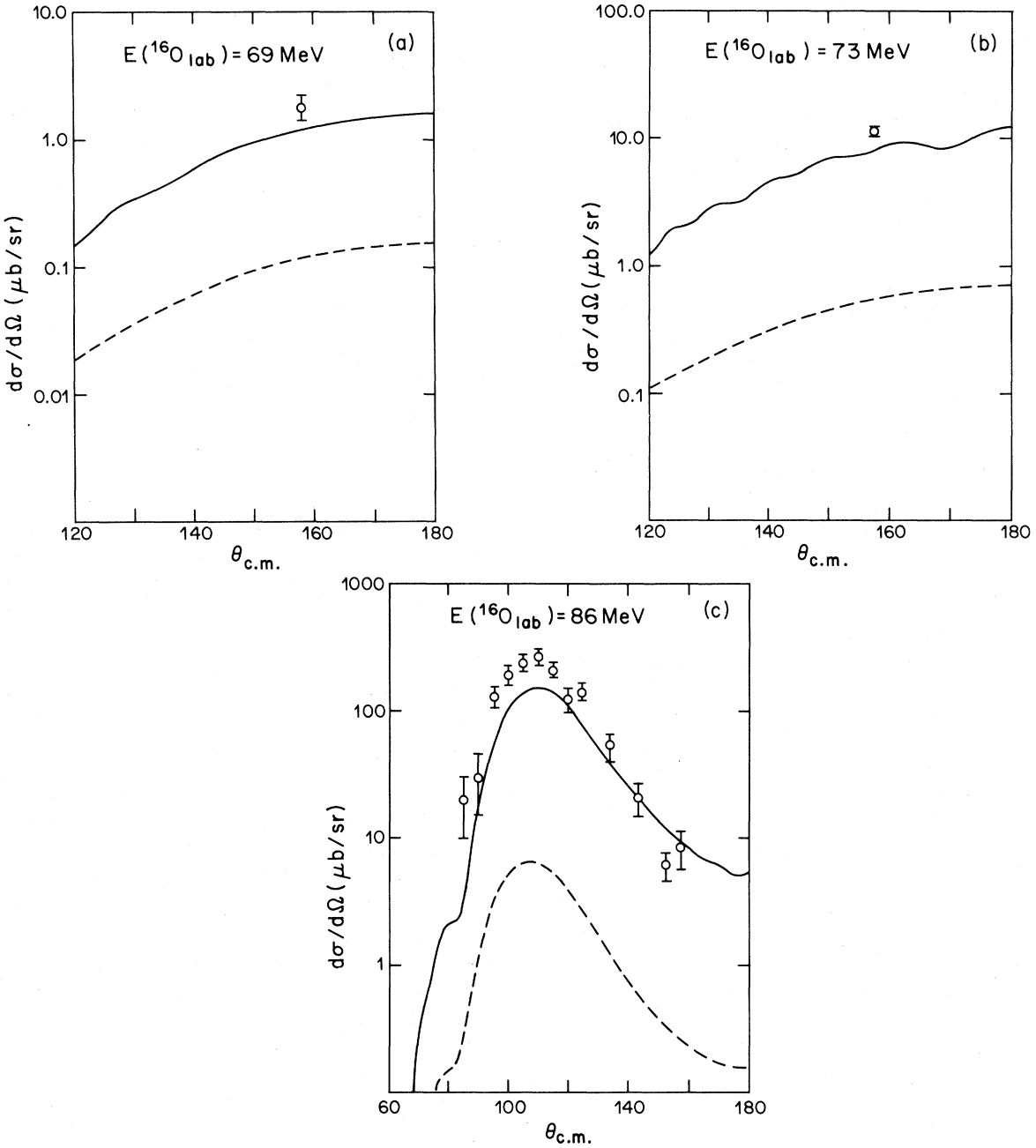

FIG. 4. Comparison of calculated differential cross sections and experimental data. The dashed lines give the result of the one-step calculation, and the solid lines the result of the one-step-plus-two-step calculation. The data are from Refs. 21 and 22.

well parameters $W=-45$ MeV, $r_0=1.34$ fm, $a=0.33$ fm. These parameters give a good account of the observed elastic scattering angular distributions.[21,22]

The optical parameters used for the calculation of the optical wave functions needed in (16) and (17) are given in Table I. They give a good account of the observed elastic scattering angular distributions.[21,22] At 69 and 73 MeV, these angular distributions differ little from the Rutherford scattering law over the entire angular range: At 86 MeV, the Rutherford scattering law is obeyed out to about 100°. Thus the 69 and 73 MeV ^{16}O projectiles are in the sub-Coulomb range, and the optical wave functions used in (16) and (17) will be almost pure Coulomb wave functions. The 86 MeV wave func-

tions will differ somewhat from pure Coulomb wave functions, and this makes our 86 MeV calculation somewhat less reliable.

The classical distances of closest approach of the ^{16}O and ^{208}Pb are 14.7, 13.9, and 11.8 fm at ^{16}O bombarding energies of 69, 73, and 86 MeV, respectively. The sum of the nuclear radii $[1.3\times(16^{1/3}+208^{1/3})]$ is about 11 fm. Thus we expect the transfer to take place when the neutrons are in regions governed by the exponential tails of their bound state wave functions. What is required in (11) is the product of a one-nucleon-transfer spectroscopic amplitude times an asymptotic bound-state wave function for each of the four pairs (B,C), (C,A), (a,c), and (c,b). These same products are required in sub-Coulomb one-nucleon transfer studies, and have been measured by Franey, Lilley, and Phillips[21] for all the cases needed here. The spectroscopic amplitudes given in Table I, used in conjunction with the bound-state wave functions calculated in the well specified in the caption of Table I, will give $S\psi^j$ products that agree with the measurements of Franey, Lilley, and Phillips. Thus the bound state aspect of the *two-step* two-nucleon transfer process is uniquely determined by one-nucleon transfer data, at least at sub-Coulomb energies. Unfortunately, we have no such assistance in determining the one-step spectroscopic amplitudes $C_{j_f}(B,A)$ and $C_{j_i}(a,b)$ needed in (11a), or the bound state radial functions needed there. We have to rely on the results of shell-model calculations[23,24] for the spectroscopic amplitudes. We take these to be

$$C_{5/2}(^{18}\mathrm{O},^{16}\mathrm{O})=0.893\ ,\quad C_{1/2}(^{18}\mathrm{O},^{16}\mathrm{O})=0.450\ ,$$

$$C_{1/2}(^{208}\mathrm{Pb},^{206}\mathrm{Pb})=0.769\ ,$$

$$C_{5/2}(^{208}\mathrm{Pb},^{206}\mathrm{Pb})=0.477\ ,$$

and

$$C_{3/2}(^{208}\mathrm{Pb},^{206}\mathrm{Pb})=0.426\ .$$

We have calculated the single-nucleon radial wave functions using the same well geometry as was used for $T_{2\mathrm{NT}}$, choosing the depth of the well to bind each nucleon with half the two-neutron separation energy. It has been shown by Kunz *et al.*[25] and Feng *et al.*[13] that this may underestimate $T_{2\mathrm{NT}}^{1\,\mathrm{step}}$ by a factor of the order of $\sqrt{2}$, compared to a calculation in which the two-nucleon tail is treated more carefully. It will be seen in the next section that such a factor will not change the essential conclusions of this paper.

The range of relative angular momentum l values included in the partial wave sum in (13a) was zero to 69, 74, and 84 for bombarding energies of 69, 73, and 86 MeV, respectively.

Figures 4(a)–(c) show calculated angular distributions and available experimental data.[21,22] At 69 MeV, the measured 157° differential cross section is 1.8 ± 0.4 μb/sr. The one-step calculation yields 0.12 μb/sr, too small by a factor of 15. When one- and two-step contributions are added together, the result is 1.15 μb/sr. This is still somewhat smaller than the measured value, but it is probably as close as we could reasonably expect to come considering the limited set of oxygen and lead configurations we included. However, our calculation shows that at this angle, and over the entire angular range shown in Fig. 4, the two-nucleon transfer process is dominated by the two-step component. The situation is similar when the bombarding energy is 73 MeV. Here the measured 157° differential cross section is 11.5 μb/sr, compared to the one-step calculated value of 0.55 μb/sr. The one-plus-two step calculated value of 7.9 μb/sr is smaller than the data by about the same factor (0.7) as for the 69 MeV case.

At a bombarding energy of 86 MeV, which is over the Coulomb barrier, the cross sections are larger and the data are more plentiful. Figure 4(c) shows that our one-plus-two-step calculation gives a rather good representation of the absolute magnitude of the differential cross section over the entire measured angular range. Our calculation underpredicts the data points by about a factor of 2 for $\theta_{\mathrm{c.m.}}\lesssim 130°$. We have noted that the 86 MeV case is the only one we have studied where there is sensitivity to optical potentials, and this may account for part of the discrepancy. Figure 4(c) shows that the one-step calculation underpredicts the cross section by about a factor of 40.

If the nonorthogonality terms $T_{2\mathrm{NT}}^{\mathrm{NO}}$ had been omitted from (11a), the effect on the solid curves of Fig. 4 would have been to raise them by about 15%. Thus there is rather little overlap between the two terms used in (6) to approximate the exact wave function.

Much of the information that has been extracted from one-step analyses of two-nucleon transfer data has relied upon predictions of the relative cross section for transfer into or out of different single-particle states.[3] Thus it is of some interest to ascertain whether inclusion of two-step contributions affects these predictions of the relative cross section. This information is summarized in Table II. The notation $\sigma_{j_ij_f}$ refers to the 157° differential cross section for transfer from a pure $(j_i^2)^{0+}$ state in ^{18}O to a pure $(j_f^2)^{0+}$ state in ^{208}Pb. Since we are interested here in ratios, the $\sigma_{j_ij_f}$ have been renormal-

ized to make $\sigma_{s_{1/2}p_{1/2}}=1$. It is clear from the comparison presented in Table II that at each energy, the pure one-step calculation predicts relative $\sigma_{j_i j_f}$ in rather good agreement with those predicted by the one-plus-two-step calculation. This observation has also been made by Feng *et al.*[13] Thus we find no reason to question spectroscopic conclusions that have relied on ratios computed with one-step codes.

V. CONCLUSIONS

The transfer of two neutrons in the ^{206}Pb(^{16}O,^{18}O)^{206}Pb ground state transition occurs predominantly by a two-step mechanism. This is equivalent to the statement that the transition amplitude is dominated by a region of configuration space where the first-order Born approximation provides a poor approximation to the exact wave function. It is thus important to improve the approximation to the wave function before we calculate the transition amplitude. Our main assumption is that we get a useful improvement by including components corresponding to the relative motion of ^{207}Pb and ^{17}O in low-lying single-particle states. This seems to be a reasonable assumption, and is supported by the fact that our calculated one-plus-two-step cross sections are fairly close in absolute magnitude to measured values over an energy range of 69 to 86 MeV (which spans the Coulomb barrier). We emphasize that we do not imply that first-order DWBA provides an inadequate approximation to the wave function everywhere in configuration space. We believe it to be adequate in the region required for one-nucleon transfer, but not in the region required for two-nucleon transfer.

An example of a simple system that exhibits a similar one-step versus two-step behavior is provided by one-dimensional scattering in a double δ-function potential:

$$V(x)=V_0\left[\delta\left[x-\frac{a}{2}\right]+\delta\left[x+\frac{a}{2}\right]\right].$$

The amplitude of the reflected wave is easily shown to be

$$r=-\frac{2\mu V_0}{ik\hbar^2}\frac{\cos ka+i\left[\frac{\mu V_0}{k\hbar^2}\right]^2\sin ka}{1+\frac{2\mu V}{ik\hbar^2}-\left[\frac{\mu V_0}{k\hbar^2}\right]^2[1-e^{-2ika}]}. \tag{19}$$

Here μ is the reduced mass and k is the wave number. To get the Born series for r, it is sufficient to expand (19) in a power series in V_0. The first two terms in this expansion are

$$r=-\frac{2\mu V_0}{ik\hbar^2}\cos ka \tag{20a}$$

$$+\left[\frac{2\mu V_0}{ik\hbar^2}\right]^2\left[\cos ka+\frac{i}{2}\sin ka\right]\cdots. \tag{20b}$$

If $2\mu V_0/\hbar^2 k \ll 1$ and ka is not too near $(m+1/2)\pi$ ($m=0,1,2,\ldots$), then r is approximately given by the first Born term (20a). If ka is near $(m+1/2)\pi$, (20a) can be smaller than the $\sin ka$ term in (20b). Higher orders of the Born approximation are reduced by additional powers of $2\mu V_0/\hbar^2 k$. Thus, whether or not ka is close to $(m+1/2)\pi$, the two terms (20a) and (20b) should provide a good approximation to r, and it is not necessary to go beyond second order. Of course this one-dimensional potential problem is much simpler than the transfer reaction considered in this paper. However, it does illustrate a situation in which both the first and second Born terms contribute, and in which the second Born term dominates and provides a good approximation to the exact scattering amplitude.

All the calculations in this paper have referred to a particular two-neutron transfer reaction, over a limited energy range. Clearly, we are not entitled to infer the dominance of multistep processes for all other multiparticle transfer reactions. However, we believe that the qualitative discussion of Sec. II strongly suggests that multistep processes will always be of importance in multiparticle transfer reactions, even if they are not dominant. Our calculation in the particular case studied here, and calculations by other workers in this field, appear to support this contention.

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