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Anomalous 3*d* shifts and widths in heavy π^- atoms

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We argue that 3*d* shifts and widths in heavy π^- atoms are probably not anomalous, contrary to recent experimental and theoretical claims. The observed shifts are small because the π^--nucleus interaction is expected to become repulsive in an atom close to ^{209}Bi. No plausible microscopic explanation has been found for the small widths and it is stressed that the experimental evidence for the anomaly, particularly of the widths, is not strong.

[NUCLEAR REACTIONS Strong-interaction shifts and widths in π^- atoms; π^--nucleus optical potential.]

Unexpectedly small shifts and widths have been observed[1] in a CERN experiment in the 3*d*-state π^- atoms of heavy nuclei, ^{181}Ta, Re, and ^{209} Bi; and the ^{181}Ta case seems to be confirmed[2] by a Rutherford group. The measured values are a factor of 2 or more smaller than the value calculated[1–5] using the conventional Ericson-Ericson optical potential,[6] though the potential is known to reproduce successfully all the other numerous π^--atom data available to date.[7] Strangely, the 4*f*-state data in these atoms do agree well with the theoretical values. The anomaly suggests that we may have overlooked some important pion dynamics in the low-energy pion-nucleus potential. Recently Ericson has proposed[5] including an explicit energy dependence in the momentum-independent part. In this Communication we show (1) that Ericson's proposal is largely equivalent to a renormalization of the potential parameters in the conventional potential, (2) that the π^--nucleus interaction in the 3*d* state becomes repulsive in atoms close to ^{209}Bi and thus the 3*d* shift data are probably not anomalous, and (3) that no plausible microscopic explanation has been found for the small width data. (4) We stress that the experimental evidence for the anomaly, particularly of the widths, is not strong and urge that careful measurements and analyses be carried out in the atoms of ^{208} Pb and even-even nuclei.[8] In the former no complication exists due to the nuclear ground-state deformation and the neutron distribution is better known. In the latter this advantage is less but the 4*f*-3*d* peak could be seen more easily.

(1) It is well known that the isoscalar momentum-independent parameter is small as a consequence of the nearly complete cancellation between π^--proton and -neutron scattering lengths. The proposal[5] is to include in this parameter the effective range part which is claimed not to be proportional to the square of the momentum but to the square of the local energy. The claim corresponds to a particular choice of the π-nucleon Lagrangian yielding this off-mass-shell behavior, but we do not dispute adequacy of the choice. Rather we show that an inclusion of either form of the effective range part corresponds largely to a renormalization of the conventional potential parameters. If we adopt the momentum form, the potential acquires a new term proportional to $-\vec{\nabla}\rho\cdot\vec{\nabla}-\frac{1}{2}(\vec{\nabla}^2\rho)$ which is clearly included by a renormalization of the parameters[7] as seen below.

Let us elaborate on the energy form, the claim of Ref. 5: The potential V becomes a sum of the conventional potential V_0 and a new energy-dependent term as

$$V=V_0+[(\omega-V_C)^2-\mu^2]s(r)/2\mu\ ,$$

where μ is the pion rest mass, and V_C is the (finite-size-modified) Coulomb potential, and $s(r)$ is proportional to the effective range times the nuclear density $\rho(r)$. Because of the V_C the potential now *appears* to have an extra atomic-number Z dependence so as to provide better agreement with the anomalous data for large Z. However, the Klein-Gordon equation

$$(-\nabla^2+\mu^2+2\mu V)\psi=(\omega-V_c)^2\psi$$

for the above potential V can be written in an identical form for an equivalent potential $\tilde{V}$ after grouping the energy terms on both sides of the equation.[9] Here V is energy independent and is defined to be

$$2\mu\tilde{V}=[s/(s-1)]\nabla^2+[2\mu/(1-s)]V_0\ .$$

By comparing V and $\tilde{V}$, and noting $0<s(r)<1$, we see that the role of the energy-dependent potential amounts to shifting some strength of the (attractive) momentum-dependent part to that of the (repulsive) momentum-independent part in the conventional potential. That is, the extra Z dependence is now buried in the pion wave function but will appear through the potential in a way similar to the already existing Z dependence; thus it is not "extra." Consequently, when the potential parameters in the energy-dependent V are readjusted to become best fit to all other data, the improved agreement with the anomaly will largely disappear.[10] Here we emphasize that it is important to adjust either all or the essential potential parameters[7]: A readjustment of a limited number of the parameters selected with a bias would still show the improved agreement with the anomaly as seen in Ref. 5. The sizes of the readjustment in the parameters are small because of a large cancellation for the $3d$ shifts as discussed below.

We can sharpen the argument using the radial part of the Klein-Gordon equation obtained via the Krell-Ericson transformation.[11] For $2\mu V_0 = q(r) - \nabla\alpha(r)\cdot\nabla$, redefinitions of $\alpha(r)$ and $q(r)$ [e.g., $\alpha \rightarrow (\alpha+s)/(1-s)$] yield an equation identical to the one for $s=0$. The new α and q are expressed in power series of $\rho(r)$ but each of them can be replaced effectively[7] by a term proportional to $\rho(r)$ with a coefficient obtained using replacements such as $\rho^2(r) \rightarrow \rho_e\rho(r)$. Here ρ_e is the (constant) effective nuclear density. The coefficient thus obtained will be numerically close to that for $s=0$. Note that in the leading order the new α and q are $\alpha+s$ and $q-\frac{1}{2}(\nabla^2 s)$, respectively. This $-\frac{1}{2}(\nabla^2 s)$ correction has the same form as the so-called angular transformation terms (ATT) but with the opposite sign. We observe that, when the ATT are included in the absence of the energy-dependent term, the $3d$ shifts in the heavy atoms are increased by about 45% but the $3d$ widths, and $4f$ shifts and widths, by much lesser amounts. However, the increase disappears when we use the potential parameters refitted[7] to all other π^--atom data including the ATT but still excluding the energy-dependent term. Thus we conclude that $\vec{\nabla}^2 s$ or $\nabla^2\rho$ is safely included by a renormalization of the potential parameters.

(2) To the best of our knowledge, no careful examination has been made of the dependence on the nuclear density distributions of the shifts and widths in the heavy $3d$ atoms. In order to examine this dependence, we took ^{209}Bi as a test case. In the conventional potential we increased the half-density radius of the neutron distribution c_n in the two-parameter Fermi form from 6.67 fm ($=c_p$ of the proton distribution), with $t=2.3$ fm fixed. An increase of about 0.30 fm was found[12] to be needed for obtaining the observed shift 18.6 ± 1.3 keV (Ref. 1) or about 47% reduction in the calculated shift. However, no reasonable c_n was found to yield the observed width 24.4 ± 5.3 keV.[1] For example, for $c_n=7.50$ fm the shift is about -7 keV and the width is about 62.0 keV. In contrast, the $4f$ shift and width are decreased only by about 7% and 19%, respectively over the range of c_n considered. These calculations demonstrate that the shifts in the heavy $3d$ atoms are quite sensitive to the neutron distribution and that the use of a realistic neutron distribution could explain the small shifts but not the small widths.

The negative $3d$ shift means that the interaction in the $3d$ state becomes repulsive, as was observed in the $2p$ state,[13] because the momentum-independent part of the potential starts to dominate the momentum-dependent part in nuclei of large radii. The repulsive $3d$ interaction is not due to our particular choice of the isovector parameter values: We have confirmed it using another set of the parameter values[14] which include an order of magnitude smaller isovector momentum-dependent parameter and are the best fit to the data including the recent Ca and Ti isotope data. Figure 1 illustrates the general trend of the cancellation between the two parts of the potential. We see that the $3d$ shifts in the heavy atoms are indeed a result of large cancellations.

(3) We have examined other possible explanations of the anomaly but have found no plausible one for the small width data. We have found various causes which would change the $3d$ shifts by non-negligible amounts but the $4f$ shifts by quite small amounts. The effect of these causes should be included in careful analyses in the future. However, we do regard the neutron distribution described in (2) to be the first candidate for the possible explanation of the small shifts. Since a detailed account of the examination of the various causes will be presented in the full paper in preparation, let us just summarize its results

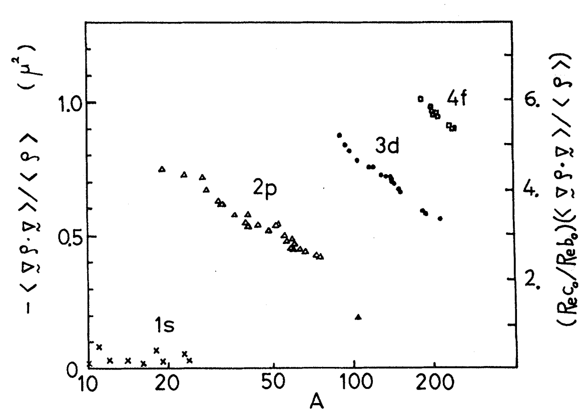

FIG. 1. The effective pion momentum $k_e^2 = -\langle\vec{\nabla}\rho\cdot\vec{\nabla}\rangle/\langle\rho\rangle$ in nuclei (the left ordinate) and the ratio of contributions from the momentum-dependent and -independent parts of the π^--nucleus optical potential (the right ordinate). $\langle\rho\rangle$ denotes an expectation value of the nuclear density ρ using the numerically integrated pion wave function, and the right ordinate equals the left multiplied by $\mathrm{Re}b_0/\mathrm{Re}c_0$, the ratio of the best-fit parameters without the Lorentz-Lorenz effect.

here, as (a)–(d), in the following:

(a) The electromagnetic interaction seems to be too weak to reduce appreciably the shifts and widths as exemplified in mixings of other states via $E2$ mixings[4] and the Weisskopf and Wigner model.[15]

(b) Higher momentum dependence in the optical potential can arise directly from the π^--nucleon higher partial contributions and from the finite range or form factor of the π^--nucleon interaction. The d- and f-wave isoscalar π^--nucleon scattering lengths are about a few thousandths in pion mass unit[16] and are negligible. The finite range or form factor of the π^--nucleon interaction seems to reduce the strength of the momentum-dependent part of the potential only by $10-20\%$, which was estimated using $(1+k_e^2/\Lambda^2)^{-1}$ for $\Lambda = 2 \sim 7$ and $k_e \sim 0.7\mu$ (from Fig. 1). Furthermore, the reduction is expected to be larger in lighter $3d$ atoms as also seen in the figure. We expect that at the phenomenological level the effect of such a form factor could be incorporated into an effective nuclear radius in ρ just as had been done in one of our previous analyses.[17] A recent study[4] in selected nonanomalous π^- atoms seems to confirm this expectation.

(c) Off-mass-shell effect: The large k_e as shown above implies that *in nuclei the pion effective mass is quite small and the pion is far off shell.* We have found that the off-shell pion does yield an additional higher momentum dependence but the dependence is weak.

The off-shell effect could appear as multipion states due to the strong (nearly $\frac{1}{2}\mu$) potential and also as direct dynamical effect. For the former, we found that the effect would not yield a new potential structure because the Lorentz-Lorenz parameter is nearly momentum independent.[18] As to the latter, a field theoretic model[19] of the π-nucleon interaction is found to yield the same V in (1) except $s(r)$ is now momentum dependent. However, $s(r)$ has a small coefficient of about $0.006\mu^5$ (contributed mostly from the N^* pole diagram in the s channel) and thus the dynamical off-mass-shell effect is small. Note that the on-the-mass-shell isoscalar π-nucleon p-wave effective range[20] also gives a small $s(r)$ with the coefficient of about $0.01\mu^5$.

(d) If pion condensate should set in, a new momentum-dependent repulsive term[21] is to be added to the potential and, an adequate magnitude of the order parameter could explain the anomaly. However, in view of unsuccessful attempts to observe this phenomenon, we have considered it to be a remote possibility. Hadronic structure effects [such as a polarization of the pion[22] and a (multi-) gluonic[23] or dyonic[24] long-range force] and nuclear polarization effect[22] are expected to increase the atomic binding energy; thus they are not an explanation of the anomaly. Note that the effects would be much larger in the $3d$ state than in the $4f$ state because of the radial, negative-power dependence of these potentials.

(4) Even assuming that the nuclear quadrupole moment is accurately known, the $4f$-$3d$ peaks observed seem to be too broad and inaccurate to yield simultaneously reliable information on the quadrupole and monopole parts of the strong interaction shifts and widths. Consequently, the anomaly is decided on the basis of a subtle statistical argument; e.g., Ref. 2 shows an anomalous result with $1.13\chi^2$ per degree of freedom and a nonanomalous result with $1.16\chi^2$ per degree of freedom, both for approximately 120 degrees of freedom. Furthermore, because of weighted averaging of various transition lines, the precise values of the monopole shift and width would be appreciably influenced by various effects.[3,25] We think that the widths are particularly hard to determine reliably because of the Lorentzian shape of the peaks. The relative intensity can provide a consistency check but there is no assurance that all other peaks underneath are identified; e.g., the Ta peaks fitted by the two groups are different.

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