Relations among the quenching strengths of magnetic transitions due to the Δ isobar excitation in $N \neq Z$ nuclei

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A possible excitation scheme of particle-hole states in $N \neq Z$ nuclei is worked out within the isospin formalism. Particle-hole states with a definite isospin are decomposed into 1p-1h (particle-hole), 2p-2h, and 3p-3h states. This decomposition enables us to relate transition strengths from the ground state to the excited states and those in the adjacent nuclei. The same thing is done for Δ isobar-hole states. Using these results, we obtain the relations among the quenching strengths of magnetic transitions in $N \neq Z$ nuclei. In particular, we apply these relations to the $M1$ states around ^{48}Ca and ^{90}Zr and compare the results with experimental data obtained by (p,n), (p,p'), and (e,e') experiments on ^{48}Ca and ^{90}Zr.

[NUCLEAR STRUCTURE Isospins, Δ isobar polarization.]

I. INTRODUCTION

Recent developments in measuring high energy neutrons have resulted in clean and systematic data on the magnetic properties of nuclei.[1–3] These (p,n) data together with (e,e') (Ref. 4) and (p,p') (Refs. 5 and 6) data seem to suggest that magnetic transition strengths are considerably quenched. The quenching strength $1-B(M\lambda)_{\rm exp}/B(M\lambda)_{\rm th}$ is mass and spin dependent and goes up to ~ 0.5.[7]

Stimulated by this finding, several authors emphasized the importance of the Δ isobar degree of freedom as the quenching mechanism.[8–10] This is because there are no restrictions on the Δ isobar states being excited by the magnetic λ pole $(M\lambda)$ operators, whereas the nuclear states which have magnetic strengths in the long wavelength limit are blocked due to the Pauli effect. It was demonstrated also that this mechanism provides the desired mass and spin dependence.[10] Further contributions should be, however, expected from the more complete description of the nuclear wave functions[12] and possibly mesonic currents.[13]

Most of the calculations for the Δ isobar-hole mechanism were carried out in the isospin formalism either in nuclear matter or in light nuclei with $N=Z$. A few cases were reported for $N \neq Z$ nuclei where protons and neutrons were treated explicitly.[11,14] The many body Hamiltonian suggests, however, that the isospin should be a good quantum number even in heavy nuclei with $N \neq Z$. In fact, Bohr and Mottelson demonstrated that the isospin impurities in ground states are as little as 0.3% or even less.[15] Therefore, in this paper we would like to develop a formalism where the isospin is treated as a good quantum number for $N \neq Z$ nuclei. By doing so, we would like to find relationships among the polarization strengths to different isospin states with the same spatial configurations in the low lying excitation spectra.

Of special interest in this regard are the $M1$ states found by (p,n),[16] (p,p'),[5,6] and (e,e') (Ref. 4) reactions on ^{48}Ca and ^{90}Zr. In a naive shell model where $j_>$ states $f_{7/2}$ in ^{48}Ca and $g_{9/2}$ in ^{90}Zr are completely occupied by neutrons, we expect one $M1$ state in the double magic nuclei and three $M1$ states in the neighboring odd-odd nuclei with one proton more and one neutron less. Experiments identify those states with possible fragmentation of the $M1$ strength into more complicated configurations.[3,16] The $M1$ strengths are provided with reasonable accuracy. We shall therefore calculate those transition strengths within the isospin formalism and compare the results with experiment.

We organize this paper as follows: In Secs. II and III, we discuss the level scheme for the nucleon-hole excitations and Δ isobar-hole states. Then, in Sec. IV, these results are combined to provide the polarization strengths to the low-lying nucleon-hole excited states due to the Δ isobar-hole intermediate excitations. In Sec. V, we compare the calculated results with experiment on ^{48}Ca and ^{90}Zr obtained by (p,n), (p,p'), and (e,e') reactions. We will summarize our results in Sec. VI.

II. CLASSIFICATION OF PARTICLE-HOLE STATES

The classification of particle-hole states according to isospin in the $N \neq Z$ nuclear system has been worked out by several authors.[24,25] Among them the work by Soga[24] is most elegant, in which particle-hole creation operators with good isospin are introduced. Unfortunately, he did not work out many examples and cases met in the recent (p,n) experiments, where there seems to be a little confusion on the transition strengths among different isospin states. Others are also not satisfactory in this respect.[25] Therefore in this section, we would like to write explicitly wave functions for all possible isospin states around a double magic nucleus in such a way that these wave functions can be used for the later discussions.

We start with the ground state of a double magic nucleus, which has Z protons and N neutrons. Defining $t_z|p\rangle = -\frac{1}{2}|p\rangle$, $t_z|n\rangle = \frac{1}{2}|n\rangle$, the total isospin of the ground state is $T = T_Z = \frac{1}{2}(N-Z) \equiv T_0$. Now we would like to make a particle-hole excitation from the ground state. We have three cases to treat separately: (i) particle-hole excitations from an occupied shell by both protons and neutrons to an open shell [Fig. 1(a)]; (ii) from a valence shell to an open shell [Fig. 1(b)] or from an occupied shell to a valence shell [Fig. 1(c)]; (iii) from a valence shell to a valence shell [Fig. 1(d)]. We shall construct particle-hole states with good isospin for each case.

(i) Occupied→open (oo). Since we are not interested in a specific spin state, we do not write the total spin and parity explicitly for any particle-hole states. The particle-hole excitation is generated by a particle-hole creation operator $c^\dagger c$. In this case, the particle-hole excitation carries a definite isospin $\tau = 1$ or 0. In order to achieve a good isospin as a whole, this particle-hole excitation has to further couple with the core isospin resulting in a definite T spin:

$$|\text{oo};\tau TT_z\rangle = \sum_{m_\tau} (\tau m_\tau T_0 T_z - m_\tau | TT_z) \times [c^\dagger c]_{\tau m_\tau} | T_0 T_z - m_\tau\rangle \ . \tag{2.1}$$

Here $|T_0 T_z - m_\tau\rangle$ denote the core state for $T_z - m_\tau = T_0$ and its isobaric analog states for $T_z - m_\tau \neq T_0$. In the $\tau = 1$ case, we have several states. For example, $[c^\dagger c]_{11} | T_0 T_0 - 1\rangle$ corresponds to a 2 particle-2 hole state and $[c^\dagger c]_{11} | T_0 T_0 - 2\rangle$ corresponds to a 3 particle-3 hole state as shown in Figs. 1(e) and 1(f), respectively. We may therefore write particle-hole states with good isospin as

$$|\text{oo};\tau TT_z\rangle = \sum_{n=1}^{3} \alpha_i(T,n) \, | i; np - nh, T_z\rangle \ . \tag{2.2}$$

Here i stands for different cases. We list the coefficients α for all the possible isospins in Table I. The $\tau = 0$ case provides a simple 1 particle-1 hole state with $T = T_z = T_0$.

(ii) Valence→open or occupied→valence (ov). The particle-hole excitation does not carry a definite isospin due to the lack of protons in the valence shell $(v \to o)$ or of neutron space in the valence shell $(o \to v)$. However, we know that the particle-hole

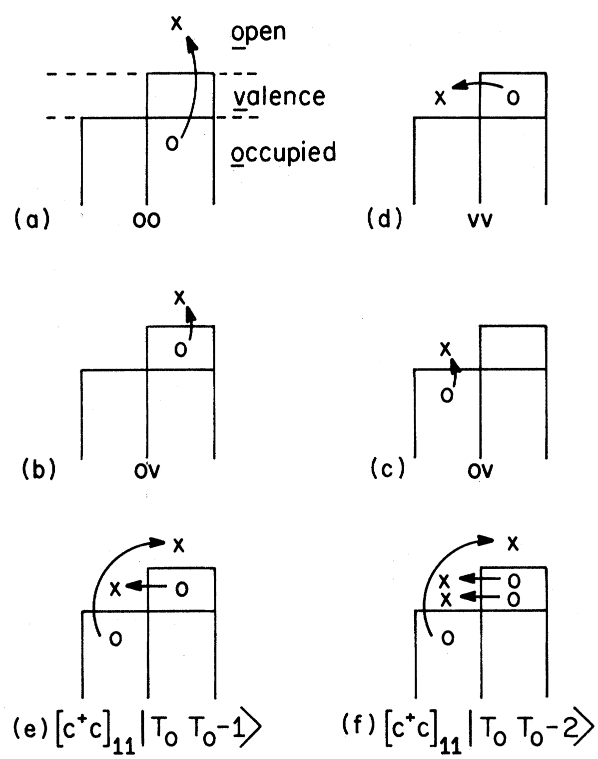

FIG. 1. The excitation modes for the three categories, oo, ov, and vv. The bottom figures demonstrate examples of the 2 particle-2 hole and 3 particle-3 hole excitations.

TABLE I. The expansion coefficients $\alpha_i(T,n)$ in Eq. (2.2) for nucleon particle-hole states with good isospin T in each case $i=1$, 2, and 3 for oo, ov, and vv, respectively. T_0 denotes the T_z of the double magic nucleus; $T_0=\frac{1}{2}(N-Z)$.

T	T_z	$\alpha(T,1)$	$\alpha(T,2)$	$\alpha(T,3)$
		$i=1$ (occupied $\rightarrow$ open)		
T_0+1	T_0+1	1		
T_0+1	T_0	$\sqrt{1/(T_0+1)}$	$\sqrt{T_0/(T_0+1)}$	
T_0	T_0	$-\sqrt{T_0/(T_0+1)}$	$\sqrt{1/(T_0+1)}$	
T_0+1	T_0-1	$\sqrt{1/(2T_0+1)(T_0+1)}$	$\sqrt{4T_0/(2T_0+1)(T_0+1)}$	$\sqrt{(2T_0-1)T_0/(2T_0+1)(T_0+1)}$
T_0	T_0-1	$-\sqrt{1/(T_0+1)}$	$-(T_0-1)/\sqrt{T_0(T_0+1)}$	$\sqrt{(2T_0-1)/T_0(T_0+1)}$
T_0-1	T_0-1	$\sqrt{(2T_0-1)/(2T_0+1)}$	$-\sqrt{(2T_0-1)/T_0(2T_0+1)}$	$\sqrt{1/T_0(2T_0+1)}$
		$i=2$ (Valence $\rightarrow$ open or occupied $\rightarrow$ valence)		
T_0	T_0	1		
T_0	T_0-1	$\sqrt{1/2T_0}$	$\sqrt{(2T_0-1)/2T_0}$	
T_0-1	T_0-1	$-\sqrt{(2T_0-1)/2T_0}$	$\sqrt{1/2T_0}$	
		$i=3$ (Valence $\rightarrow$ valence)		
T_0-1	T_0-1	1		

state has to have $T=T_0$, since the operation of the raising operator T_+ on this particle-hole states does not provide a new state. Hence,

$$|\,\mathrm{ov};T,T_0\rangle=(c^\dagger c)_{\tau_z=0}\,|\,T_0T_0\rangle\delta_{TT_0}\ , \qquad (2.3)$$

where the bracket just means a particle-hole creation operator with a definite τ_z but not a definite τ. The $T_z=T_0-1$ state can be obtained by the isospin lowering T_- operation on this state (2.3). Since the number of ways to shift the neutron into the open proton orbit ($v\rightarrow o$) or the proton hole into the occupied neutron orbit ($o\rightarrow v$) is 1 and for the valence neutrons is $2T_0-1$, we get with a proper normalization

$$|\,\mathrm{ov};T_0,T_0-1\rangle=\frac{1}{\sqrt{2T_0}}(c^\dagger c)_{-1}\,|\,T_0T_0\rangle+\left[\frac{2T_0-1}{2T_0}\right]^{1/2}(c^\dagger c)_0\,|\,T_0T_0-1\rangle\ . \qquad (2.4)$$

The $T=T_0-1$ state should be orthogonal to this state.

(iii) Valence→valence (vv). We have only a $T=T_z=T_0-1$ state;

$$|\,\mathrm{vv};T_0-1,T_0-1\rangle=(c^\dagger c)_{-1}\,|\,T_0T_0\rangle\ . \qquad (2.5)$$

All the possible isospin states are tabulated in Table I.

Now we would like to relate the transition probabilities to those states caused by, e.g., (p,p'), (p,n), (n,p), (e,e'), ..., etc., reactions. Generally, the transition operations associated with these reactions may be written as

$$O=O_1(\vec{r},\vec{\sigma})\vec{\tau}+O_0(\vec{r},\vec{\sigma})\ . \qquad (2.6)$$

The exchange term in hadron scattering cannot strictly be written in this manner. However, we may use the pseudopotential prescription[17] and include this effect into O_1 and O_0 in Eq. (2.6). The one body operator (2.6) can only connect the ground state to the 1p-1h components of the wave functions. This is true also for the exchange process, although one cannot write the corresponding operator as in (2.6). Therefore the transition probabilities are proportional to the square of the 1p-1h amplitudes. In each case we can relate the transition probabilities as shown in Fig. 2, where the strong transitions are indicated by solid lines for $\tau=1$ and by dashed lines for $\tau=0$ excitations.

One may notice a rule from Fig. 2. For nuclei with $N\neq Z$, in particular $N\gg Z$, the transition from the ground state occurs strongly to $T=T_z$ states in all cases. This rule may seem strange at first glance. Why does the $\vec{\tau}$ operator not favor $T=T_z+1$ states? The answer is given by the way vectors couple. When $T_0=\frac{1}{2}(N-Z)\gg 0$, the addition of a vector $(\tau\tau_z)=(10)$ (this vector is supposed to be for the transition operator) to the (T_0T_0) vector for the ground state results in a vector whose length hardly changes from the original

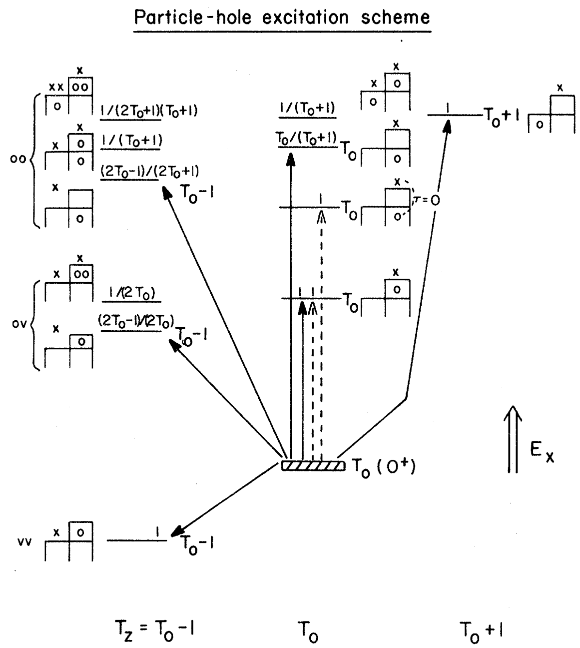

FIG. 2. The particle-hole excitation scheme with good isospin. The same T spin states with different T_z are arranged at the same height and the isospin is denoted beside the right most state. The main configuration of each state for large T_0 is depicted beside each state. The excitation strengths from the ground state, $T_0(O^+)$, are given above each state, where the strengths are normalized to 1 for the largest isospin state in each case. The states most strongly excited among the same T_z states in each case are indicated by arrows; the solid lines for the isovector excitation, the dashed lines for the isoscalar excitation.

vector (T_0T_0). Hence, the resultant states should be mostly $T=T_0$ states. If a vector $(\tau\tau_z)=(1-1)$ is added, the resultant vector should be mainly the $T=T_z=T_0-1$ state. As T_0 decreases, the length of the resultant vector starts to differ from that of the T_0 vector; hence the $\vec{\tau}$ operator starts to excite $T=T_0+1$ states, and finally at $T_0=0$, the addition of the (10) vector only results in a $T=\tau=1$ state.

III. CLASSIFICATION OF Δ ISOBAR-HOLE STATES

In recent years, it has become more and more apparent that the Δ isobar excitation plays an important role in quenching the magnetic transition strengths in nuclei in the long wavelength limit.[8–11] In order to establish the relations among the quenching strength for different T spin states, we would like to construct the Δ isobar-hole states with the isospins treated explicitly as has been done for particle-hole states.

In the Δ isobar-hole case, we have two cases: Δ isobar-hole excitations from an occupied orbit to a Δ isobar state and from a valence orbit to a Δ isobar state.

(i) Occupied→Δ isobar state (Δ*o*). As in the oo case, the isobar-hole excitation carries a definite isospin; in this case $\tau=1$ or 2. Of particular interest for us is the $\tau=1$ case. Hence, we have

$$|\Delta o;\tau=1TT_z\rangle=\sum_{m_\tau}(1m_\tau T_0T_z-m_\tau\,|\,TT_z)\times[\Delta^\dagger c]_{1m_\tau}\,|\,T_0T_z-m_\tau\rangle\ . \tag{3.1}$$

Since the same Clebsch-Gordan coefficients appear as for the oo case, the weighting factors are the same as those discussed there.

(ii) Valence→Δ isobar state (Δ*v*). This case differs from the nucleon particle-hole case, because $t_\Delta=\frac{3}{2}$ instead of $t=\frac{1}{2}$ for nucleons. This fact allows us to construct $T=T_0+1$ states as

$$|\Delta v;T_0+1,T_0+1\rangle=(\Delta^\dagger c)_{m_\tau=1}\,|\,T_0T_0\rangle\ , \tag{3.2}$$

although the Δ isobar-hole excitation does not couple to a definite isospin. All the other states with $T_z=T_0$ and T_0-1 are constructed by two procedures: the T_- operation and orthogonalization. When T_- is operated, we should note that Δ has the isospin $\frac{3}{2}$; i.e., $t_-\,|\,\frac{3}{2}m_t\rangle=\sqrt{3}$, 2, $\sqrt{3}$, and 0 for $m_t=\frac{3}{2},\ \frac{1}{2},\ -\frac{1}{2}$, and $-\frac{3}{2}$, respectively. The results are tabulated in Table II.

Considering a one body operator, which causes Δ isobar-hole excitations, one can construct a similar excitation scheme for these Δ isobar-hole states, as shown in Fig. 3. Although the excitation energies are large, $E_x\sim\omega_\Delta=2.3m_\pi$, this direct feeding of the Δ isobar-hole states from the ground state has been shown to have an important role on the magnetic transition strengths in the low lying spectrum due to the availability of many Δ isobar-hole configurations and the strong coupling strength which connects the two configurations.[8,11]

IV. Δ ISOBAR INDUCED POLARIZATION

The Δ isobar degree of freedom is important when magnetic transitions take place in the low ly-

TABLE II. The expansion coefficients $\alpha_i(T,n)$ in Eq. (2.2) for Δ isobar-hole states with good isospin T in each category $i=4$ and 5.

T	T_z	$\alpha(T,1)$	$\alpha(T,2)$	$\alpha(T,3)$
		$i=4$ (occupied $\rightarrow\Delta$ isobar)		
T_0+1	T_0+1	1		
T_0+1	T_0	$\sqrt{1/(T_0+1)}$	$\sqrt{T_0/(T_0+1)}$	
T_0	T_0	$-\sqrt{T_0/(T_0+1)}$	$\sqrt{1/(T_0+1)}$	
T_0+1	T_0-1	$\sqrt{1/(2T_0+1)(T_0+1)}$	$\sqrt{T_0/(2T_0+1)(T_0+1)}$	$\sqrt{(2T_0-1)T_0/(2T_0+1)(T_0+1)}$
T_0	T_0-1	$-\sqrt{1/(T_0+1)}$	$-(T_0-1)/\sqrt{T_0(T_0+1)}$	$\sqrt{(2T_0-1)/T_0(T_0+1)}$
T_0-1	T_0-1	$\sqrt{(2T_0-1)/(2T_0+1)}$	$-\sqrt{(2T_0-1)/T_0(2T_0+1)}$	$\sqrt{1/T_0(2T_0+1)}$
		$i=5$ (valence $\rightarrow\Delta$ isobar)		
T_0+1	T_0+1	1		
T_0+1	T_0	$\sqrt{3/2(T_0+1)}$	$\sqrt{(2T_0-1)/2(T_0+1)}$	
T_0	T_0	$-\sqrt{(2T_0-1)/2(T_0+1)}$	$\sqrt{3/2(T_0+1)}$	
T_0+1	T_0-1	$\sqrt{3/(2T_0+1)(T_0+1)}$	$\sqrt{3(2T_0-1)/(2T_0+1)(T_0+1)}$	$\sqrt{(2T_0-1)(T_0-1)/(2T_0+1)(T_0+1)}$
T_0	T_0-1	$-\sqrt{(2T_0-1)/T_0(T_0+1)}$	$-(T_0-2)/\sqrt{T_0(T_0+1)}$	$\sqrt{3(T_0-1)/T_0(T_0+1)}$
T_0-1	T_0-1	$\sqrt{(2T_0-1)(T_0-1)/T_0(2T_0+1)}$	$-\sqrt{4(T_0-1)/T_0(2T_0+1)}$	$\sqrt{3/T_0(2T_0+1)}$

ing spectrum. The spin-isospin dependent part of the magnetic operators causes excitation of the Δ isobar-hole state and then the spin-isospin dependent interaction brings them back to the low lying states. This "nucleonic" polarization phenomenon is sometimes called dimesic polarization[18,19] due to the analogy with the dielectric phenomenon in order to distinguish it from the "nuclear" core polarization.

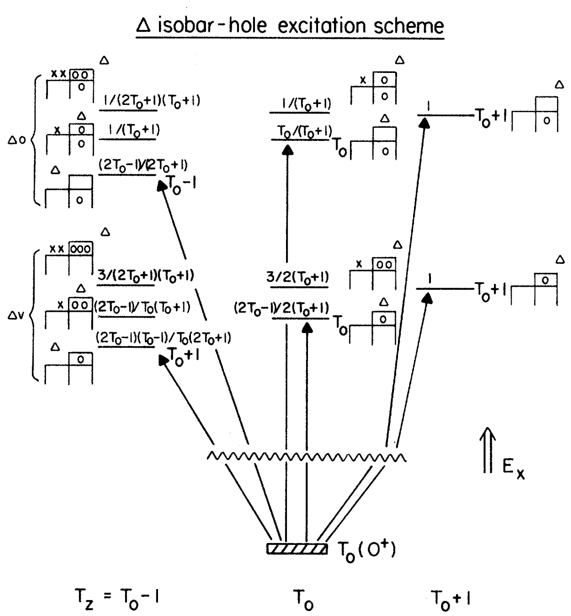

FIG. 3. The Δ isobar-hole excitation scheme with good isospin. See the caption of Fig. 2 for details.

The purpose of this section is to express the polarization strength due to the Δ isobar-hole excitations with good isospin in terms of 1 particle-1 hole matrix elements which are calculated in a simple manner.

To start with, let us work out the Δ isobar induced polarization in the first order perturbation.

$$\delta Q(i;TT_z)=\sum_j \langle i;TT_z \mid V \mid j;TT_z\rangle \frac{-1}{E_j-E_i} \times\langle j;TT_z \mid O \mid T_0T_0\rangle \, , \quad (4.1)$$

where $i=1$, 2, 3 stands for the three cases (oo,ov,vv) and $j=4$, 5 for the 2 Δ isobar-hole cases ($\Delta o,\Delta v$). τ in Eq. (2.1) has been dropped, since we are only interested in the $\tau=1$ part. The two body interaction and the magnetic operators are denoted by V and O. We will first connect the interaction matrix elements with those of the 1 particle-1 hole states. Decomposing $\mid i;TT_z\rangle$ into 1p-1h, 2p-2h, and 3p-3h states, we obtain a matrix equation for $\langle i;TT_z \mid V \mid j;TT_z\rangle$

$$\hat{A}_i\hat{M}_{ij}\hat{A}_j^T=\hat{W}_{ij} \, , \quad (4.2)$$

where

$$(\hat{A}_i)_{lm}=\alpha_i(T=T_z+l-1,m) \quad (4.3)$$

and

$$(\hat{M}_{ij})_{lm}=\langle i;l\text{p-}l\text{h},T_z \mid V \mid j;m\text{p-}m\text{h},T_z\rangle \ , \tag{4.4}$$

$$(\hat{W}_{ij})_{lm}=\langle i;T=T_z+l-1,T_z \mid V \mid j;T=T_z+m-1,T_z\rangle \ . \tag{4.5}$$

Here $(\hat{W}_{ij})_{lm}$ is diagonal due to the isospin conservation. Since $\hat{A}_i^{-1}=\hat{A}_i^T$ due to the orthonormalization conditions, we can easily solve for $\hat{M}_{ij}$,

$$\hat{M}_{ij}=\hat{A}_i^T\hat{W}_{ij}\hat{A}_j \ . \tag{4.6}$$

Since we are interested in connecting the matrix elements of states with good T spins to those of 1 particle-1 hole states, we explicitly write the (1,1) component of the above equation,

$$\langle i,1\text{p-}1\text{h},T_z \mid V \mid j;1\text{p-}1\text{h},T_z\rangle = \sum_{l=1}^{3} \alpha_i(T=T_z+l-1,1)\alpha_j(T=T_z+l-1,1) \times\langle i;T_z+l-1,T_z \mid V \mid j;T_z+l-1,T_z\rangle \ . \tag{4.7}$$

This equation together with the fact that

$$\langle i;T,T_z \mid V \mid j;T,T_z\rangle$$

does not depend on T_z relates the matrix elements with good isospin to those of 1 particle-1 hole states.

We would now like to relate the 1 particle-1 hole matrix elements with different T_z's so that those with good isospin are related. For this purpose we introduce a model for the interaction V as $V=V(\vec{r},\vec{\sigma},\vec{S})\vec{\tau}\cdot\vec{T}$, which is supposed to include the exchange contribution like that of the Landau-Migdal interaction.[20] Here σ and τ are the Pauli spin and isospin operators while S and T are the transition spin and isospin operators connecting nucleon states to Δ isobar states. With this model for the interaction we can write the 1 particle-1 hole matrix elements as

$$\langle i;1\text{p-}1\text{h},T_z \mid V(\vec{r},\vec{\sigma},\vec{S})\vec{\tau}\cdot\vec{T} \mid j;1\text{p-}1\text{h},T_z\rangle=\langle i;1\text{p-}1\text{h} \mid V(\vec{r},\vec{\sigma},\vec{S}) \mid j;1\text{p-}1\text{h}\rangle\gamma_i(T_z)\gamma_j(T_z) \ , \tag{4.8}$$

where

$$\gamma_i(T_z)=\sqrt{2} \text{ for } i=1 \text{ (oo) and for } T_z=T_0-1 \text{ of } i=2 \text{ (ov) and } i=3 \text{ (vv)} \ ,$$

$$\gamma_i(T_z)=1 \text{ for } T_z=T_0 \text{ of } i=2\text{(ov)} \ ,$$

$$\gamma_j(T_z)=\sqrt{4/3} \text{ for } j=4 \ (\Delta o) \ ,$$

and

$$\left.\begin{array}{r}\gamma_j(T_0+1)=1\\ \gamma_j(T_0)=\sqrt{2/3}\\ \gamma_j(T_0-1)=\sqrt{1/3}\end{array}\right\} \text{ for } j=5(\Delta v) \ . \tag{4.9}$$

Furthermore, we wish to relate $Q(i;1\text{p-}1\text{h},T_z)$ with different T_z's in order to relate the polarization strengths for each i,

$$\begin{aligned}Q(i;1\text{p-}1\text{h},T_z)&=\langle i;1\text{p-}1\text{h},T_z \mid O(\vec{r},\vec{\sigma})\tau_{1\mu} \mid T_0T_0\rangle \ ,\\ &=\langle i;1\text{p-}1\text{h} \mid O(\vec{r},\vec{\sigma}) \mid O\rangle\gamma_i(T_z) \ ,\end{aligned} \tag{4.10}$$

$$\begin{aligned}Q(j;1\text{p-}1\text{h},T_z)&=\langle j;1\text{p-}1\text{h},T_z \mid O(\vec{r},\vec{S})T_{1\mu} \mid T_0T_0\rangle \ ,\\ &=\langle j;1\text{p-}1\text{h} \mid O(\vec{r},\vec{S}) \mid O\rangle\gamma_j(T_z) \ .\end{aligned} \tag{4.11}$$

Combining the above relations, (4.7)–(4.11), we can easily relate the polarization strengths

$$\chi(i;T,T_z)\equiv\delta Q(i;T,T_z)/Q(i;T,T_z)$$

with different T's for each i. First, in the case of one isospin for T_z, such as $(\text{oo},T_z=T_0+1)$, $(\text{ov},T_z=T_0)$, and $(\mathrm{vv}, T_z = T_0 - 1)$,

$$\chi(i;T,T_z) = \sum_j \frac{1}{\alpha_j(T_z,1)} \langle i;1\text{p-1h},T_z \mid V \mid j;1\text{p-1h},T_z\rangle \left[\frac{-1}{E_j - E_i}\right]$$

$$\times \langle j;1\text{p-1h},T_z \mid O \mid T_0T_0\rangle \alpha_j(T_z,1) / \langle i;1\text{p-1h},T_z \mid O \mid T_0T_0\rangle$$

$$\equiv \sum_j \chi(i,j;T,T_z) \; . \tag{4.12}$$

This result indicates that when the low lying nuclear state with good isospin is a pure 1 particle-1 hole state (no 2 particle-2 hole and 3 particle-3 hole components), we can take just 1 particle-1 hole intermediate states without considering the isospin and obtain the polarization strength. In the case of two isospins for T_z, such as $(\mathrm{oo}, T_z = T_0)$ and $(\mathrm{ov}, T_z = T_0 - 1)$,

$$\chi(i;T,T_z) = \sum_j x_{ij}(T)\chi(i,j;T,T_z) \; , \tag{4.13}$$

where

$$x_{ij}(T) = \frac{1}{\{\alpha_i(T_z,1)\}^2}\left[\frac{\gamma_i(T_z)\gamma_j(T_z)}{\gamma_i(T_z+1)\gamma_j(T_z+1)} - \frac{\alpha_i(T_z+1,1)\alpha_j(T_z+1,1)}{\alpha_j'(T_z+1,1)}\right]\frac{\gamma_j(T_z)\gamma_i(T_z+1)}{\gamma_j(T_z+1)\gamma_i(T_z)} \; . \tag{4.14}$$

Here $\alpha_j'(T_z+1,1)$ denotes the $\alpha_j(T_z+1,1)$ coefficient of the system where the z component of the isospin is T_z+1. Finally, in the case of three isospins for T_z, such as $(\mathrm{oo}, T_z = T_0 - 1)$, we obtain a more complicated expression;

$$\begin{aligned} x_{ij}(T) = \frac{1}{\{\alpha_i(T_z,1)\}^2} \Bigg[& \frac{\gamma_i(T_z)\gamma_j(T_z)}{\gamma_i(T_z+2)\gamma_j(T_z+2)} - \frac{\alpha_i(T_z+1,1)\alpha_j(T_z+1,1)}{\alpha_i'(T_z+1,1)\alpha_j'(T_z+1,1)}\frac{\gamma_i(T_z+1)\gamma_j(T_z+1)}{\gamma_i(T_z+2)\gamma_j(T_z+2)} \\ & + \frac{\alpha_i(T_z+1,1)\alpha_j(T_z+1,1)\alpha_i'(T_z-2,1)\alpha_j'(T_z+2,1)}{\alpha_i'(T_z+1,1)\alpha_j'(T_z+1,1)} \\ & - \alpha_i(T_z+2,1)\alpha_j(T_z+2,1)\Bigg]\frac{\gamma_j(T_z)\gamma_i(T_z+2)}{\gamma_j(T_z+2)\gamma_i(T_z)} \; , \end{aligned} \tag{4.15}$$

we note that $x_{ij}(T)$ does not depend on T_z. The coefficient x_{ij} comes out to be one for most of the cases except the following:

$$x_{i=1,j=5}(T_0) = \frac{2T_0-1}{3T_0} \; ,$$

$$x_{i=1,j=5}(T_0-1) = \frac{T_0-1}{3T_0} \; , \tag{4.16}$$

$$x_{i=2,j=5}(T_0-1) = \frac{T_0-1}{2T_0-1} \; .$$

Finally, we mention that the ground state correlation [random-phase approximation (RPA) backward amplitudes] can be added by replacing

$$\left[\frac{-1}{E_j - E_i}\right]$$

by

$$\left[\frac{-1}{E_j - E_i} + \frac{-1}{E_j + E_i}\right]$$

as usual. Furthermore, if we wish to iterate the polarization up to all orders, we can work out the isospin part in the same spirit as the above.

V. COMPARISON WITH EXPERIMENT

We have worked out in the previous sections the polarization strengths for magnetic states with good isospin around the double magic $N \neq Z$ nuclei and related them with those calculated by 1 particle-1 hole descriptions. In this section we shall make a comparison of the calculated values with experiment. Of particular interest are the $M1$ states seen by (p,n), (p,p'), and (e,e') experiments on ^{48}Ca and

^{90}Zr. Let us assume that these two nuclei have simple shell structures; i.e., the $f_{7/2}$ orbit is occupied by 8 neutrons for ^{48}Ca and the $g_{9/2}$ orbit by 10 neutrons for ^{90}Zr. Then we have two cases which can produce 1^+ states which carry the $M1$ strength in the long wavelength limit; $(ph)=j_< - j_>{}^{-1}$ and $j_> - j_>{}^{-1}$ states where $j_{\gtrless} = l \pm \frac{1}{2}$. They correspond to the cases ov and vv, respectively, in Sec. II. Moreover, since these two states have the same spatial wave functions, the transition strengths are simply related by the $9j$ coefficients. Therefore, we are able to compare the transition strengths for four states: ov; (T,T_z) is equal to (T_0,T_0), (T_0,T_0-1), (T_0-1,T_0-1), and vv; (T_0-1,T_0-1) states where $T_0=\frac{1}{2}(N-Z)$ of the double magic nucleus.

We first relate the $M1$ strengths

$$S(i;T,T_z) \equiv |\langle i;T,T_z || \sum_k \sigma^{(k)} t^{(k)}_{\pm,z} || T_0 T_0 \rangle|^2 \tag{5.1}$$

for these four states before considering the polarization effect. We follow here the definition for the transition strength from the work of Goodman *et al.*[21] For this purpose, we relate $S(\text{vv};T_0-1,T_0-1)$ to $S(\text{ov};T_0,T_0)$,

$$\frac{S(\text{vv};T_0-1,T_0-1)}{4S(\text{ov};T_0,T_0)} = \left| \frac{\langle j_>,j_>;1\,|\,L=0,S=1;1\rangle}{\langle j_<,j_>;1\,|\,L=0,S=1;1\rangle} \right|^2 . \tag{5.2}$$

The factor 4 in the denominator appears due to the use of t in Eq. (5.1). This definition for the transition strength is nice because the isovector part of the cross sections at $q=0$ is proportional to the S's with the same factor for the (p,n) and (p,p') reactions.

In order to see how the simple shell model results compare with experiment, we extract the $M1$ strengths from the experimental data. We take the $B(M1)=4\mu_v{}^2$ obtained by electron scattering on ^{48}Ca for $S_{\text{ex}}(\text{ov},T=4,T_z=4)$. The value $S_{\text{ex}}=1.48$ results by assuming that only the isovector part is renormalized while the isoscalar part is unchanged.[14] The remaining numbers for ^{48}Ca are derived from the (p,n) cross sections at $E_p=160$ MeV (Ref. 3); i.e., $d\sigma/d\Omega\,|_{\theta=0^\circ}=2.22$, 33.9, and 7.4 mb/sr for $(i;T,T_z)=(\text{ov};4,3)$, $(\text{ov};3,3)$, and $(\text{vv};3,3)$, respectively, using the same distortion factor derived by Goodman *et al.*[21] On the other hand, $S_{\text{ex}}(\text{ov},T=5,T_z=5)$ in ^{90}Zr is extracted from the (p,p') cross section taken at $E_p=200$ MeV, where 25% of the cross section is assumed to arise from the isoscalar excitation[22] and again the same distortion factor of Goodman *et al.*[21] is used. S_{ex} for other states in ^{90}Zr are obtained using the (p,n) data at $E_p=120$ MeV taken by Sterrenberg *et al.*[16] The method used here to extract the $M1$ strengths from the experimental data might cause systematic errors, but these extracted values should be fairly good estimates for the transition strengths and provide ideas on how various models compare with experiment.

We show the comparison of the simple shell model predictions with experiment in the first and second columns of Table III. We notice two things. First, the simple shell model overestimates the $M1$ strengths. Second, this deviation is much too large for $S(\text{vv},T=T_0-1,T_z=T_0-1)$. The second part originates from the nuclear structure effect where the two states ov and vv, with the same isospin $T=T_z=T_0-1$, mix strongly due to the identical spatial wave functions. Bertsch *et al.*[23] have made a systematic study of the giant Gamow-Teller states and found that a zero range interaction with a reasonable strength which reproduces the excitation energies of the Gamow-Teller states results in a value of about 20% for the ratio between the transition strengths of the Gamow-Teller states, mainly $(\text{ov};T=T_0-1,T_z=T_0-1)$, and the lower $M1$ states, mainly $(\text{vv};T=T_0-1,T_z=T_0-1)$. The states are therefore the linear combinations of the two wave functions,

$$\begin{aligned} |\widetilde{\text{ov}};T_0-1,T_0-1\rangle &= \sqrt{1-\beta^2}\,|\text{ov};T_0-1,T_0-1\rangle + \beta\,|\text{vv};T_0-1,T_0-1\rangle , \\ |\widetilde{\text{vv}};T_0-1,T_0-1\rangle &= \beta\,|\text{ov};T_0-1,T_0-1\rangle - \sqrt{1-\beta^2}\,|\text{vv};T_0-1,T_0-1\rangle . \end{aligned} \tag{5.3}$$

The small amplitude β is fixed so as to reproduce the ratio

$$S_{\text{ex}}(\text{vv};T_0-1,T_0-1)/S_{\text{ex}}(\text{ov};T_0,T_0-1)+S_{\text{ex}}(\text{ov};T_0-1,T_0-1) ,$$

which is determined well by experiment. The calculated results after this correction are shown in the third column in Table III. Now, the calculated results are commonly about a factor of 3 too large as compared to the experimental numbers.

TABLE III. Experimental and theoretical $M1$ strengths defined in Eq. (5.1). The experimental values S_{ex} are derived from various sources with the procedures described in the text. $S_{ex}(\text{ov};4,4)$ in ^{48}Ca is derived from $B(M1)=4\mu_\nu^2$ obtained by the (e,e') experiment (Ref. 4). The others in ^{48}Ca come from the (p,n) data taken at $E_p=160$ MeV (Ref. 3). $S_{ex}(\text{ov};5,5)$ in ^{90}Zr is obtained by using the (p,p') cross section at $E_p=200$ MeV (Ref. 5). The other strengths for $T_z=T_0-1$ around ^{90}Zr are derived from the unpublished data on the (p,n) cross sections taken at $E_p=120$ MeV (Ref. 16). The calculated values are the following: S = simple shell model precitions; $S(\beta)=M1$ strengths where the two bottom states are admixed with the amplitude β; $\tilde{S}=$ the Δ isobar polarization is added on S; $\tilde{S}(\beta)=$ the Δ isobar polarization is added on $S(\beta)$.

Case	T	T_z	S_{ex}	S	$S(\beta=0.33)$	$\tilde{S}$	$\tilde{S}(\beta=0.33)$
				^{48}Ca			
ov	4	4	1.48	3.42	3.42	1.71	1.71
ov	4	3	0.57	1.71	1.71	0.86	0.86
ov	3	3	8.69	11.97	18.70	6.47	9.92
vv	3	3	1.90	10.29	3.56	5.15	1.70
				^{90}Zr			
Case	T	T_z	S_{ex}	S	$S(\beta=0.23)$	$\tilde{S}$	$\tilde{S}(\beta=0.23)$
ov	5	5	2.10	4.45	4.45	2.23	2.23
ov	5	5	0.63	1.78	1.78	0.89	0.89
ov	4	4	7.55	16.01	22.07	8.42	11.51
vv	4	4	2.07	12.24	6.18	6.12	3.03

We believe that the major part of this common factor originates from the Δ isobar polarization mechanism. This polarization strength has been calculated by Härting *et al.*[14] for ^{48}Ca where protons and neutrons are treated explicitly. Therefore, we have already the results on the polarization strength for the state $|\text{ov}; T_0,T_0\rangle$. Since the shell model calculations by McGrory and Wildenthal[12] showed that about 30% of the full quenching strength is due to configuration mixing, we take in this paper that the Δ isobar polarization mechanism produces 50% quenching strength, which is obtained with $g'=0.65$.[14] We can now calculate the quenching strengths of the other states using the relations developed in the previous section with further small assumptions. We assume that the polarization strength for $|\text{vv}, T_0-1,T_0-1\rangle$ is equal to that for $|\text{ov}; T_0,T_0\rangle$, since the spatial wave functions are the same. Furthermore, the polarization strength ratios x_{ij} in Eq. (4.13) are averaged over by weighting the number of nucleons in the occupied shells and in the valence shell for the Δo and Δv contributions. The results calculated with and without the admixture of the wave functions under these assumptions are listed in the fourth and fifth columns in Table III. If we consider the possible additional contribution from the shell model configuration mixing (about 30% quenching)[12] and the ambiguities caused by the procedures to extract the $M1$ strengths, the results are very satisfactory.

The ratio

$$S(\text{ov};T_0-1,T_0-1)/S(\text{ov};T_0,T_0-1)$$

is $2T_0-1$; i.e., 7 for ^{48}Ca and 9 for ^{90}Zr in the simple shell model. On the other hand, the ratio is 15.2 for ^{48}Ca and 12.0 for ^{90}Zr experimentally. The latter value is not as well determined because the two peaks overlap in the (p,n) spectra. After adding the two states admixture and the Δ isobar induced polarization effect, this ratio becomes 11.5 for ^{48}Ca and 12.9 for ^{90}Zr. The desired improvement is achieved by the mechanisms considered here for this ratio, although this ratio might change experimentally due to the difficulty in separating the two states in the (p,n) spectra.

VI. CONCLUSION

Motivated by the recent systematic (p,n) and (p,p') experiments in the medium and heavy mass regions, we have worked out the possible excitation scheme of particle-hole excitations in $N\neq Z$ nuclei within the isospin formalism. We have also analyzed the Δ isobar-hole excitations within the isospin formalism and worked out the polarization strengths for the magnetic transitions in the low-lying spectrum through the Δ isobar intermediate excitations. In this way, we relate the Δ isobar induced polarization strengths for different low-lying $M\lambda$ transitions.

We have applied these relations to the $M1$ states around ^{48}Ca and ^{90}Zr found by the (p,n), (p,p'), and (e,e') experiments. Note that the long anticipated[26] giant Gamow-Teller state in $T_z = T_0 - 1$ nuclei was discovered for the first time by Galonsky *et al.*[27] in the ^{90}Zr (^{3}He,t) spectrum. Assuming the simple shell model for the ground state where the $f_{1/2}$ shell is completely occupied by 8 neutrons in ^{48}Ca and the $g_{9/2}$ shell by 10 neutrons in ^{90}Zr, we have two kinds of excitations; one from the valence shell to the open shell (ov) and one from the valence shell to the valence shell (vv), for the $M1$ states. Since the two cases have the same spatial wave functions, we can relate the transition strengths to those states (ov and vv) through the $9j$ coefficients in the long wavelength limit. There are two major discrepancies found by comparing the simple shell model predictions with experiment. First, the transition strengths calculated are in general too large. Second, the lower $M1$ states in the $T_z = T_0 - 1$ nucleus with T_0 being the isospin of the double closed shell nucleus in particular have strengths which are about a factor of 10 too large compared to the experimental values.

The second discrepancy has been removed partly by considering the admixture of the ov and vv states with $T = T_z = T_0 - 1$ with a reasonable residual interaction which reproduces the excitation energies. Then the achieved common quenching factors (a factor of ~ 3) are considered to originate mainly from the Δ isobar polarization mechanism. By assuming the Δ isobar polarization strength of 0.5 for the $|\text{ov}; T = T_0, T_z = T_0\rangle$ state in the closed shell nucleus, which has been provided by Härting *et al.* in ^{48}Ca, we calculated all the $M1$ strengths using the relations for the polarization strengths. The calculated results compare very well with the experimental values. Although the methods used to extract the $M1$ strengths from the experimental data might have systematic erorrs, we seem to have a good understanding of the $M1$ states around the double magic $N \neq Z$ nuclei.

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