Angular momentum sum-rule analysis of $T=\frac{1}{2}$ mirror nuclei in the $2s$-$1d$ shell

J. A. Hall* and I. P. Johnstone
Department of Physics, Queen's University, Kingston, Ontario, Canada K7L 3N6
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Empirical data from single-particle transfer reactions on the stable members of $T=\frac{1}{2}$ mirror pairs in the $2s$-$1d$ shell are analyzed using sum rules to obtain information about the expectation values of neutron and proton angular momenta, and spins of states in neighboring nuclei. The analysis provides estimates of the renormalization required for the various data sets, and of the errors in relative spectroscopic strengths. The angular momentum expectation values are compared with values calculated from measured magnetic moments and β-decay ft values, and used in an attempt to deduce information concerning exchange-current corrections to nucleon g factors and the β-decay axial vector coupling constant. For mirror pairs with only one measured magnetic moment, the particle transfer data are used to predict the unknown moment. Occupation numbers for the $d_{5/2}$, $s_{1/2}$, and $d_{3/2}$ subshells are deduced from the renormalized data sets.

NUCLEAR STRUCTURE $T=\frac{1}{2}$ mirror nuclei, $2s$-$1d$ shell, sum rule analysis, single particle transfer data. Calculated proton, neutron J; μ, ft value; exchange current corrections. Calculated proton, neutron J; J^{π}; occupation numbers; unmeasured μ.

I. INTRODUCTION

In principle, sum rules applied to empirical data from single-particle transfer reactions (SPTR) can give a wealth of information about nuclear structure. Following the pioneering work of French and McFarlane,[1] Clement[2–4] and Clement and Perez[5–7] have developed the theory of these sum rules, and have applied them to a number of specific nuclei. Despite these demonstrations of their efficacy, the sum rules have largely been ignored in the analysis of experimental work. The only one commonly used is the relationship between spectroscopic strength and the number of particles or holes in the target nucleus.

The present paper provides further illustration of the large amount of information available from single particle transfer data by reporting some results of an extensive analysis[8] for the $2s$-$1d$ shell $T=\frac{1}{2}$ mirror nuclei. Interest is focused on the angular momentum sum rules (AMSR) for the proton and neutron total angular momentum $\underline{J}_p$ and $\underline{J}_n$. To overcome difficulties associated with incomplete or inconsistent data, use is also made of partial non-energy-weighted angular momentum sum rules (PSR). The analysis allows a large number of spin assignments to states in the neighboring nuclei to be made with varying degrees of confidence, and also provides estimates of the errors in relative spectroscopic strengths for the various data sets available.

The expectation values of $\underline{J}_p$ and $\underline{J}_n$ resulting from the sum rule analysis can be compared with values deduced from the magnetic moments of the mirror pairs and the ft value of the β decay linking them. Since the latter depend on the nucleon g factors and β-decay axial vector coupling constant, comparison of the two sets of values can, in principle, give information about exchange-current corrections. One of the objectives of the present calculations was to determine whether a sum-rule analysis of currently-available SPTR data is sufficiently sensitive to provide evidence for such corrections. In cases where only one of the two magnetic moments has been measured, the expectation value of $\underline{J}_p$ or $\underline{J}_n$ deduced from the SPTR data is used to provide an estimate of the unknown moment. Occupation numbers for the $d_{5/2}$, $s_{1/2}$, and $d_{3/2}$ subshells are deduced from the renormalized data sets.

The sum rules, and methods used for applying them, are described in Sec. II. Results are presented in Sec. III.

II. CALCULATIONS

A. Data renormalization and spin assignments

Although the relative spectroscopic strengths as measured in a particular SPTR are generally regarded as having small errors, the absolute strengths are subject to much larger uncertainties. Before using empirical data in any sum rule which requires absolute strengths, it is therefore essential to have some method of renormalizing the data. The procedure followed for the present calculations made use of the PSR derived by Clement[3,4] and used by Clement and Perez.[5] The derivation of these partial sum rules, and of the AMSR discussed below, is straightforward and only their final forms will be given here.

For transfer of a single particle with quantum numbers nlj, here labeled simply by j, the partial sum of empirical spectroscopic strengths to bound states with a particular spin is

$$S_{J_\alpha}^-(j)=\sum_{\substack{\alpha\\ J_\alpha \text{ fixed}}} C^2 S_{\alpha,R}(j) \tag{1}$$

for pickup, and

$$S_{J_m}^+(j)=\frac{(2J_m+1)}{(2J_R+1)} \times \sum_{\substack{m\\ J_m \text{ fixed}}} C^2 S_{R,m}(j) \tag{2}$$

for stripping. Here, R and J_R refer to the A-particle mirror nucleus, α and J_α refer to the $(A-1)$-particle nucleus, and m and J_m refer to the $(A+1)$-particle nucleus. In addition to S^+ there is in stripping a strength contribution $S_{J_m}^c(j)$ to continuum states which is generally unobserved in a stripping experiment.

The sum rule for total strength gives the relation

$$n^+(j)\sum_{J_m} S_{J_m}^+(j)+\sum_{J_m} S_{J_m}^c(j)+n^-(j)\sum_{J_\alpha} S_{J_\alpha}^-(j)=(2j+1)(1+2/A)\,, \tag{3}$$

where $n^+(j)$ and $n^-(j)$ are renormalization factors required for the stripping and pickup data, and the factor $(1+2/A)$ is the center-of-mass correction for $2s$-$1d$ shell nuclei discussed by Clement.[3,4]

The PSR imply the vanishing for each J_m of

$$Q_{J_m}(j)=n^+(j)S_{J_m}^+(j)+S_{J_m}^c(j) +(2J_m+1)\left[\sum_{J_\alpha}\begin{Bmatrix} J_\alpha & j & J_R\\ J_m & j & J_R\end{Bmatrix} n^-(j)S_{J_\alpha}^-(j)-\frac{1}{(2J_R+1)}(1+2/A)\right] \tag{4}$$

if the spectroscopic factors are exact. If the Q_{J_m} are nonzero, they imply a proportional error $\sigma(j)$ in the relative spectroscopic strengths given by

$$\sigma^2(j)=\sum_{J_m} Q_{J_m}{}^2(j)\Bigg/\left[\sum_{J_m} n^+(j)^2 S_{J_m}^+(j)^2+\sum_{J_m} S_{J_m}^c(j)^2+(2J_m+1)\sum_{J_\alpha}\frac{n^-(j)^2 S_{J_\alpha}^-(j)^2}{(2J_\alpha+1)}\right]. \tag{5}$$

The procedure followed for the present calculations, given particular sets of proton or neutron stripping and pickup data, was to determine those renormalization factors $n^+(j)$ and $n^-(j)$ satisfying Eq. (3) which minimized the values of $\sigma(j)$, and hence implied errors in relative spectroscopic strengths which were as small as possible. The continuum contributions were assumed to be negligible after test calculations[8] performed for ^{39}K and ^{35}Cl confirmed the conclusions of Clement and Perez[3,5,6] that only with very restrictive assumptions about the spin dependence of $S_{J_m}^c$ can more than about 10% of the stripping strength be in the continuum without giving unreasonably large proportional errors $\sigma(j)$.

Because the intent here was only to investigate sum rule behavior, spectroscopic data from various sources were used as presented without examination of the details of its calculation. Recent representative data that use a standard method of analysis and seem to see most of the expected levels are used. The data may not be the most recent as calculations were redone when further data became available only if significant changes were likely. Whenever possible, calculations for both the neutron and proton stripping data were done using analog data renormalized to the first two or three $T=1$ levels to correct the problem of incomplete $T=1$ data.

The magnitude of the minimum σ was used as the criterion for making spin assignments to levels of unknown spin strongly excited in the neighboring nuclei. Usually σ could be found with sufficient accuracy by allowing either n^+ or n^- to range from 0.5 to 1.5 with an increment of 0.1 in calculations. Occasionally, calculations with an extended range or finer grid were necessary, but only rarely was there not a minimum value for σ at a reasonable renormalization. For a range of possible assignments to strong levels, the minimum σ given by a particular data set could vary by 0.5 or more. Changes in spin assignments to weak levels usually had little effect. The strength of weak unassigned levels was summed and split equally among the possible spins, unless the summed strength was considerable. In that case, unequal distribution of the summed strength among the possible spins was considered. Changes in σ due to using differing data sets were usually less than 0.2.

In practice, spins (and data sets) giving a minimum σ of greater than 0.1 were rejected. Occasionally further selection among the data sets satisfying this criterion was made if there was a large difference in the minimum σ, or if the renormalization corresponding to the minimum gave very unreasonable total pickup or stripping strength. This use of the renormalizations associated with the minimum value of σ to calculate the renormalized spectroscopic strengths differs somewhat from the approach of Clement *et al.*[5–7] They propose that the renormalization should only be constrained within a region given by a cutoff value of σ (0.1). However, use of a single renormalization is more convenient here, and it is acceptable if it is realized that the resulting spectroscopic strengths, although more correct for being renormalized, are only determined within the range of values bounded by σ.

An unsatisfactory feature of the calculations was that with current data it was necessary to assume that $l=2$ transfer is either pure $d_{3/2}$ or pure $d_{5/2}$. Usually, shell-model considerations allowed the most probable j to be assigned, but when a strong level was involved with the dominant j doubtful, the calculations were carried out with a $d_{5/2}$ assignment and repeated with a $d_{3/2}$ assignment. If not given along with the data, the ratio of $d_{5/2}$ to $d_{3/2}$ single particle cross sections required to correct tabulated strengths was determined by using the appropriate optical-model parameters in a DWBA calculation.[9] It is possible that with otherwise complete sets of experimental data, when all spins of significantly-excited states in neighboring nuclei are known, the PSR could be used to obtain estimates of j mixing for the strongest levels. With current data, however, this was found to be an impossible task in almost all cases.

The generally good results obtained with the assumption of negligible j mixing appear to indicate that mixing is, in fact, small in most cases. This is supported by vector polarization data for the $2s$-$1d$ shell. The measurements of Sanderson *et al.*[10] for the ^{23}Na($\vec{d}$,^{3}He)^{22}Ne (1.27 MeV 2^+) reaction and ^{27}Al($\vec{d}$,^{3}He)^{28}Si (g.s. and 2^+) reaction indicate that possible $d_{3/2}$ admixture into the $d_{5/2}$ levels is small. Similarly, the work of Berg[11] for the ^{37}Cl($\vec{d}$,t)^{36}Cl reaction indicates there are no arguments for $d_{3/2}$ and $d_{5/2}$ admixtures. Although the situation in the $1f$-$2p$ shell may be different, it is perhaps significant that when Kocher and Haeberli[12] measured vector analyzing powers for the ^{53}Cr($\vec{d}$,p)^{54}Cr and ^{57}Fe($\vec{d}$,p)^{58}Fe reactions, they also found that a single j value contributed to the great majority of ($l=1$ or $l=3$) transitions.

B. Neutron and proton angular momenta

After the PSR had been used to renormalize the stripping and pickup data sets, and to assign spins in the neighboring nuclei, the renormalized strengths were used in the AMSR to determine expectation values of neutron and proton angular momenta in the mirror nuclei ground states. Denoting the reduced matrix element of $\underline{J}_n$ in the neutron-rich mirror nucleus by $\langle J_n \rangle$, the AMSR using renormalized neutron pickup strengths is

$$\langle J_n \rangle = \sum_{j,J_\alpha} \frac{(2J_R+1)}{2} \frac{[j(j+1)+J_R(J_R+1)-J_\alpha(J_\alpha+1)]}{[J_R(J_R+1)(2J_R+1)]^{1/2}} n^-(j) S^-_{J_\alpha}(j)$$

$$\equiv \sum_{j,J_\alpha} A_{J_\alpha}(j) n^-(j) S^-_{J_\alpha}(j) , \qquad (6)$$

TABLE I. $\langle J_n \rangle$ for selected $T=\frac{1}{2}$ mirror nuclei in the 2*s*-1*d* shell.

	$\langle J_n \rangle$			
A	Pickup[a]	Stripping[b]	One body[c]	Rel+ 1-π[d]
17	+8.90(49)	+7.54(55)	+7.95(06)	+7.46(06)
19	−1.06(02)	−1.05(04)	−1.10(05)	−0.82(04)
25	+11.40(94)	+9.81(69)	+13.47(06)	+11.87(05)
29	+1.29(06)	+1.45(02)	+1.58(05)	+1.50(04)
31	+0.83(43)	+0.07(13)	−0.32(08)	−0.13(06)
35	+1.02(29)	+0.32(03)	+0.66(05)	+0.95(04)
39	+0.66(61)		+0.59(06)	+0.92(05)

[a]Calculated using the AMSR and neutron pickup data (Ref. 8).
[b]Calculated using the AMSR and neutron stripping data (Ref. 8).
[c]Calculated using magnetic moments and ft values (Ref. 11) without exchange current corrections to the magnetic moments.
[d]Calculated using magnetic moments and ft values (Ref. 17) with exchange current corrections included.

while the analogous sum rule using neutron stripping strengths is

$$\langle J_n \rangle = \sum_{j,J_m} \frac{(2J_R+1)}{2} \frac{[j(j+1)+J_R(J_R+1)-J_m(J_m+1)]}{[J_R(J_R+1)(2J_R+1)]^{1/2}} n^+(j) S^+_{J_m}(j)$$

$$\equiv \sum_{j,J_m} B_{J_m}(j) n^+(j) S^+_{J_m}(j) \ . \tag{7}$$

Similar expressions using the renormalized proton pickup or stripping strengths give $\langle J_p \rangle$.

The contributions to $\langle J_n \rangle$ or $\langle J_p \rangle$ from filled shells sum to zero. For the present analysis, only 2*s*-1*d* shell transfers were considered. Relative errors $\sigma(j)$ from the PSR were used to provide estimates of the uncertainties σ_J in $\langle J_n \rangle$ or $\langle J_p \rangle$. Since σ is a variance,

$$\sigma_J{}^2 = \sum_{j,J_\alpha} [A_{J_\alpha}(j)\sigma(j) S^-_{J_\alpha}(j)]^2 \tag{8}$$

or

$$\sigma_J{}^2 = \sum_{j,J_m} [B_{J_m}(j)\sigma(j) S^+_{J_m}(j)]^2 \ . \tag{9}$$

Independent estimates of $\langle J_n \rangle$ and $\langle J_p \rangle$ were obtained from the magnetic moments of the mirror pairs and the ft value of the β decay connecting them. In terms of orbital and spin angular momenta, the isovector part of the magnetic moment is

TABLE II. $\langle J_p \rangle$ for selected $T=\frac{1}{2}$ mirror nuclei in the 2*s*-1*d* shell.

	$\langle J_p \rangle$			
A	Pickup[a]	Stripping[b]	One body[c]	Rel+ 1-π[d]
17	0.0	−0.45(208)	−0.70(06)	−0.21(06)
19	+2.05(04)	+2.28(19)	+2.32(05)	+2.04(04)
21	+2.81(38)	+1.14(69)	+0.47(09)	+0.58(07)
29	−0.31(03)	−0.07(22)	−0.35(05)	−0.27(04)
31	+1.10(54)	+1.25(23)	+1.54(08)	+1.35(06)
35	+3.48(23)	+3.37(18)	+3.21(05)	+2.92(04)
39	+2.91(19)	+3.49(11)	+3.28(06)	+2.96(05)

[a]Calculated using the AMSR and proton pickup data (Ref. 8).
[b]Calculated using the AMSR and proton stripping data (Ref. 8).
[c]Calculated using magnetic moments and ft values (Ref. 17) without exchange current corrections to the magnetic moments.
[d]Calculated using magnetic moments and ft values (Ref. 17) with exchange current corrections included.

$$\underline{\mu}^{(1)}=\Delta g_l(\underline{L}_p-\underline{L}_n)+\Delta g_s(\underline{S}_p-\underline{S}_n)$$
$$=2\Delta g_l\underline{J}_p+(\Delta g_s-\Delta g_l)(\underline{S}_p-\underline{S}_n)$$
$$-\Delta g_l\underline{J}\ , \qquad (10)$$

where $2\Delta g_l=g_{l_p}-g_{l_n}$ and $2\Delta g_s=g_{s_p}-g_{s_n}$ are differences of orbital and spin gyromagnetic ratios for protons and neutrons. The isospin convention that $t_z=+\frac{1}{2}$ for neutrons is used.

Since

$$\langle\mu^{(1)}\rangle=\left[\frac{(2J_R+1)(J_R+1)}{J_R}\right]^{1/2}\Delta\mu\ , \qquad (11)$$

where $\Delta\mu$ is one-half the difference between the magnetic moments of the neutron-rich and proton-rich mirror nuclei, and

$$|\langle S_p-S_n\rangle|^2=\frac{(2J_R+1)}{4}|M_{\mathrm{GT}}|^2\ , \qquad (12)$$

where M_{GT} is the Gamow-Teller matrix element for the β decay, $\langle J_p\rangle$ and $\langle J_n\rangle=\langle J\rangle-\langle J_p\rangle$ can be evaluated as a function of Δg_l, Δg_s, and the axial-vector coupling strength. The vector and axial vector coupling constants of Kropf and Paul[13] were used, as the use of more recent values[14] had no significant effect on calculations. The sign of $\langle S_n-S_p\rangle$ is not determined by the ft value, so there are two solutions for $\langle J_p\rangle$ and $\langle J_n\rangle$. In practice, a sign selection was made by comparison with the values obtained independently in the AMSR calculation.

Mesonic exchange-current effects in nuclei are expected to modify both the nucleon gyromagnetic ratios and the effective axial-vector coupling strength. To test the sensitivity of the results to these effects, $\langle J_n\rangle$ and $\langle J_p\rangle$ were calculated both with the free-nucleon values and with modified parameters. Following the simple prescription of Miyazawa[15] that the main mesonic effect is to modify g_l by about $+0.1\tau_3$, Δg_l was changed from 0.5 to 0.6. The axial vector coupling strength was renormalized by a variable factor $\rho=G_{Ae}/G_A$ to allow for the mesonic correction. In addition, the relativistic correction to the magnetic moments and to the axial vector coupling strength as calculated by Ohtsubo[16] were considered, but they were found only to be significant for the magnetic moment.

TABLE III. $\rho=G_{Ae}/G_A$ for selected $T=\frac{1}{2}$ mirror nuclei in the $2s$-$1d$ shell.

A	Proton data[a]	ρ Neutron data[b]	Combined
17	1.01 ± 0.29	1.26 ± 0.22	1.14 ± 0.25
19	1.04 ± 0.06	1.08 ± 0.03	1.06 ± 0.04
21	0.63 ± 0.15		0.63 ± 0.15
25		$2.7\ \pm2.8$	2.70 ± 2.79
29	0.94 ± 0.14	0.89 ± 0.07	0.91 ± 0.10
31	0.83 ± 0.37	0.64 ± 0.16	0.74 ± 0.26
35	0.63 ± 0.13	0.83 ± 0.26	0.73 ± 0.19
39	0.90 ± 0.11	0.87 ± 0.29	0.89 ± 0.17

[a]Using the data from Table II.
[b]Using the data from Table I.

III. RESULTS

There are 12 $T=\frac{1}{2}$ mirror pairs in the $2s$-$1d$ shell. The sum rules cannot be applied to $A=37$, because ^{37}Ar is unstable and there are no SPTR data. In addition, it was found that incompleteness of current data (mainly the large number of unassigned spins) made it impossible to do anything other than restrict the value of $\langle J_n\rangle$ to within wide limits for $A=21$ and 23, while $\langle J_p\rangle$ could not be obtained for $A=23$ and 25 because it was not possible to carry out the PSR calculations. For $A=23$,

TABLE IV. Predicted values for selected magnetic moments.

Nucleus	$\mu(\mu_N)$ One body[a]	Rel + 1-π[b]	$\langle J_p\rangle$	$\langle J_n\rangle$
^{27}Si	-1.26 ± 0.81[c]	-1.80 ± 0.97[c]		-0.63 ± 1.13[c]
	or	or		or
	-1.32 ± 0.38	-1.87 ± 0.45		-0.71 ± 0.50
^{33}S	$+1.08\pm0.19$	$+1.27\pm0.22$	$+0.36\pm0.22$[d]	

[a]Calculated without exchange current corrections to the magnetic moment.
[b]Calculated with exchange current corrections to the magnetic moment.
[c]Two values exist because both neutron pickup and stripping data (Ref. 8) are used. Both are quoted, instead of the average, because of the remarkable consistency in the resulting magnetic moments.
[d]Proton pickup (Ref. 8).

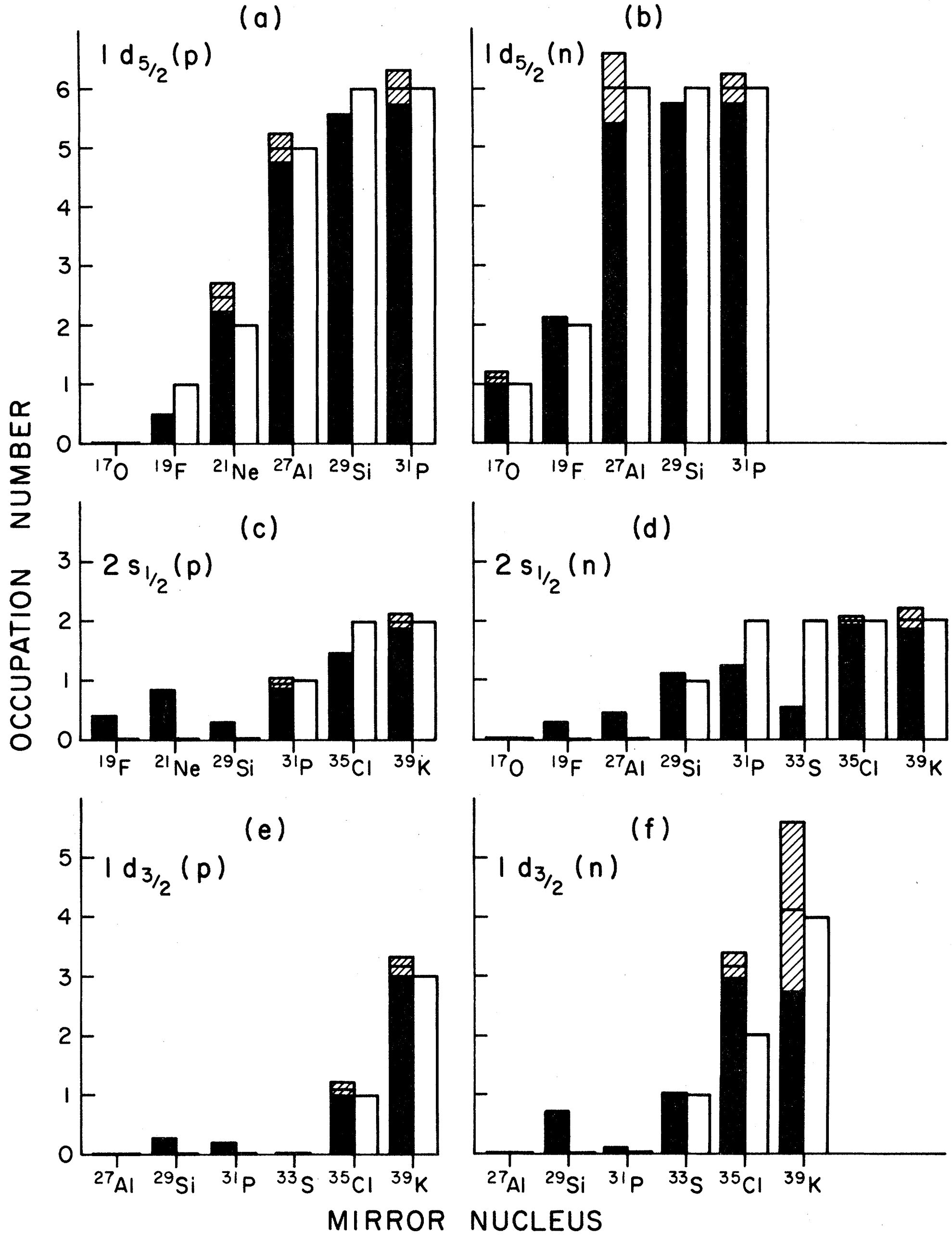

FIG. 1. Occupation numbers for the $1d_{5/2}$, $2s_{1/2}$, and $1d_{3/2}$ subshells. The white bars represent the independent particle shell model estimate. The black bars represent the values calculated from data. The crosshatching represents the uncertainty, if any, in the calculated values.

27, and 33 the magnetic moment of the β-unstable nucleus has not been measured, therefore only $\langle J_p \rangle$ and $\langle J_n \rangle$ from sum rule calculations were available.

Results of the calculations for $\langle J_p \rangle$ and $\langle J_n \rangle$ for the remaining cases where there are two independent sets of values for $\langle J_n \rangle$ and $\langle J_p \rangle$ are given in Tables I and II. Values of $\langle J_n \rangle$ and $\langle J_p \rangle$ calculated from the sum rules are compared with values deduced from magnetic moments and ft values. The latter are calculated (a) with free nucleon parameters, and (b) with the relativistic corrections plus modifications of Δg_l as discussed in Sec. II. The relativistic corrections are small, being of the order ± 0.1.

As the tables indicate, the effects of the change in Δg_l are smaller than, or comparable to, the uncer-

TABLE V. Spin assignments for selected levels.

Nucleus	E_x (MeV)	J^π, T	SPTR	l	Spin assignment
^{20}F	2.195	$(1-4)^+,1$	p.pu.[a]	2	4^+
	5.46	$(2,3)^+,1$	n.str.[b]	2	3^+
^{22}Na	1.937d	$(1^+,2^+),0,1$	p.str.[c]	$0+2$	50% 1^+, 50% 2^+
	6.18	$(0-4)^+,0$	p.str.	2	4^+
^{22}Ne	8.85	$(0-4)^+,1$	n.str.	2	4^+
^{26}Mg	4.33t	$(4^+,2^+,3^+),1$	p.pu.	2	4^+
	7.28	$(0-5)^+,1$	p.pu.	2	2^+
^{28}Si	11.42	$(2,3)^+,1$	p.str.	0	2^+
^{30}Si	4.81d	$(2^+,3^+),1$	p.pu.	2	3^+
^{34}S	4.07, 4.12d	$(1,2)^+$	p.pu.	0	90% 1^+, 10% 2^+
^{34}Cl	3.33	$(1-3)^+,0$	n.pu.[d]	2	2^+
^{38}Ar	5.55	$(1,2)^+,1$	p.pu.	0	1^+
	7.14	$(1-4)^+,1$	p.pu.	2	3^+
^{39}K	5.85	$(1,2)^+,0$	n.pu.	$0+2$	1^+

[a]Proton pickup.
[b]Neutron stripping.
[c]Proton stripping.
[d]Neutron pickup.

tainties in the AMSR calculations. It is therefore not possible from the present calculations to obtain definitive information about the magnitude of mesonic corrections. However, a statistical analysis of the results is quite suggestive. Angular momentum values $\langle J_n \rangle_1$ and $\langle J_p \rangle_1$ obtained from the AMSR were compared with values $\langle J_n \rangle_2$ and $\langle J_p \rangle_2$ deduced from magnetic moments and ft values using simple linear regression. If

$$\langle J_\tau \rangle_1 = c + d \langle J_\tau \rangle_2$$

perfect agreement would give $c=0$, $d=1$, and a correlation coefficient $R^2=1$. For both sets (a) and (b) the correlation is high, giving $R^2>0.95$, but whereas c and d with free-nucleon parameters have the unsatisfactory values of 0.30 and 0.83, with the modifications they improve to 0.13 and 0.95.

Table III lists the values of $\rho = G_{Ae}/G_A$ required to bring the $\langle J_n \rangle$ and $\langle J_p \rangle$ values from parameter set (b) into agreement with the AMSR values. The sum rule values used were the average of those from pickup and stripping. Raman *et al.*[18] also have obtained estimates of the effective axial-vector coupling strength for some of these nuclei, using $\langle S_n + S_p \rangle$ deduced from the isoscalar magnetic moments, $G_{Ae}\langle S_n - S_p \rangle$ deduced from ft values, and values for the relatively small quantity

$$\langle S_n + S_p \rangle \pm \langle S_n - S_p \rangle$$

calculated from shell-model wave functions including core-polarization corrections. Their value for ρ, which in principle should be true mesonic renormalizations free of core polarization effects, are not in good agreement with the present results for $A=17$, 19, 35, and 39. In particular, whereas they deduced a value of $\rho = 1.29 \pm 0.05$ for $A=35$, Table III suggests that ρ should be significantly less than unity. Restricting attention to $2s$-$1d$ shell transfers for the AMSR undoubtedly means that some effects of core polarization are being ignored in the present type of analysis, but the effect on $\langle J_n \rangle$ and $\langle J_p \rangle$ is difficult to gauge.

For two of the three mass numbers having only one measured magnetic moment, it was possible to obtain fairly well-defined values of $\langle J_n \rangle$ or $\langle J_p \rangle$ from the sum-rule analysis, and hence to predict the unknown moment. The predictions using parameter sets (a) and (b) are shown in Table IV, together with the angular momentum values resulting from the AMSR. As can be seen from the table, the calculation of $\mu(^{27}Si)$ was quite sensitive to the exchange current corrections. In addition, the consistency of the pickup and stripping values is good, despite the large uncertainties in the calculation. Therefore measurement of this moment in conjunction with improved SPTR data could provide an interesting test of exchange current corrections.

Occupation numbers for the $d_{5/2}$, $s_{1/2}$, and $d_{3/2}$ subshells, calculated from data sets renormalized by the PSR analysis, are compared with independent-particle shell model (IPSM) estimates in Fig. 1. To facilitate the comparison, the factor $(1+2/A)$ is removed from the renormalized data to give an integral number of particles. Although the sum rules used imposed a restriction on the total number of particles plus holes in each subshell, there was no restriction on the total number of particles within the major shell.

As is seen by examining the figure, there is little evidence for large deviations from the occupancies expected in the IPSM. At the same time, there does not appear to be complete closure of the subshells. Both the $2s_{1/2}$ and $1d_{3/2}$ subshells begin filling at a lower mass than expected in the IPSM, while the $1d_{5/2}$ and $2s_{1/2}$ subshells remain unfilled at or above the masses where closure is expected in the IPSM.

Spin assignments were made with varying degrees of confidence for over 100 levels[8] on the basis of the combined PSR and AMSR analysis. The use of two sum rule analyses increased the number of assignments possible by more than 50%, when compared to the assignments possible on the basis of the PSR analysis alone. Assignments became available[19] for 51 of these levels after the calculations presented here were completed. The resulting agreement for more than 90% of these levels provides further support for the use of sum rule techniques as a method for assigning the spins of energy levels. Some typical assignments which could be made with considerable confidence, but were unassigned,[19] are given in Table V.

IV. CONCLUSION

Further evidence of the great power of sum rule techniques for providing valuable information on nuclear structure from SPTR data has been provided. It is unfortunate that existing data were not sufficiently complete for definite conclusions about mesonic exchange current corrections or $l=2$ mixing. However, the usefulness of combining the j-dependent TSR and PSR with the J-dependent AMSR is well demonstrated.

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*Present address: Reactor Control Branch, Atomic Energy of Canada Limited, Research Company, Chalk River Nuclear Laboratories, Chalk River, Ontario, Canada KOJ 1J0.

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