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# Spin observables and nuclear geometry

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The new measurements of polarization $P$ and spin rotation function $Q$ in 500 MeV $p$-$^{40}$Ca elastic scattering require geometric differences among the various parts of the $p$-nucleus interaction. We present a general analytic formalism which defines these differences and shows how the new experiments can be interpreted as interferometric determinations of them.

[NUCLEAR REACTIONS Closed form spin-dependent $p$-nucleus scattering amplitudes. Data-to-data relations for polarization and spin rotation.]

## I. INTRODUCTION

The connections between the structures observed in the angular distributions of polarizations and cross sections have long been an object of study. In this work we wish to continue the exploration begun in Ref. 1. [Amado, McNeil, and Sparrow (AMS)] using the analytic methods of Ref. 2 [Amado, Dedonder, and Lenz (ADL)]. In ADL it is shown that for typical nuclear densities, intermediate energy scattering processes are dominated by the nuclear edge. In particular, the singularity (normally a pole) of the effective interaction density associated with the surface dominates the momentum transfer, $q$, dependence. It is primarily this singularity position which characterizes the elastic scattering. Having an analytic form for the amplitudes with isolatable $q$ dependence allows one to derive connections between the polarization and cross section angular distributions. Since the relations are between observables, they are easily converted from theoretical to empirical or "data-to-data" relations, and it is in this form that they have their greatest utility.

In AMS, and other works, it is shown that for identical central and spin-orbit geometries,[3] both the polarization $P$ and spin rotation function $Q$ (Ref. 4) are linear in momentum transfer to first order in the spin-orbit strength. This minimal theory predicts that $P+iQ$ is a simple function of only the ratio of spin orbit to central strengths ($w$) and is independent of the nuclear shape

$$P+iQ=2iqw^* \ .$$

This result means that $P$ and $Q$ are structureless, or, in our data-to-data language, independent of the cross section $\sigma$, in striking disagreement to the data.

If the central and spin orbit interactions are independent, the simplest analytic generalization would be for them to have different pole positions in their interaction densities

$$b_{\text{central}}=b_0 \ ,$$

$$b_{\text{spin}}=b_0+\delta \ ,$$

where $b_0$ is the position of the nearest singularity in the central $r$-space density. In this case we find

$$P=P(w,\delta,\sigma)$$

and the data-to-data aspect of the relations becomes nontrivial.[5]

The differences in geometry introduce structures in $P$, which are constructable from the cross section $\sigma$. Very roughly these structures are the log derivative and reciprocal of the cross section. The additional information in $P$ not contained in $\sigma$ is $w$, characteristic of the spin-orbit to central strength ratio, and $\delta$ characteristic of the spin-orbit versus central geometry difference. This minimal generalization of the equal geometry case is remarkably successful at describing elastic and inelastic polarizations.[1,6,7] For a recent application see Baker *et al.*[8] In this most simple case structure is introduced in $Q$ as well as $P$, but the structures in $P$ and $Q$ are not independent; in fact $P$ and $\sigma$ together can be used to determine $Q$. This, of course, is not a general result—in principle, $\sigma$, $P$, and $Q$ are independent; however, with only these two lengths present in the scattering, they are not. AMS concluded by predicting $Q$ in terms of $P$ and $\sigma$ for 800 MeV $p$-$^{208}$Pb scattering.

 

Recently $\sigma$, $P$, and $Q$ have been measured for 500 MeV $p$-$^{40}$Ca scattering and the results are in qualitative disagreement with the relation of $P$, $Q$, and $\sigma$ implied by AMS. There is additional information in the $Q$ measurements. This implies the existence of an additional dynamical term, in either the central or the spin-orbit interaction or both. In Sec. II we consider both for generality. This additional term must have an independent (complex) strength and an independent (complex) effective geometry. (A second term with a new strength but the same geometry is simply a parameter change which will not suffice.) Unfortunately, although a new strength and range are both required, they cannot be cleanly separated by the data, and only two combinations are determined which are two lengths, $\lambda_1$ and $\lambda_2$ replacing $\delta$. In terms of these lengths we may write

$$P = P(w, \lambda_1, \lambda_2, \sigma) ,$$

$$Q = Q(w, \lambda_1, \lambda_2, \sigma) .$$

In other words, measurement of both $P$ and $Q$ at 500 MeV determines three complex lengths in addition to the information already contained in the cross section. We wish to stress that the quantities $w$, $\lambda_1$, and $\lambda_2$ determined via the data-to-data approach characterize the data and are the fundamental phenomenological quantities. By using these data-to-data relations, we isolate the new information in $P$ and $Q$ over and above that contained in $\sigma$. The relations show how $P$ and $Q$ determine these two new lengths, and nothing else. These lengths play, for polarization, a role analogous to effective range and scattering length in low energy scattering. It is, at least initially, surprising that the data may be so simply characterized over the intermediate momentum transfer region. The possibility of further information being contained at either very low or very high momentum transfer certainly warrants further attention.

In Sec. II we show how these lengths may be related to terms in a potential model of the interaction or a conventional multiple scattering treatment. In Sec. III we extract these from the 500 MeV data and relate them to two models. Our conclusions are presented in Sec. IV.

## II. GEOMETRIC MODEL OF SPIN-DEPENDENT SCATTERING

For completeness we review our previous analytic treatment of spin-dependent scattering before generalizing it to include two central and spin-orbit dynamics. The elastic scattering of a proton from a spin zero target nucleus may be written as

$$F = F_1 + \vec{\sigma} \cdot \hat{n} F_2 , \tag{1}$$

where $\vec{\sigma}$ is the projectile (proton) spin operator and $\hat{n}$ is a unit vector normal to the scattering plane. Expressed in these amplitudes the polarization $P$, spin rotation function $Q$, and unpolarized cross section are given by

$$\sigma = |F_1|^2 + |F_2|^2, \quad P + iQ = 2F_1 F_2^* / \sigma . \tag{2}$$

The general expressions for $F_1$ and $F_2$ in the Fourier-Bessel representation are

$$\begin{aligned} F_1 &= ik \int_0^\infty db\, b J_0(qb) \\ &\quad \times [1 - e^{i\chi_c(b)} \cosh \chi_s'(b)] , \\ F_2 &= -k \int_0^\infty db\, b J_1(qb) e^{i\chi_c(b)} \sinh \chi_s'(b) , \end{aligned} \tag{3}$$

where $k$ is the incident wave number, $q$ the momentum transfer, and primes denote differentiation with respect to $b$. In the eikonal approximation in the "short range" limit we write

$$\begin{aligned} i\chi_c(b) &\simeq -\gamma \int_{-\infty}^{\infty} dz\, \rho_c(b,z) \equiv -\gamma t_c(b) , \\ \chi_s(b) &\simeq w\gamma \int_{-\infty}^{\infty} dz\, \rho_s(b,q) \equiv w\gamma t_s(b) , \end{aligned} \tag{4}$$

where the subscript on the thickness function $t(b)$ allows for differences between the central and spin orbit geometries not normally associated with the short range limit, but which are essential to understanding spin dependent phenomena.[3,9] There is, of course, only one nuclear matter density. Central to understanding the geometric origins of spin dependent observables is the realization that *whatever* processes go into making up the central and spin orbit optical potentials, there is no reason to expect them to have exactly the same shape as the matter density. For example, differences in the ranges of the central and spin orbit fundamental interactions give rise to different central and spin orbit geometries. The phenomenological fact that such range differences alone do not quantitatively give the correct effective geometries indicates that this microscopic interpretation may be wanting; fortunately this is not an impediment to understanding the geometric character of spin dependent phenomena.

The central and spin orbit strengths $\gamma$ and $w$ are taken from the isospin averaged forward nucleon-nucleon amplitude, $t_{nn}$

$$t_{nn}(q)=A(q)+iq\vec{\sigma}\cdot\hat{n}C(q)\ ,$$
$$\gamma=-i\frac{2\pi}{k}A(0)\ , \quad (5)$$
$$w=C(0)/A(0)\ .$$

Extracting the threshold $q$ dependence from the spin orbit term guarantees that our results will have the correct threshold behavior and coincidently gives $w$ the dimensions of length. Our results are, strictly speaking, only valid for large $q$. However, explicit inclusion of the threshold $q$ dependence in the context of a normalized ($|P|^2\leq 1$) data-to-data expression seems quite successful for the lowest $q$ values yet studied. In principle, therefore, there could be further information in the low-$q$ and high-$q$ regions of $P$ (or of $Q$); however, this seems to us somewhat unlikely.

Since in practice $w$ is small compared to the nuclear radius (the dominant characteristic length), all the essential features of $P$ and $Q$ emerge by treating it to first order. Thus we have

$$F_1\simeq -ik\int_0^\infty db\, bJ_0(qb)e^{-\gamma t_c(b)}$$
$$F_2\simeq -k\int_0^\infty db\, bJ_1(qb)e^{-\gamma t_c(b)}w\gamma t_s'(b)\ , \quad (6)$$

where we have dropped the 1 in (3) as it contributes only at $q=0$.

To go further we require some model for $\rho_c$ and $\rho_s$. We consider this task in varying degrees of generality. The simplest situation we can have is the same geometry for both the central and spin-orbit thickness functions, $t_c=t_s$. We then find after integration by parts and use of a Bessel function identity

$$F_2=-iqwF_1\rightarrow P+iQ\simeq 2iqw^*\ . \quad (7)$$

This is the structureless linear rise mentioned earlier. This result is quite general, independent of the functional form chosen, but true to first order in $w$ only (note that corrections lead to terms third order in $w$ since $P$ and $Q$ must be odd in $w$).

To retrieve the empirical structure in $P$ and $Q$ we must permit differences in the geometries leading to $\chi_s$ and $\chi_c$. Such differences could arise from range differences, but we stress again that the need for such geometric differences is manifest in the data and our formalism extracts them phenomenologically without recourse to postulating a dynamical origin. To discuss nuclear geometry in hadron-nucleus scattering we use the analytic methods of ADL. The first step is to separate the interfering parts of the scattering amplitude as follows:

$$F_1=-\frac{ik}{2}\int_0^\infty db\, be^{i\chi_c(b)}$$
$$\times[H_0^{(1)}(qb)+H_0^{(1)*}(qb)]$$
$$=-ik[G(q,\gamma)+G^*(q,\gamma^*)]\ , \quad (8)$$

where $H_0(qb)$ is a Hankel function.

The characteristic diffraction pattern of elastic scattering arises from the interference of $G$ and $G^*$. For large $qb$ the integral for $G$ is dominated by the inflection point of the density and can be evaluated by the method of steepest descent. With standard phenomenological forms for the density the integrals are dominated by the nearest singularity of the density $b_0=c+i\pi\beta$, where for the familiar Fermi, $c$ is the half density radius and $\beta$ the diffusivity. (Generalization to other functional forms of similar analytic structure is given in Ref. 10 and discussed in Ref. 11.) In any case, $G$ is given by a slowly varying function of $q$ multiplied by $e^{iqc}e^{-\pi\beta q}$, leading to the familiar oscillation and exponential fall of the cross section. The minimal geometric change we can make to generate structure in $P$ and $Q$ is to shift the singular point of the spin-orbit density function with respect to the central,

$$b_{0,\text{spin}}=b_{0,\text{central}}+\delta\ ,$$

where $\delta$ is in general complex. That is, we shall describe both geometries with the same functional form but with different shape parameters. This shift restores the familiar structure of intermediate energy polarization and gives an excellent description of elastic and collective inelastic polarization phenomena at 800 MeV with $|\delta|$ of the order of $0.1-0.2$ fm$^2$. The polarization measurements determine $\delta$, but cannot ascribe this small difference to any underlying dynamics.

Once $\delta$ is determined by the polarization, the analytic property of the closed form amplitudes fixes $Q$ as well. If the single shape difference (minimal) model were adequate there would be nothing new to be learned by measuring $Q$. What makes the recent $Q$ measurements at 500 MeV particularly interesting is that they clearly show this minimal picture to be inadequate—something new is being measured. What is it and how can we understand it?

Having an analytic form, one can explicitly tract the impact of initial assumptions through to the final result. An inspection of the analytic structure of the spin dependent amplitudes reveals that the new $Q$ measurements require the minimal geometric model to be extended to include at least one more independent dynamical term of different geometry in either the central or spin-orbit terms (or both). As mentioned above, in principle, very low $q$ measurements could contain independent information and therefore require additional independent terms. Similarly the existence of data at larger momentum transfer ($qw>1$) might require the existence of additional independent dynamical terms.

The empirical need for this further geometric difference is clear in the data, as we shall see. Furthermore, our analytic or data-to-data formalism can easily be generalized to include it without recourse to any particular underlying dynamical or microscopic origin for the difference. However, since such an approach is contrary to the customary microscopic multiple scattering starting point and since in any case the purpose of the data-to-data phenomenology is to distill from the data the features that must ultimately find their origin in some microscopic theory, we will introduce those differences in terms of an obvious dynamical candidate for their origin—neutron-proton differences. In this way we will have a model context for the equation and also see how the many parameters of a microscopic model combine to yield in addition to $w$ the two (complex) lengths determined by the data.

Consider treating the neutron and proton contributions to the eikonal phase separately. The different strengths and ranges of the *pn* and *pp* interactions will result in two effective central and spin-orbit terms. Where previously two strengths and two lengths characterized the interaction, we now have four of each—16 real parameters altogether. As we shall see, only certain combinations are accessible from the data and ambiguities must arise in attempting a unique microscopic interpretation of the results. Taking neutrons and protons separately as a prototype microscopic theory for the two dynamics, and assuming with ADL that only the position of the singular point of the density is crucial, we have the following extension:

$$\begin{aligned} t_c(b,b_0) &\to \frac{Z}{A}\gamma_p t(b,b_0+\delta_p) \\ &\quad + \frac{N}{A}\gamma_n t(b,b_0+\delta_n)\,, \\ t_s(b,b_0) &\to \frac{Z}{A}w_p\gamma_p t(b,b_0+\delta'_p) \\ &\quad + \frac{N}{A}w_n\gamma_n t(b,b_0+\delta'_n)\,. \end{aligned} \tag{9}$$

Treating all $\delta$'s to first order we find by Taylor's theorem

$$\begin{aligned} t_c(b,b_0) &\simeq \gamma t\left[b,b_0+\delta+\frac{\Delta\gamma}{\gamma}\epsilon\right], \\ t_s(b,b_0) &\simeq w\gamma t\left[b,b_0+\delta'+\frac{\Delta w}{w}\epsilon'\right], \end{aligned} \tag{10}$$

where

$$\begin{aligned} &\delta=\frac{\delta_p+\delta_n}{2}\,,\quad \epsilon=\frac{\delta_p-\delta_n}{2}\,, \\ &\delta'=\frac{\delta'_p+\delta'_n}{2}\,,\quad \epsilon'=\frac{\delta'_p-\delta'_n}{2}\,, \\ &\gamma=\frac{Z\gamma_p+N\gamma_n}{A}\,,\quad w=\frac{Zw_p\gamma_p+Nw_n\gamma_n}{A\gamma}\,, \\ &\Delta\gamma=\frac{Z\gamma_p-N\gamma_n}{A}\,,\quad \Delta w=\frac{Zw_p\gamma_p-Nw_n\gamma_n}{A\gamma}\,. \end{aligned} \tag{11}$$

We now redefine $b_0$ to include the shift $\delta+\epsilon\delta\gamma/\gamma$ and reexpand to find

$$\begin{aligned} &\gamma t_c(b)=\gamma t(b,b_0)\,, \\ &w\gamma t'_s(b)=(1+\mu)w\gamma t'(b,b_0)\,, \end{aligned} \tag{12}$$

where by using the explicit form (ADL) for the important (singular) part of $t(b,b_0)$ we have

$$\begin{aligned} \mu(\gamma,w,\Delta\gamma,\Delta w,\delta,\delta',\epsilon,\epsilon') &= \frac{-3}{b_0}\left[\delta'-\delta+\epsilon'\frac{\Delta w}{w}-\epsilon\frac{\Delta\gamma}{\gamma}\right] \\ &\quad\times\left[\frac{qb_0}{2\pi\beta\gamma\rho_0}\right]^{2/3} e^{-i(2/3)\pi}\,. \end{aligned} \tag{13}$$

Following AMS, we note that to first order in $w$ the spin-orbit term of (12) simply shifts $b$ by $(1+\mu)w\gamma$. The resulting form is equivalent to that treated in ADL so we find to first order in $w$ the analytic result

$$F_2 \simeq kqw[(1+\mu)G(q,\gamma)+(1+\bar{\mu})G^*(q,\gamma^*)]\,, \tag{14}$$

where

$$\bar{\mu}=\mu^*(\gamma^*,w^*,\Delta\gamma^*,\Delta w^*,\delta,\delta',\epsilon,\epsilon') \tag{15}$$

and $G(q,\gamma)$ is defined in Eq. (7) and given approximately for asymptotic $qc$ by[2]

$$\begin{aligned} &G(q,\gamma)=b_0{}^2\left[\frac{2\pi\beta\gamma\rho_0}{3^{3/2}q^4b_0{}^4}\right]^{1/3}\exp[iqb_0+i\frac{5\pi}{6}+\tfrac{3}{2}(2\pi\beta\gamma\rho_0)^{2/3}(qb_0)^{1/3}e^{i\pi/6}-\gamma\tilde{t}]\,, \\ &\tilde{t}\simeq 1.46\rho_0(2\pi\beta b_0)^{1/2}\,. \end{aligned} \tag{16}$$

Note that the asymptotic dependence is carried in the factor $e^{iqb_0}=e^{-\pi\beta q}e^{iqc}$ which accounts for the oscillating and exponentially falling cross sections.

It is vital to note that $\mu^*\neq\bar{\mu}$, in general. Aside from $\gamma$, $w$, $b_0$, and $\rho_0$, there are 12 real parameters in the specification of $\mu$, but in fact only four combinations have an observable impact. These are conveniently represented by the two complex lengths

$$\lambda_1\equiv(\delta'-\delta+\epsilon'\Delta w/w-\epsilon\Delta\gamma/\gamma)$$

and

$$\lambda_2\equiv[\delta'-\delta+\epsilon'(\Delta w/w)^*-\epsilon(\Delta\gamma/\gamma)^*] .$$

The combination $\delta'-\delta$ is the single $\delta$ of AMS while the complex combination

$$\eta=\epsilon'\Delta w/w-\epsilon\Delta\gamma/\gamma$$

is new. Note how the length and strength factors are entangled in $\eta$. This shows the ambiguities which can arise in trying to fit data by adjusting range and strength factors separately. If these factors are separable at all it would require momentum transfer sufficient to probe the range differences $\epsilon$, $q\epsilon\gg 1$, and they may not be separable even then. To elucidate the structure of $F_2$ define

$$C=\tfrac{1}{2}[G(q,\gamma)+G^*(q,\gamma^*)] ,$$
$$S=\frac{1}{2i}[G(q,\gamma)-G^*(q,\gamma^*)] . \qquad (17)$$

Since $G$ behaves asymptotically as $e^{-i\pi\beta q}e^{iqc}$, $C$ and $S$ are seen to be cosinelike and sinelike, respectively. In terms of these we can write

$$F_1\simeq 2ikC ,$$
$$F_2\simeq 2qwk(\Delta_1 C-\Delta_2 S) , \qquad (18)$$

where

$$\Delta_1=1+(\mu+\bar{\mu})/2$$

and

$$\Delta_2=(\mu-\bar{\mu})/2i .$$

As we found in AMS, the spin observables $P$ and $Q$ will contain three terms each: a linear term in $q$ which persists even when the geometries are equal, and two oscillating terms which only appear for unequal geometries; a term in $\mathrm{Re}CS^*/C^*C$ which yields the familiar tangentlike structure in polarization; and a term in $\mathrm{Im}CS^*/C^*C$ which is new to polarization phenomenology and oscillates like $1/\sigma$. Assuming $\eta=0$, i.e., only one geometry, the three structure terms in $P$ and $Q$ are related in a fashion violated by the new 500 MeV $p$-$^{40}$Ca data. This result is more general than the eikonal or analytic approximations and follows solely from the assumption of one central and one spin-orbit term. We are forced empirically to consider the more general case of $\eta\neq 0$. In terms of the tangent and $1/\sigma$ structure terms we have from (18)

$$P+iQ=2iqw^*(A+iB)/\bar{\sigma} , \qquad (19)$$

with

$$A=\xi_1\cos\varphi_1-\xi_2\left[\frac{\mathrm{Re}CS^*}{CC^*}\cos\varphi_2+\frac{\mathrm{Im}CS^*}{CC^*}\sin\varphi_2\right] ,$$

$$B=-\xi_1\sin\varphi_1+\xi_2\left[\frac{\mathrm{Re}CS^*}{CC^*}\sin\varphi_2-\frac{\mathrm{Im}CS^*}{CC^*}\cos\varphi_2\right] , \qquad (20)$$

$$\bar{\sigma}=1+q^2(w)^2\left[|\Delta_1|^2+\frac{SS^*}{CC^*}|\Delta_2|^2-\frac{2\,\mathrm{Re}(CS^*\Delta_1\Delta_2^*)}{CC^*}\right] ,$$

where $(\xi_i,\varphi_i)$ are the magnitude and phase of $\Delta_i$. The minimal $(\eta=0)$ theory corresponds to $\phi_1=\phi_2=0$. The three polarization structures are still present in Eq. (19); the difference is that $\phi_1$ and $\phi_2$ rotate their weights between $P$ and $Q$.

The forms in Eq. (20) depend only on the existence of three nuclear geometries with relatively complex phases (so that $\bar{\mu}\neq\mu^*$). Equation (20) is far more general than the pedagogic example based on neutron-proton differences used to construct it. In the data-to-data form, which fixes $C$ and $S$ from the elastic data, $P$ and $Q$ determine only the two complex nuclear geometry differences $\Delta_1$ and $\Delta_2$ (or equivalently $\lambda_1$ and $\lambda_2$) and the spin-orbit strength $w$. The $\Delta$'s are determined interferometrically. That is, it is the shift by $\Delta$ in the diffraction structure between the spin channel cross sections that results in the characteristic oscillations in $P$ and $Q$. The data require only that such a shift be present. The microscopic origin of the shift is an important, but separate, question.

In AMS we showed how the tangentlike and $1/\sigma$-like structures can be extracted directly from the unpolarized cross sections yielding a data-to-data formula for the spin observables in terms of the measured unpolarized cross section and the various length and strength factors. Thus Eqs. (19) and (20) can be viewed as a data-to-data formula as well. In the calculations to follow we adopt this data-to-data approach as a means of including automatically processes and corrections left out of the analytic form. Quite similar results emerge, however, if the analytic expression itself is used.

## III. POLARIZATION AND SPIN ROTATION IN 500 MeV $p$-$^{40}$CA SCATTERING

The first measurements of $Q$ were recently made from 500 MeV $p$-$^{40}$Ca scattering.[12] We analyzed the elastic data from two perspectives. First we attempt an *a priori* "prediction" based on the empirical nucleon-nucleon amplitudes of Arndt[13] as input to a standard Glauber calculation. Second, we fix the isospin averaged strengths from Arndt's amplitudes and fit the two lengths, $\lambda_1$ and $\lambda_2$, to the $P$ and $Q$ data. The unpolarized cross sections of Hoffman *et al.*[14] provide the needed structure input for the data-to-data relations. We have elected to present the data-to-data version of our analysis, although similar results follow from using the analytic form of Eq. (19). We have chosen to emphasize the data-to-data relations in this way, based on our belief that these relations survive the approximations used to derive them and separate questions of spin dynamics from the total mechanism. The understanding gained by such analysis is not complete but stands intermediate to that of a full microscopic calculation and a purely *ad hoc* phenomenology. We liken this understanding to that provided by effective range theory in low energy scattering—it provides a simple representation of the essential physics.

For the applications to $p$-$^{40}$Ca we fix the radius and diffusivity from electron scattering, $b_0=(3.7+i0.58)$ fm. The isospin averaged nucleon-nucleon strength factors determined from Arndt's amplitudes are

$$\gamma=(1.62-i0.30)\ \mathrm{fm}^2$$

and

$$w=(0.05+i0.42)\ \mathrm{fm}\ .$$

In Figs. 1 and 2, the dashed curves show the $P$ and $Q$ predictions of the Glauber calculation compared with the data. The agreement with the data is poor.

Next we attempt an effective geometric understanding of the data. We fix $b_0$, $\gamma$, and $w$ as before, but knowing that the remaining parameters involve differences among small numbers we treat them as unknown—the most efficient combination to search on being $\lambda_1$ and $\lambda_2$. In this way we obtain the solid curves of Figs. 1 and 2 corresponding to the values:

$$\lambda_1=-0.14+i0.065\ \mathrm{fm}\ ,$$

$$\lambda_2=0.20+i0.24\ \mathrm{fm}\ .$$

The agreement with the data is excellent.

One could attempt repairs of the "*a priori*" calculation; for example, pretend the neutron density is an unknown and, believing all the other microscopic assumptions of the previous calculations, fit the

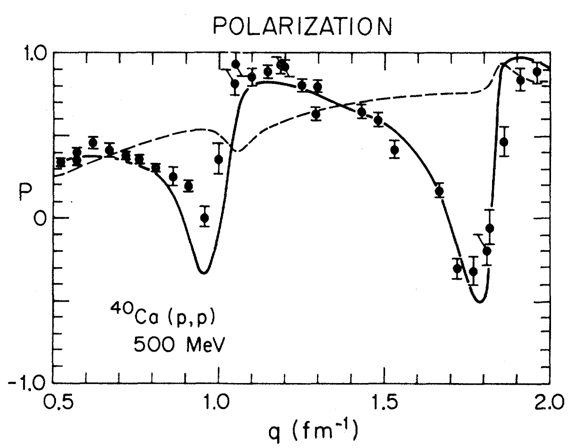


FIG. 1. Calculations of 500 MeV $p$-$^{40}$Ca polarization, using the data-to-data form of Eq. (19), are compared to the data (Ref. 12). The dashed curve is a standard Glauber calculation, where all the dynamic parameters are determined from the empirical nucleon-nucleon amplitudes. The solid curve is a fit of $\lambda_1$ and $\lambda_2$ (four real parameters) to the $P$ and $Q$ data.

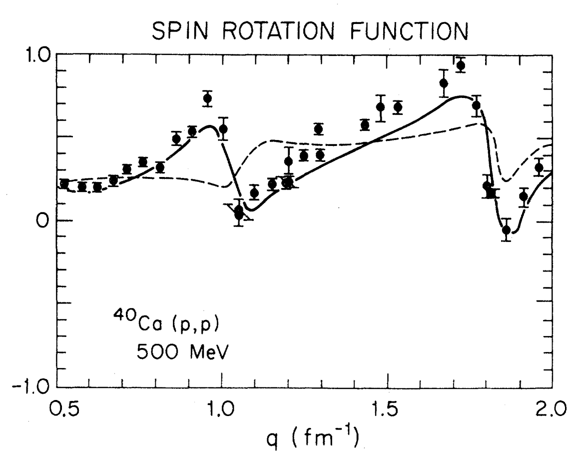


FIG. 2. Calculations of 500 MeV $p$-$^{40}$Ca spin rotation function, $Q$, using the data-to-data form of Eq. (19), are compared to the data (Ref. 12). As in Fig. 1, the dashed curve is a standard Glauber calculation, where all the dynamic parameters are determined from the empirical nucleon-nucleon amplitudes. The solid curve is a fit of $\lambda_1$ and $\lambda_2$ (four real parameters) to the $P$ and $Q$ data.

neutron density. In this way one interprets the $P$ and $Q$ data as "measurements" of the neutron density parameters. This may or may not be valid interpretation; at this stage it is an uncertain procedure. Nevertheless, the resulting $\lambda_1$ and $\lambda_2$ given in any hypothetical microscopic search must agree with those of the geometric model given above. Thus the geometric model should be thought of as on a different level than the microscopic theories. It is in some sense more general since it is more removed from dynamic details. The understanding gained by treating the data in the geometric model is complementary to that of microscopic calculations.

Extension to inelastic spin-dependent phenomena is straightforward. So long as the projectile spin is not of necessity involved in the transition, it can be treated in the distortion just as here for elastic scattering. Similar data-to-data relations will follow but with slight modifications due to the different envelope function of the unpolarized cross section.[6]

## IV. CONCLUSION

Here and in AMS we have presented a new way of looking at spin-dependent observables in intermediate energy proton-nucleus scattering. The relationship between the spin observables and the unpolarized differential cross section is given explicit form in the data-to-data relations derived from an asymptotic analytic approximation of the eikonal amplitude. This relationship is best understood in a geometric sense as arising from small shape differences in the effective central and spin-orbit optical potentials. We show that understanding the new $Q$ data at 500 MeV requires at least two dynamical contributions (relatively complex) in either the central or spin-orbit eikonal phase with different shapes. In the analytic framework these differences are conveniently parametrized by two small complex lengths $\lambda_1$ and $\lambda_2$. For the 500 MeV $p$-$^{40}$Ca example analyzed we find

$$\lambda_1 = -0.14 + i\,0.065 \text{ fm}$$

and

$$\lambda_2 = 0.20 + i\,0.24 \text{ fm} .$$

The measurements of $P$ and $Q$ provide an interferometriclike determination of these small lengths which summarize the data without microscopic theoretical bias. Indeed our analysis shows that the various length and strength factors of our example are entangled so as to render an unambiguous microscopic interpretation difficult. Certainly it remains an important theoretical challenge to understand the underlying hadronic theory and here, it is hoped, the geometric model will provide guidance and, in the interim, a context for understanding intermediate energy spin observables.

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