Energy dependence of the p-^{40}Ca optical potential: A Dirac equation perspective

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The energy dependence of a relativistic optical model potential consisting of a mixture of Lorentz scalar and Lorentz vector components is determined from the analysis of p-^{40}Ca elastic scattering experiments from 26 to 1040 MeV.

[NUCLEAR REACTIONS Dirac equation based analysis of $\vec{p}$-^{40}Ca elastic scattering, E_p = 26 to 1040 MeV. Energy dependence of empirical relativistic optical potential.]

In this work new experimental results from the IUCF[1] and TRIUMF[2] for $\vec{p}$-^{40}Ca elastic scattering in the transition energy region (100–500 MeV) are analyzed using a Dirac equation based optical model approach. These results, in combination with those from analysis at lower[3] and higher[2,4] energies, allow an investigation of the energy dependence of a nuclear optical model potential within a relativistic framework.

The description of the nuclear scattering problem using the Dirac equation rather than the Schrödinger equation is one of the necessary parts of a relativistic description of the nucleon-nucleus interaction. An optical model treatment appropriate for use in the Dirac equation has been developed and applied to the description of nucleon-nucleus scattering over a wide energy range.[3–6] A critical feature of this model is the consideration of the Lorentz transformation character of the potential, usually taken to be a mixture of Lorentz scalar and Lorentz vector interactions. Duerr[7] considered such a model sometime ago; and, during the past few years, relativistic models have been applied successfully to the description of both finite nuclei and nuclear matter.[1–35] These works generally use a one-boson exchange potential (OBEP) description in obtaining an effective nucleon-nucleon interaction which, in the simplest case, includes only the exchange of scalar and vector bosons. In such a case the relativistic single particle potential for treating the nuclear many-body problem contains both Lorentz scalar U_s, and

Lorentz vector, U_0, potentials.

The complex potentials used in this analysis of $\vec{p}$-^{40}Ca elastic data are written

$$U_0(r)=V_0 f_0^R+iW_0 f_0^I\ , \tag{1}$$

and

$$U_s(r)=V_s f_s^R+iW_s f_s^I\ , \tag{2}$$

with the form factors chosen to be two-parameter Fermi functions $[1+\exp(r-c)/z)]^{-1}$. Thus, taken as a strictly phenomenological model, there are 12 adjustable parameters, the same number as for a phenomenological Schrödinger-equation based optical model using Fermi functions for the complex central potential and derivatives of Fermi functions for the complex spin-orbit potential. In the relativistic model all spin dependent effects are implicit in the Dirac equation. These potentials are incorporated in the Dirac equation ($\hbar=c=1$)

$$\{\vec{\alpha}\cdot\vec{p}+\beta[m+U_s(r)] +[U_0(r)+V_C(r)]\}\psi(\vec{r})=E\psi(\vec{r})\ , \tag{3}$$

where $V_C(r)$ is the Coulomb potential for protons determined from the empirical nuclear charge distribution, m the proton mass, and E the proton total energy in the c.m. frame. This equation is solved by partial wave analysis to obtain the elastic differential cross sections and analyzing powers.

As a starting point for this analysis of $\vec{p}$-^{40}Ca experiments, we consider a folding or Dirac-Hartree construction of the real parts of U_0 and U_s given by[6]

$$V_0(r)=\mathrm{Re}U_0(r)=\int v_0(|\vec{r}-\vec{r}'|)\tilde{\rho}_0(\vec{r}')d\vec{r}'\ , \tag{4}$$

$$V_s(r)=\mathrm{Re}U_s(r)=\int v_s(|\vec{r}-\vec{r}'|)\tilde{\rho}_s(\vec{r}')d\vec{r}'\ . \tag{5}$$

The density $\tilde{\rho}_0$ is found from a double folding of projectile and target nucleons with the nuclear matter density ρ_0. The baryon density ρ_0 is taken from an empirical formula of Negele[36] and the scalar density is approximated by $\tilde{\rho}_s(r)=[\rho_s/\rho_0]_{nm}\tilde{\rho}_0(r)$, where $[\rho_s/\rho_0]_{nm}$ is the scalar-to-baryonic density ratio in nuclear matter. The effective interaction is approximated by $v(r)=tf(r)$, where $f(r)$ is the appropriate meson form factor and t is the volume integral of the effective interaction in nuclear matter. The density ratio and the values of t_0 and t_s are taken from Walecka's relativistic mean field theory of nuclear matter.[22] These potentials are parametrized in terms of Fermi equivalent shapes as described in Ref. 1. The vector and scalar form factor parameters, given by $c_0=3.474$ fm, $z_0=0.668$ fm, $c_s=3.453$fm, and $z_s=0.692$ fm, are kept fixed in the analysis. The strengths of V_0 and V_s, as determined from Eqs. (4) and (5), were essentially suitable for representing experiment at the two lowest energies, 26 and 30 MeV.[3] For higher energies, the strengths were treated as free parameters.

The imaginary parts of U_0 and U_s are also parametrized with Fermi shapes or, at energies below 50 MeV, derivatives of Fermi shape form factors.[3] Thus, the imaginary optical potentials contain, at most, six free parameters. In general, we have found that acceptable fits to experiment could be obtained by allowing the real and imaginary strengths and two of the imaginary geometry parameters to vary.

One of the purposes of the present work is to investigate in a systematic way the empirical energy dependence of the individual Lorentz scalar and Lorentz vector potentials. An earlier work,[5] which also employed the Dirac equation, analyzed $\vec{p}$-^{4}He elastic scattering experiments at a number of energies above 500 MeV. The authors of Ref. 5 found that the ratio R_R, defined by

$$R_R=\frac{J_0}{J_s}=\frac{\int V_0(r)d\vec{r}}{\int V_s(r)d\vec{r}}\ , \tag{6}$$

was well determined by their analysis. This parameter is also well determined in the present work where we obtain the parameter R_R from an analysis of $\vec{p}$-^{40}Ca data at eight energies, from 26 to 1040 MeV. This ratio gives a clear signature of the energy dependence of V_0 and V_s as well as functioning as a characteristic parameter describing the mixture of different Lorentz character potentials. Figure 1 shows this ratio as determined from the $\vec{p}$-^{40}Ca analysis at the eight energies considered. The energy dependence of R_R is approximated quite well by the expression $R_R(T_P)=a+bT_p$, with $a=-0.798$ and $b=0.260\times10^{-3}$. The corresponding ratio from Ref. 5 for $\vec{p}$-^{4}He is also shown in Fig. 1, and again a linear energy dependence is suitable. In this case, $a=-0.814$ and $b=0.891\times10^{-4}$. In both cases, as shown in Table I, the empirical values of R_R at zero energy are in agreement with values obtained from Walecka's relativistic mean field theory. However, the energy dependence of R_R is greater for ^{40}Ca than for ^{4}He.

TABLE I. Values for the ratio R_R determined from relativistic mean field theory in Ref. 6 for two different Fermi wave numbers, and a binding energy of 15.75 MeV. The corresponding zero energy empirical values are from Ref. 5 and this work.

Mean field theory		Empirical	
K_f (fm^{-1})	R_R	A	$R_R(0)$
1.42	−0.793	4	−0.814
1.31	−0.819	40	−0.798

The Dirac-Hartree potentials calculated from Eqs. (4) and (5) are energy independent. For ^{40}Ca, using mass $m_\omega = 780$ MeV and $m_\sigma = 560$ MeV, the potential strengths are $V_0 = 383$ MeV and $V_s = -470$ MeV so that $R_R = -0.8$ independent of energy. The energy dependence of this ratio as obtained from the phenomenological analyses is thus a measure of nonstatic corrections to the Hartree approximation which can be anticipated within the framework of the model.[6] For example, recent Hartree-Fock calculations by Jamion and Mahaux[37] yield an energy dependence for R_R that is in qualitative agreement with the empirical results for energies below the transition region where the influence of absorption on the real part of the potential appears to be of secondary importance. Similar conclusions can be inferred from calculations by Müller.[38]

We now turn to another aspect of the energy dependence of a Dirac equation based optical model. It has been shown[1–6,34] that the reduction of a Dirac equation, which contains both U_s and U_0, to second order form, produces an equation for which a Schrödinger equation equivalent optical

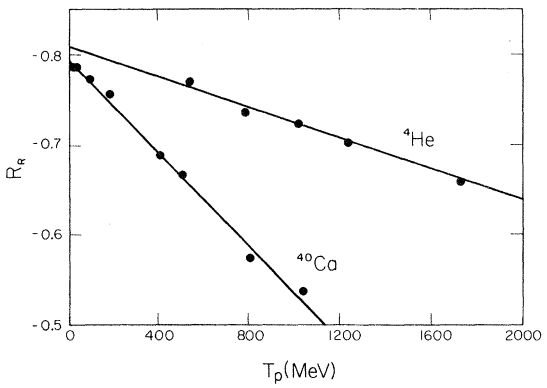

FIG. 1. Values of the ratio R_R defined in Eq. (1) from Ref. 5 for ^{4}He and the present analysis for ^{40}Ca.

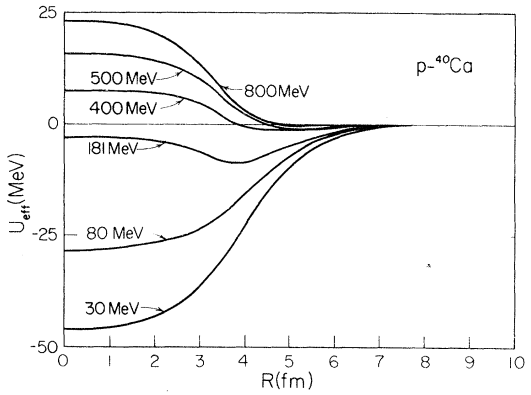

FIG. 2. The real part of the Schrödinger equivalent central potential U_{eff}, determined from the Dirac equation based analysis of $\vec{p}$-^{40}Ca elastic scattering experiments (Refs. 1, 2, and 49–55). The Darwin term is omitted.

potential may be defined. This Schrödinger equation equivalent potential, which contains both central and spin orbit components, has a real part which changes from attraction at low energies to repulsion at high energies. In the transition energy region the radial shape of this potential deviates considerably from the Fermi-type shape usually associated with nuclear densities, even when the individual Lorentz scalar and Lorentz vector potentials have Fermi shapes. That an unorthodox shape could be required to fit experimental data in this energy region was noted by Elton[39] and has been confirmed by recent analysis using both relativistic[1] as well as nonrelativistic[40] treatments.

Figure 2 shows the real Schrödinger equivalent central potentials found from the Lorentz scalar and vector potentials determined from the $\vec{p}$-^{40}Ca analysis. In the transition energy region there is a considerable deviation from the radial shape of the nuclear matter distribution. The origin of this wine-bottle-bottom shape lies both in the presence of square terms in the Schrödinger equivalent central potential and in its explicit energy dependence.[1,31] Quite similar energy dependence of the optical potential is found for Brueckner-Hartree-Fock (BHF) calculations,[41–45] where the effect is due to cancellation of direct and exchange terms in the optical potential calculated in the local density approximation. Recent calculations[46] using Fermi-hypernetted and single-operator-chain summation techniques also indicate that the nucleon-nucleus optical potential would, in a local density approximation, also have a wine-bottle-bottom shape in the transition energy region.

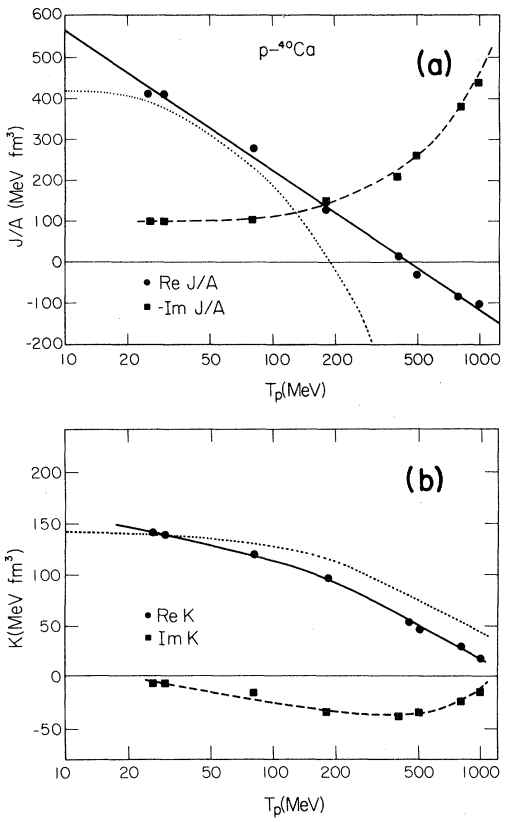

FIG. 3.(a) Values of the volume integrals of the real and imaginary parts of the Schrödinger equivalent central potential for ^{40}Ca. The smooth line is given by Eq. (7). The dashed line is to guide the eye. The dotted line gives the Dirac-Hartree values. (b) Values of the volume integrals of the real and imaginary parts of the Schrödinger equivalent spin-orbit potential for ^{40}Ca. The smooth and dashed lines are to guide the eye. The dotted line gives the Dirac-Hartree values.

The volume integrals per nucleon, J_R/A and J_I/A, of the Schrödinger equivalent central potentials are shown in Fig. 3(a). The energy dependence of J_R/A can be represented by the equation

$$J_R(T_P)/A = C + B \ln T_p \ , \tag{7}$$

with $C = -$ 916 MeV fm^3 and B = 150 MeV fm^3. The energy, 452 MeV, at which J_R/A passes through zero, is higher than that found from a simple Dirac-Hartree calculation, shown in Fig. 3 (a), and reflects the energy dependence of U_0 and U_s found in this work. A similar logarithmic energy dependence was also found by the authors of Ref. 5 in their Dirac equation analysis of $\vec{p}$-^{4}He experiments. However, the energy at which the real effective volume integral passed through zero was 360 MeV, indicating the effect of target mass on the energy dependence already noted in the discussion of R_R. Logarithmic energy dependence of J_R/A is also a characteristic of Schrödinger equation based analysis.[47] Finally, we note that the energy variation of J_I/A is reasonable and, as expected, its magnitude increases rapidly as the pion production threshold is passed.

In Fig. 3(b) we give the values found in this work for K_R and K_I, the Schrödinger equation equivalent spin-orbit volume integrals divided by $A^{1/3}$. The energy dependence is smooth in contrast to the rapid energy variation of both K_R and K_I found from Schrödinger equation based analysis in the transition energy region.[48] We suggest that this latter behavior in the nonrelativistic analysis is due to the choice of Fermi or derivatives of Fermi shapes for the phenomenological potentials. Figure 2, as well as theoretical calculations, suggests that less restrictive radial shapes are appropriate in the transition energy region.

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