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Excitation energy at scission in thermal-neutron-induced fission

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For the thermal-neutron-induced fission of ^{235}U and ^{233}U, we extract the internal excitation energy at the scission point by use of two different methods. The first method uses experimental data for $^{235}\mathrm{U} + n_{\mathrm{th}}$ on the neutrons and gamma rays emitted from doubly magic fission fragments, where the extra stability associated with shell closures makes the deformation energy small. Under the assumption that both fragments have equal temperatures, this yields an upper limit of 9.9 MeV for the total internal excitation energy at scission. The second method uses experimental data for $^{233}\mathrm{U} + n_{\mathrm{th}}$ on odd-even effects in fission-fragment mass distributions and yields an average value of 5.6 MeV for the total internal excitation energy at scission when integrated over all fission-fragment kinetic energies. In both cases the internal excitation energy at scission is significantly smaller than that predicted by the one-body-dissipation theory of Swiatecki and co-workers and is instead much closer to that predicted by ordinary two-body viscosity.

[NUCLEAR REACTIONS, FISSION $^{235}\mathrm{U} + n_{\mathrm{th}}$, $^{233}\mathrm{U} + n_{\mathrm{th}}$; extracted internal excitation energy at scission point. Neutrons and gamma rays from doubly magic fission fragments, odd-even effects in fission-fragment mass distributions, one-body dissipation, two-body viscosity, temperature-dependent BCS pairing theory.]

I. INTRODUCTION

Despite considerable work during the past few years on the dynamics of fission,[1–13] the amount of energy dissipated during the descent from the saddle point to the scission point is still not known. For example, average experimental fission-fragment kinetic energies for the fission of nuclei throughout the Periodic Table at high initial excitation energy, where single-particle effects are unimportant, do not discriminate between two extreme pictures of the dynamical descent. At one extreme, the one-body wall-and-window dissipation theory of Swiatecki and co-workers[6–10] predicts a slow, highly dissipative descent to a compact scission configuration, with most of the final kinetic energy arising from the Coulomb repulsion of the fragments after scission. At the other extreme, ordinary two-body viscosity[3,6,7] predicts a rapid, slightly dissipative descent to an elongated scission configuration, at which point the fragments have already acquired appreciable kinetic energy. These two radically different pictures reproduce average experimental fission-fragment kinetic energies for the fission of nuclei at high initial excitation energy equally well.[7]

In the present work we use two different methods to extract the internal excitation energy at the scission point for the thermal-neutron-induced fission of ^{235}U and ^{233}U. The first method, which is described in Sec. II, uses experimental data for $^{235}\mathrm{U} + n_{\mathrm{th}}$ on the neutrons and gamma rays emitted from doubly magic fission fragments. The second method, which is described in Sec. III, uses experimental data for $^{233}\mathrm{U} + n_{\mathrm{th}}$ on odd-even effects in fission-fragment mass distributions. In both cases the internal excitation energy at scission is found to be moderately small. However, as discussed in Sec. IV, the small dissipation for these cases could be the result of single-particle and/or superfluidic effects rather than being a general feature of the dynamics of large-amplitude collective nuclear motion.

II. NEUTRONS AND GAMMA RAYS FROM DOUBLY MAGIC FISSION FRAGMENTS

Experimental data have existed for many years on the average number of neutrons[14,15] and energy of the gamma rays[16] emitted from fission fragments as functions of their mass number. Although these data are directly related to the average excitation energy of the fission fragments at infinity, for most mass divisions they do not provide useful information concerning the scission configuration, since at that point the energy can be in the form of either internal excitation energy or deformation energy. However, by specializing to the region of doubly magic fission fragments, where the extra stability associated with shell closures makes the deformation energy small, we can obtain an important upper limit on the internal excitation energy at scission. Of course, the limitation of this approach is that the extracted internal excitation energy could be strongly influenced by single-particle effects.

We consider the thermal-neutron-induced fission of ^{235}U, where the sawtooth neutron-emission curve has been measured by Apalin *et al.*[15] and the gamma-energy curve has been measured by Pleasonton *et al.*[16] For a given fission fragment with mass number A_1, the total average excitation energy at infinity associated with both neutrons and gamma rays is

$$E_1^{\text{tot}} = \nu_1(B_n + \epsilon_n) + E_1^{\gamma} , \tag{1}$$

where ν_1 is the average number of neutrons and E_1^{γ} is the average energy of the gamma rays emitted by this fragment. For the neutron separation energy B_n we use calculated values[17] averaged over even and odd neutron and proton divisions, and for the average kinetic energy ϵ_n of the emitted neutron we use the experimental data of Bowman *et al.*[18] for the spontaneous fission of ^{252}Cf, which should be very close to that for the thermal-neutron-induced fission of ^{235}U.

Upon inserting into Eq. (1) the values appropriate to a fission fragment with mass number $A_1 = 132$, we obtain

$$E_1^{\text{tot}} = [0.3(6.3+1.6)+1.9]\ \text{MeV} = 4.3\ \text{MeV} .$$

The combined error associated with this quantity should be less than 1 MeV. Since at the scission point this energy can be in the form of either excitation energy or deformation energy, it follows that

$$E_1^{\text{ex}} + E_1^{\text{def}} = 4.3\ \text{MeV} ,$$

from which we immediately obtain the inequality

$$E_1^{\text{ex}} \le 4.3\ \text{MeV} .$$

On the basis of the Fermi-gas model, where the excitation energy of a fragment is related to its level-density parameter and temperature by $E_i^{\text{ex}} = a_i T_i^2$, we can also obtain an upper limit for the excitation energy of both fragments at scission by assuming that the two fragments have equal temperatures. For the level-density parameters we use the values[19] $a_1 = 132/(10\ \text{MeV})$ for the nearly spherical doubly magic fragment and $a_2 = 104/(6\ \text{MeV})$ for the deformed midshell complementary fragment. This yields

$$E^{\text{ex}} = \left[1 + \frac{a_2}{a_1}\right] E_1^{\text{ex}} = \left[1 + \frac{10(104)}{6(132)}\right] E_1^{\text{ex}} \le 9.9\ \text{MeV}$$

III. ODD-EVEN EFFECTS IN FISSION-FRAGMENT MASS DISTRIBUTIONS

A. Qualitative considerations

The yields of fission fragments in the thermal-neutron-induced fission of ^{233}U show that fragments with even numbers of protons or neutrons are more frequent than those with odd numbers.[20–22] The odd-even effect for neutrons is smaller by about a factor of 5 than that for protons.

As a reasonable measure of the strength of the odd-even effect we may introduce the fraction f defined by

$$f = \frac{P_u}{\frac{1}{2}(P_g + P_u)} ,$$

where P_g and P_u are the yields of the even (gerade) and odd (ungerade) fragments, respectively. We often use the equivalent quantity

$$g = \frac{P_g}{\frac{1}{2}(P_g + P_u)} = 2 - f . \tag{2}$$

The absence of odd-even effects would imply that $f = g = 1$. Experiment shows that the odd-even effect becomes larger for higher kinetic energies of the fragments.[20–22]

B. Specific model

A simple explanation of the odd-even effects in fission fragments can be given in the framework of the temperature-dependent pairing theory.[23] The possible importance of the temperature-independent pairing in the energy dissipation of fissioning nuclei has been emphasized by Wilets.[24]

As the excitation energy of an even-even fissioning nucleus increases, some pairs start to break up, giving rise to quasiparticle excitations. This is the intuitive picture that we use for the descent from the saddle point of the originally cold nucleus to the scission configuration. As the excitation energy increases, the pairing gap $\Delta(T)$ diminishes and the odd-even effects in the prompt mass distributions decrease.

We denote by $\hat{N}$ the number of quasiparticles above the Fermi sea at a given temperature T and by N the number of nucleons above the Fermi sea at the same temperature in the absence of the pairing interaction. In the spirit of the above intuitive picture it is natural to make the identification

$$f=\frac{\hat{N}}{N}\ , \tag{3}$$

since odd-even effects in the mass distributions have their origin in the presence of quasiparticles far from closed shells, where the attractive pairing force is not restricted by the Pauli exclusion principle.

We show in the Appendix that we approximately may write

$$\frac{\hat{N}}{N}=\frac{2}{1+\exp[\Delta(T)/T]}\ . \tag{4}$$

Then, by inserting Eqs. (3) and (4) into Eq. (2), we obtain

$$\frac{\Delta(T)}{T}=\ln\frac{g}{2-g}\ . \tag{5}$$

We note that the expression on the right-hand side of Eq. (4) is equal to the ratio of the attenuation coefficient of ultrasonic waves in superconductors to the attenuation coefficient in a normal conductor.[23]

Equation (5) demonstrates that if g has been measured experimentally, the temperature T of the nucleus at its scission point can be determined, provided that the functional dependence of the pairing gap $\Delta(T)$ is known. The solution of Eq. (5) for the temperature T can then be found by either graphical or analytical methods.

The temperature dependence of $\Delta(T)$ has a particularly simple form in the BCS theory applied to a uniform set of single-particle levels with constant pairing interaction.[25] The gap becomes zero at the critical temperature T_c that is related to the gap at zero temperature by[25]

$$\frac{\Delta(0)}{T_c}=1.7639\ .$$

For the value of $\Delta(0)$ we use[26] the result $\Delta(0) = 12\ \text{MeV}/\sqrt{A}$, which gives 0.784 MeV for the fissioning nucleus ^{234}U. We neglect the increase in $\Delta(0)$ that would occur in a dynamical theory when the original nucleus separates into two fragments.

The experimental results[20–22] give a mean value for the pairing of protons $g_p = 1.25$ and for the pairing of neutrons $g_n = 1.05$ when integrated over all kinetic energies. For these values we show in Fig. 1 the graphical solutions of Eq. (5). The experimental values for g determine straight lines for protons and neutrons in the $\Delta - T$ plane. The intersections with the $\Delta(T)$ curve give the solutions of Eq. (5). Analytical solutions can be obtained also, since $\Delta(T)/\Delta(0) = 1.74[1 - (T/T_c)]^{1/2}$ for temperatures[25] near the critical temperature T_c. The results for protons are $T_p = 0.433$ MeV and $\Delta_p(T_p) = 0.227$ MeV, while for neutrons they are $T_n = 0.444$ MeV and $\Delta_n(T_n) = 0.044$ MeV. Both the proton temperature and neutron temperature are close to the critical temperature $T_c = 0.445$ MeV. Whereas neutrons and protons have only slightly different temperatures, the gap parameters are very different because the temperature of the system is close to the critical temperature of the BCS theory, where minor temperature

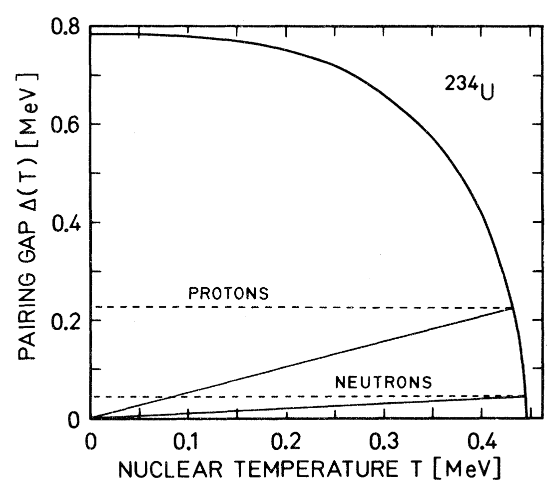

FIG. 1. Graphical solution of Eq. (5) for the pairing gap $\Delta(T)$ and nuclear temperature T at the scission point for protons and neutrons in the thermal-neutron-induced fission of ^{233}U, giving rise to the compound nucleus ^{234}U.

differences lead to rather different odd-even effects.

There are theoretical reasons why we should expect the neutrons to have a higher temperature than the protons at the point where the respective distribution is determined. First, the Coulomb forces cause a polarization of the proton fluid at the late stages of the fission process before the final neck rupture is carried out by neutrons. In addition, a possible neutron skin[27] could contribute to a higher neutron temperature.

The dependence of the odd-even effect upon the fission-fragment kinetic energy has also been studied experimentally. We have analyzed this dependence by using Eq. (5). Our results are summarized in Table I, where we also compute the total internal excitation energy at scission by use of the Fermi-gas result $E^{\text{ex}} = aT^2$ with level-density parameter[19] $a = A/(8\text{ MeV})$, which is appropriate for a distribution of fission-fragment masses containing both nearly spherical doubly magic nuclei and deformed midshell nuclei.

The excitation energies for ^{234}U at scission as derived from proton and neutron data are close, with an average value of 5.6 MeV when integrated over all kinetic energies. They show only a slight tendency to decrease with increasing kinetic energy, which implies that most of the fluctuations in the fission-fragment kinetic energy arise from fluctuations in the deformation energy and only a minor part from fluctuations in the intrinsic excitation energy. The analysis of these data within the limits of the BCS theory imply that in fission at thermal energies the fragments have relatively little internal excitation energy at scission.

The calculated values for the total intrinsic excitation energy of ^{234}U are about 43% less than the upper bounds derived in the previous section for ^{236}U. Several limitations of the BCS theory make the present estimate a lower bound. The correction due to the finite particle number in nuclear systems has the tendency to smooth out the sharp transition near the critical temperature by contributing a tail to the curve specifying the temperature dependence of the gap parameter. This effect would increase the derived temperature. A quantitative treatment, however, would require a better understanding of pairing in finite systems. Because of such uncertainties, it is not worthwhile at this stage to extend the temperature-dependent BCS calculation beyond the uniform model by using realistic single-particle levels calculated at the scission point.[28]

We finally should remark that the odd-even effect for neutrons in the secondary mass distributions is impaired by neutron evaporation from the prompt fragments, except for high kinetic energies where this is energetically forbidden. While proton emission can be neglected, the odd-neutron fragments have a slightly higher evaporation rate than the even ones. Although the experimental values for the neutron odd-even effect have not been corrected for neutron evaporation, such a correction would not affect the qualitative result that the neutron odd-even effect is smaller than the proton odd-even effect, since for sufficiently high kinetic energies of the fragments neutron evaporation is zero.

IV. DISCUSSION AND CONCLUSIONS

By considering the neutrons and gamma rays emitted from doubly magic fission fragments, we obtained in Sec. II an upper limit of 9.9 MeV for the internal excitation energy of both fragments at scission in the thermal-neutron-induced fission of ^{235}U. By considering odd-even effects in fission-

TABLE I. Experimental strength g of the odd-even effect for protons and neutrons in the thermal-neutron-induced fission of ^{233}U (Refs. 20–22), plus the extracted values at scission of the pairing gap Δ, temperature T, and total internal excitation energy E^{ex}, for three values of kinetic energy of the light fragment and integrated over all kinetic energies.

Kinetic energy (MeV)	g_p	Δ_p (MeV)	T_p (MeV)	E_p^{ex} (MeV)	g_n	Δ_n (MeV)	T_n (MeV)	E_n^{ex} (MeV)
88	1.16	0.142	0.439	5.66	1.05	0.044	0.444	5.75
98	1.28	0.247	0.430	5.41	1.07	0.062	0.444	5.75
108	1.45	0.394	0.407	4.85	1.14	0.124	0.441	5.68
Integrated	1.25	0.227	0.433	5.47	1.05	0.044	0.444	5.75

fragment mass distributions, we obtained in Sec. III an average total internal excitation energy at scission of 5.6 MeV for the thermal-neutron-induced fission of ^{233}U. These values are both significantly smaller than the value[6] of 17.6 MeV predicted by the one-body-dissipation theory of Swiatecki and co-workers[8–10] for the excitation energy at the point the neck loses stability against rupture in the fission of the compound nucleus ^{236}U. In contrast, ordinary two-body viscosity with coefficient $\mu = 0.02$ terapoise, which is the value that optimally reproduces[29] average fission-fragment kinetic energies with the most recent constants[17,18] of the macroscopic energy, yields a dissipated energy at this same point[6] of approximately 7.9 MeV.

In interpreting these results, it must be borne in mind that both calculations refer to the fission of nuclei at high initial excitation energies in the absence of single-particle effects, whereas the experimental values corresponding to thermal-neutron-induced fission are possibly influenced by single-particle and/or superfluidic effects. However, despite the possibility of a superfluidic slither[30] from the saddle point to scission, the analysis by Yannouleas *et al.*[31] demonstrates that at least near the scission point, where $\Delta(T)/T$ is small, superfluidity has little effect on the dissipation predicted by the one-body wall formula.

An obvious problem for the future is the determination of the energy dissipated during the descent from saddle to scission in the fission of nuclei at high excitation energy, where single-particle and superfluidic effects play no role. For this purpose a calculation of the widths of fission-fragment mass and kinetic-energy distributions by solving a generalized Fokker-Planck equation in the multidimensional phase space of collective coordinates and momenta[32,33] should prove useful. In another area, a proper analysis of the extra energy required to form heavy compound nuclei in heavy-ion reactions[10,30,34–36] should help decide the dissipation issue. In summary, the amount of energy dissipated in large-amplitude collective nuclear motion continues to pose an important challenge for the future.

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APPENDIX

For an ideal Fermi-gas system the mean number N of particles above the Fermi energy ϵ_F is given by

$$N=\alpha\int_{\epsilon_F}^{\infty} d\epsilon\sqrt{\epsilon}f(\epsilon)\ ,$$

with

$$f(\epsilon)=\frac{1}{1+\exp[(\epsilon-\epsilon_F)/T]}\ .$$

The constant α is determined in terms of the volume V of the system to be

$$\alpha=\frac{4\pi}{h^3}(2m)^{3/2}V\ ,$$

where m is the nucleon mass and h is Planck's constant. In the presence of the pairing interaction the number $\hat{N}$ of quasiparticles at a given temperature T reads

$$\hat{N}=\alpha\int_{\epsilon_F}^{\infty} d\epsilon\sqrt{\epsilon}\hat{f}(\epsilon)\ ,$$

with the distribution function of the quasiparticles

$$\hat{f}(\epsilon)=\frac{1}{1+\exp[e(\epsilon)/T]}\ .$$

The quasiparticle energy is given by

$$e(\epsilon)=\sqrt{(\epsilon-\epsilon_F)^2+\Delta^2(T)}\ .$$

To evaluate the above integrals approximately, we note that for $\epsilon\geq\epsilon_F$ the distribution functions $f(\epsilon)$ and $\hat{f}(\epsilon)$ have a maximum at $\epsilon=\epsilon_F$, while their width is approximately equal to the temperature T. We therefore replace these functions in the energy integrals by parallelograms of dimensions $f(\epsilon_F)\times T/2$ and $\hat{f}(\epsilon_F)\times T/2$, respectively. This leads to

$$N\cong\frac{\alpha T}{2}\sqrt{\epsilon_F}f(\epsilon_F)$$

and

$$\hat{N}\cong\frac{\alpha T}{2}\sqrt{\alpha_F}\hat{f}(\epsilon_F)\ ,$$

which upon division becomes

$$\frac{\hat{N}}{N}\cong\frac{2}{1+\exp[\Delta(T)/T]}\ .$$

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