Inelastic light ion scattering and application to $^{12}C + {}^{12}C$

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A projection operator method is presented for the calculation of low energy heavy ion inelastic scattering cross sections. The theory treats both gross and intermediate structure and is applied to the case of $^{12}C + {}^{12}C \to {}^{12}C(2^+) + {}^{12}C(g.s.)$, where particle-vibration doorways of both 2^+ and 3^- origin are included. The results are in reasonable agreement with experiment. A discussion of the possible effects of doorways of 0_2^+ origin is given.

[NUCLEAR REACTIONS $^{12}C + {}^{12}C \to {}^{12}C(2^+) + {}^{12}C(g.s.)$, $E = 10-30$ MeV c.m., projection operator intermediate structure theory, calculated σ_{total}.]

I. INTRODUCTION

The experimental observation[1] of resonance structure in $^{12}C + {}^{12}C$ reactions has lent support to the hypothesis of nuclear molecular formation. The effects of such states should be particularly evident in the inelastic and other reaction channels. Cormier and collaborators[2–4] found resonance structure in the $^{12}C + {}^{12}C$ inelastic channel. Phillips *et al.*[5] have shown that the gross structure in the inelastic channel can be interpreted in terms of a diffraction model that does not take into account molecular formation. In addition, Ref. 5 suggests that evidence for nuclear molecular phenomena requires a model capable of producing intermediate structure. Molecular models have had some success in explaining the elastic scattering results.[6–8] One would expect that the internal excitations of the ^{12}C nuclei that lead to molecular formation and the subsequent intermediate resonance structure should also be present in the inelastic channel. Kondo *et al.*[9] have applied a band crossing model to study the $^{12}C + {}^{12}C$ inelastic scattering cross section. However, they found it necessary to introduce a phenomenological reflection coefficient. In previous work[8,10] we developed a formalism to describe gross and intermediate structure in low energy heavy ion reactions in terms of a projection operator method based on the Feshbach idea of doorway states. This method has been applied[8] to the $^{12}C + {}^{12}C$ elastic channel with good results. In the present paper we extend our previous elastic scattering model to inelastic scattering. As a test case we calculate the inelastic $^{12}C + {}^{12}C$ cross section where one nucleus is left in an excited 2^+ state and the other is left in its ground state. The theory is presented in Sec. II and the results are given in Sec. III. Section IV consists of a discussion and conclusions.

II. THEORY

In Refs. 8 and 10 the theoretical development of the elastic scattering is given in detail. The breakup of the Hilbert space for inelastic scattering studies is the same as in the elastic case. We briefly review this decomposition. The Hilbert space is divided into two orthogonal subspaces representing, respectively, the shape elastic continuum space (P) of the reduced mass and the rest of the Hilbert space (Q). The projection operators P and Q satisfy

$$P + Q = 1 \; . \qquad (1)$$

Further subdivision of the subspaces associated with these operators can be made by introducing the mutually orthogonal operators p, R, d, and q so that

$$p + R = p \; , \qquad (2a)$$

and

$$d + q = Q \; . \qquad (2b)$$

The subdivision into p and R follows the procedure of Lev and Beres[11] as adapted to heavy ion resonances.[10] Here R produces the subspace of shape elastic continuum resonances of the reduced mass projectile (R) and p produces the orthogonal nonresonance subspace (p). The operator d produces the subspace that contains the doorways (d), i.e., states which involve a nuclear interaction in addition to an optical potential, and are one step more complex than (R). The operator q produces the states (q) which are more complicated than (d). We assume that the deviation of the scattering from pure potential elastic results from the coupling of (R) to (d). The reader should consult Ref. 8 for more details concerning the validity of the doorway assumptions.

In the case of inelastic scattering the T matrix is given by

$$T=\langle \chi_f^{(-)} | H_I | \psi^{(+)} \rangle , \tag{3}$$

where $\psi^{(+)}$ is the scattering wave function. This wave function may be written in terms of the projection operators as

$$\psi^{(+)}=p\psi^{(+)}+R\psi^{(+)} +d\psi^{(+)}+q\psi^{(+)} , \tag{4}$$

where $q\psi^{(+)}$ will not contribute to T because of the doorway assumption. The homogeneous part of $p\psi^{(+)}$ is the distorted wave $\tilde{\psi}^{(+)}$ satisfying

$$(E-h_{pp})\tilde{\psi}^{(+)}=0 , \tag{5}$$

with h representing a one body Hamiltonian obtained from the effective Hamiltonian in the manner of Ref. 8. The labeling h_{pp} follows the usual projection operator notation. The interaction Hamiltonian H_I is

$$H_I=V(r)+H_{12}-V_c(r) , \tag{6}$$

where $V(r)$ contains both the real nuclear potential and the effective Coulomb potential for the reduced mass particle, and H_{12} couples the relative motion to the internal degrees of freedom of each ion. The term $V_C(r)$ is the Coulomb potential chosen in a two potential formulation to generate the final state wave function $\chi_f^{(-)}$. The wave function $\chi_f^{(-)}$ is the Coulomb scattering state with one of the nuclei left in the internal excitation state $|\lambda\mu\rangle$ and is given by

$$\chi^{(-)}_{\vec{K},\lambda\mu}=\frac{4\pi}{Kr}\sum_{lm} e^{-i(\sigma_l+l\pi/2)}(-)^l F_l(Kr) \times Y^*_{lm}(\hat{K})Y_{lm}(\hat{r})\,|\lambda\mu\rangle , \tag{7}$$

where the subscript f is replaced by the particle wave number $\vec{K}$ as well as $\lambda\mu$. The resulting T matrix becomes

$$T=T_1+T_2+T_3+T_4 , \tag{8}$$

where

$$T_1=\langle \chi^{(-)}_{\vec{K},\lambda\mu} | H_{12} | \tilde{\psi}(+)\rangle , \tag{9a}$$

$$T_2=\tau_p T_R , \tag{9b}$$

$$T_3=\tau_R T_R , \tag{9c}$$

and

$$T_4=\tau_d T_R . \tag{9d}$$

The quantities T_R, τ_p, τ_R, and τ_d are

$$T_R=\frac{\langle \phi_{RJ} | -\frac{\hbar^2}{2m}\nabla^2+V(r) | \tilde{\psi}^{(+)}\rangle}{E-E_{RJ}-\Delta_{RJ}+i\Gamma_{RJ}/2} , \tag{10a}$$

$$\tau_p=\langle \chi^{(-)}_{\vec{K},\lambda\mu} | H_{12}\tilde{G}_p^{(+)}h_{pR} | \phi_{RJ}\rangle , \tag{10b}$$

$$\tau_R=\langle \chi^{(-)}_{\vec{K},\lambda\mu} | H_{12} | \phi_{RJ}\rangle , \tag{10c}$$

and

$$\tau_d=\sum_{nl}\frac{\langle \chi^{(-)}_{\vec{K},\lambda\mu} | V(r)-V_C(r) | \Psi^{\lambda\ln}_{JM}\rangle\langle \Psi^{\lambda\ln}_{JM} | H_{12} | \phi_{RJ}\rangle}{E-\epsilon_\lambda-E_{nl}+i\Gamma_{nl}/2} . \tag{10d}$$

The interpretation of these terms in Eqs. (9) and (10) differs from that of the corresponding quantities in the elastic scattering case of Ref. 8. Most importantly the inelastic final state $|\lambda\mu\rangle$, characterized by an internal excitation of one of the colliding ions, determines the contributing doorways; i.e., only doorways based on $|\lambda\mu\rangle$ contribute to the term τ_d [Eq. (10d)]. However, the width Γ_{RJ} in Eq. (10a) is not so restricted and includes the decay of the (R) states to all of the available doorways. In addition, T_1 [Eq. (9a)], τ_p [Eq. (10b)], and τ_R [Eq. (10c)] involve the coupling H_{12} rather than a one body interaction as in Ref. 8.

In Eq. (10), ϕ_{RJ} is the gross resonance state of energy E_{RJ} and total angular momentum J. The quantities Δ_{RJ} and Γ_{RJ} are the energy shift and resonance width, respectively, and the Green's function associated with Eq. (5) is $\tilde{G}_p^{(+)}$. In the example to be studied, each doorway state $\Psi_{JM}^{\lambda ln}$ will be generated by a particle-vibration interaction of the standard form. It consists of the reduced mass particle with principal quantum number n and orbital quantum number l coupled to the angular momentum λ of the vibrational excited state of one interacting nucleus. The doorway state energy is

$$E_d = \epsilon_\lambda + E_{nl} \ , \tag{11}$$

where ϵ_λ and E_{nl} represent the vibrational and single particle energies, respectively. The quantity Γ_{nl} is the continuum width of the single particle resonance.

Finally, the differential cross section is

$$\frac{d\sigma}{d\Omega} = \frac{m^2}{4\pi^2\hbar^4} \mid T \mid^2 , \tag{12}$$

where T is given by Eqs. (8)–(10) and m is the reduced mass.

III. RESULTS

We now apply our theory to the case of $^{12}C + {}^{12}C$ inelastic scattering and calculate the cross section for one nucleus left in a $\lambda = 2^+$ (4.43 MeV) final state. The experimental cross section for $^{12}C + {}^{12}C \rightarrow {}^{12}C(2^+) + {}^{12}C$(g.s.) is the largest of all the inelastic processes[2–4] and its calculation provides a good test of our method. We emphasize that $\chi'^{(-)}_{\vec{K},\lambda\mu}$ [Eq. (7)] must be properly symmetrized for the case of the two identical ^{12}C nuclei.

The optical potential $V(r)$ is real and local, and is taken to be of the Woods-Saxon form with a linear energy dependence in the depth plus an effective Coulomb potential obtained from a Fermi charge distribution. The Woods-Saxon parameters are taken as in Ref. 8 from Gobbi,[12] and the effective Coulomb parameters are also taken from Ref. 8. One may introduce an absorption term iW, as was done previously[8] via

$$iW = i\Gamma_d^{\downarrow} \ , \tag{13}$$

to account for the spreading of the doorway states into the (q) space. The quantity $\Gamma_d^{\downarrow}$ is the spreading width of the doorway and is part of the total width given by

$$\Gamma_d = \Gamma_d^{\uparrow} + \Gamma_d^{\downarrow} \ , \tag{14}$$

with $\Gamma_d^{\uparrow}$ representing the continuum width, which is just Γ_{nl} of Eq. (10d). The quantity W is treated as an averaging interval and given the reasonable value of 400 keV.

For the final state $\lambda = 2^+$, the contributing doorways to Γ_{RJ} [Eq. (10a)] are taken as all possible particle-vibration states formed from both 2^+ (4.43 MeV) and 3^- (9.64 MeV) vibrations of ^{12}C coupled to the quasibound resonance states of the reduced mass particle. The strength of the particle-vibration coupling H_{12} is obtained from the experimental[13] $B(E2)$ and $B(E3)$ values. The doorway states of energy E_d [Eq. (11)] and widths $\Gamma_d^{\uparrow}$ [Eq. (14)] are taken from Table I of Ref. 8. There are 20 of these states that couple to the gross resonances of total angular momenta 8^+, 10^+, 12^+, and 14^+ of the $^{12}C + {}^{12}C$ system in the energy range 10–30 MeV c.m. We present in Fig. 1 the calculated total cross section for inelastic scattering to the 2^+ final state of ^{12}C and include for comparison the experimental results of Cormier *et al.*[3]

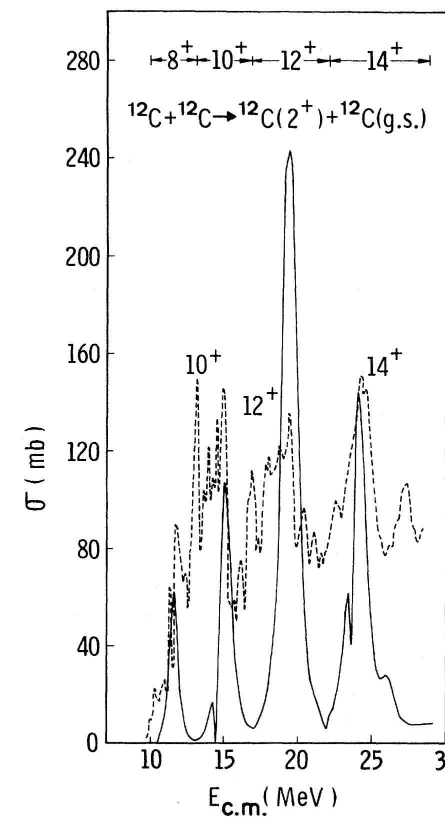

FIG. 1. Comparison of calculated and experimental (Ref. 3) total cross sections for $^{12}C + {}^{12}C$ inelastic scattering (2^+, g.s.). The solid line shows the calculated results and the dashed line the experimental. The total angular momenta from Ref. 3 are shown on the figure and the associated theoretical values are presented at the top.

IV. DISCUSSION

We note that the comparison between theory and experiment for 2^+ inelastic scattering as presented in Fig. 1 is reasonable as to the general features and assigned angular momenta of the intermediate structure resonances. Even though the doorway sum in Eq. (10d) is restricted to 2^+ as described in Sec. II, the term T_4 is the most significant contributor to the structure of the cross section. The term T_3 [Eq. (9c)] provides the gross envelope and the terms T_1 [Eq. (9a)] and T_2 [Eq. (9b)] turn out to be quite small. In calculating the cross section for the 14^+ resonance we found that the $\Gamma_d^{\uparrow} = \Gamma_{nl}$ value for the doorway $E_d = 23.15$ MeV ($\lambda, l = 2^+$, 12) was very sensitive to the Woods-Saxon well depth. This is because the single particle part of this doorway is close to being a virtual resonance rather than a quasibound state and thus leads to a relatively large width. Since the assumption of molecular resonances is based on the existence of quasibound doorways, it is unreasonable to have the cross section depend on the particular form of the interaction chosen to represent the optical potential. This problem was resolved by slightly deepening the well of Gobbi[12] to effectively make this doorway quasibound.

It is clear that our model cannot yield all of the detailed fine structure observed experimentally. At best we produce the intermediate structure inherent in the doorway interpretation of the formation of quasimolecular states. This is manifested in the averaging interval W described in Eq. (13). An increase in W from 400 keV to 2 MeV, in fact, resulted in an extensive broadening of the widths and a subsequent decrease in the cross section.

It is well known[14] that the 0_2^+ (7.68 MeV) excited state of ^{12}C is important in the various reactions that occur in $^{12}C + {}^{12}C$, especially in the alpha channel. For the inelastic 2^+ case, however, the doorways based on 0_2^+ would only contribute indirectly to Γ_{RJ} in the denominator of Eq. (10a). Since the 2^+ and 3^- excitations are much more likely than that of the 0_2^+ state, the contributions of the latter should be rather small in the present calculation and it is reasonable to exclude them.

We conclude that based on the 2^+ inelastic results reported here the projection operator method appears to provide a reasonable way of studying inelastic light ion scattering.

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[1] See, e.g., *Clustering Aspects of Nuclear Structure and Nuclear Reactions (Winnipeg, 1978),* Proceedings of the Third International Conference on Clustering Aspects of Nuclear Structure and Nuclear Reactions, edited by W. T. H. Van Oers and J. P. Svenne (AIP, New York, 1978).

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