

Degree of nonlocality of various exchange contributions to the nucleus-nucleus potential

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(Received 25 March 1981)

The degree of nonlocality introduced into the nucleus-nucleus interaction by the requirements of the Pauli principle is discussed on the basis of the properly orthogonalized equation of relative motion of the two nuclei, which is an equation for physical wave functions with a direct probability interpretation. Exact calculations of the kinetic energy exchange potential, the Coulomb exchange potential, and the pure nuclear exchange potential are performed for the α - α system. Even in this simple example, a variety of very different structures can be obtained for the nuclear exchange potential, depending on which nucleon-nucleon interaction is chosen.

[NUCLEAR REACTIONS Scattering theory, α - α calculated exact re-normalized resonating group potentials, nonlocality functions for kinetic, Coulomb, and nuclear contributions.]

I. INTRODUCTION

If we neglect the influence of the Pauli principle, then the potential describing the interaction of two composite nuclei with rigid internal configurations can easily be derived in a microscopic way by folding the nucleon-nucleon interaction v_{NN} with the internal single-nucleon densities ρ_1, ρ_2 of the nuclei. If v_{NN} is local in the nucleon-nucleon separation, then this folding potential V_f (often called the “double folding potential”) is local¹:

$$V_f(\vec{r}) = \int d\vec{r}_1 \int d\vec{r}_2 \rho_1(\vec{r}_1) \rho_2(\vec{r}_2) v_{NN}(\vec{r} - \vec{r}_1 + \vec{r}_2). \quad (1)$$

In a fully antisymmetrized theory such as the resonating group method (RGM), where the indistinguishability of all nucleons involved is treated exactly, the nucleus-nucleus interaction contains a direct contribution of the form (1) and very complicated highly nonlocal exchange terms whose origin is in the exchange of nucleons between the two nuclei.^{2,3}

However, the equation of motion of the RGM has the form of an eigenvalue equation in a nonorthogonal basis and the RGM wave functions of relative motion must not be interpreted as probability amplitudes. In the last few years there has been an increasing awareness, that the effect of this nonorthogonality must be eliminated by renormal-

izing the RGM wave function if a comparison with wave functions and potentials of a conventional Schrödinger equation is to be meaningful.⁴⁻⁸ In contrast to the “nucleus-nucleus potential” appearing in the nonorthogonal representation of the RGM, the potential in the orthogonalized representation need not *a priori* be highly nonlocal, in fact, for purely harmonic nucleus-nucleus interactions it is exactly given by the local folding potential (1).⁶ This explains why, e.g., the elastic α - α scattering data can be described by a conventional Schrödinger equation with an attractive, energy and angular momentum independent α - α potential, although the influence of the Pauli principle is definitely not negligible.⁶

Using these insights it has been possible to derive simple local nucleus-nucleus potentials which approximate the potential in the orthogonalized representation of the RGM for a number of systems,⁸⁻¹⁰ but so far the “renormalized RGM potentials” were never calculated exactly. The aim of the present paper is to study the exact structure of the nucleus-nucleus potential in the orthogonalized representation of the RGM equation in order to see whether or not the exact consideration of the requirements of the Pauli-principle does in fact lead to highly nonlocal nucleus-nucleus potentials.

The relevant formulas of the RGM and the transition to an orthogonalized representation of the equation of relative motion are summarized briefly

in Sec. II. In Sec. III the structure of the contributions of kinetic, Coulomb, and nuclear potential energy to the nucleus-nucleus interaction are discussed for the example of the α - α system. A discussion of the results and their implications is given in Sec. IV.

II. NUCLEUS-NUCLEUS POTENTIALS

According to the formalism of the resonating group method (RGM), the relative motion of two nuclei with (rigid) internal states $\phi_1(\xi_1)$ and $\phi_2(\xi_2)$, respectively, is described in the subspace of many-nucleon wave functions of the form

$$\Psi_\phi = \mathcal{A} \{ \phi_1(\xi_1) \phi_2(\xi_2) \phi(\vec{x}) \} , \quad (2)$$

where ϕ is an unknown wave function of relative motion and \vec{x} is the separation of the centers of mass of the two nuclei. The rest-antisymmetrizer \mathcal{A} accounts for all possibilities of exchanging nucleons between the two nuclei. The requirement that the full microscopic many-nucleon Hamiltonian \mathcal{H} be diagonal in the space spanned by the wave functions (2) leads to an equation of motion for the wave function ϕ . In compact operator notation this equation reads

$$H\phi = EA\phi , \quad (3)$$

where H and A are integral operators whose kernels are matrix elements of the Hamiltonian \mathcal{H} and the unit operator:

$$\begin{bmatrix} H(\vec{r}, \vec{r}') \\ A(\vec{r}, \vec{r}') \end{bmatrix} = \left\langle \phi_0(\vec{r}) \left| \begin{bmatrix} \mathcal{H} \\ 1 \end{bmatrix} \right| \phi_0(\vec{r}') \right\rangle \quad (4)$$

in the basis of many nucleon wave functions

$$\phi_0(\vec{r}) = \mathcal{A} [\phi_1(\xi_1) \phi_2(\xi_2) \delta(\vec{x} - \vec{r})] , \quad (5)$$

which span the subspace (2). Throughout this paper the energy scale is chosen such that the internal energy of two separated fragment nuclei, which is defined by the expectation values of the internal Hamiltonians $\mathcal{H}_1, \mathcal{H}_2$ in the internal wave functions ϕ_1, ϕ_2 , is zero: $E_{\text{int}} = \langle \phi_1 | \mathcal{H}_1 | \phi_1 \rangle + \langle \phi_2 | \mathcal{H}_2 | \phi_2 \rangle = 0$.

It is customary to write Eq. (3) in the form of a Schrödinger equation by splitting the RGM Hamiltonian and overlap operators, H and A , into direct and exchange parts:

$$H = T_{\text{rel}} + V_f + W , \quad (6)$$

$$A = 1 - K , \quad (7)$$

where $T_{\text{rel}} = -(\hbar^2/2\mu) \Delta$ is the kinetic energy of relative motion and V_f is the folding potential (1). In (6) and (7) W and K are short ranged, Hermitian, and highly nonlocal operators arising from the exchange parts of the matrix elements (4).

Equation (3) for ϕ may now be written as

$$(T_{\text{rel}} + V_f + W + EK)\phi = E\phi , \quad (8)$$

which looks like a Schrödinger equation with a local direct potential V_f and a highly nonlocal, explicitly energy dependent nonlocal "exchange potential"

$$W + E \cdot K . \quad (9)$$

Equation (8), is however, misleading in that the energy dependence of the "potential" is trivially due to the fact that the equation of motion (3) has the form of an eigenvalue equation in a nonorthogonal basis, which means that the "wave functions" ϕ cannot be interpreted as probability amplitudes.

We can remove the effect of this nonorthogonality by introducing renormalized wave functions⁴⁻¹⁰

$$\psi = A^{1/2} \phi , \quad (10)$$

for which the equation of motion reads^{4,6,8}

$$\Lambda H_{\text{coll}} \psi = E \psi , \quad (11)$$

with the Hermitian and energy independent collective Hamiltonian of relative motion H_{coll} defined by

$$A^{1/2} H_{\text{coll}} A^{1/2} = H . \quad (12)$$

For uniqueness $A^{1/2}$ may be chosen to be the Hermitian positive semidefinite square root of the RGM overlap operator A .

The equation of motion in the correctly orthogonalized representation (11) contains a projection operator Λ which annihilates the so-called "redundant states." These are short ranged states of relative motion forbidden by the Pauli principle; they are eigenstates of the RGM overlap operator A with vanishing eigenvalues. Matrix elements of the collective Hamiltonian H_{coll} containing one or two redundant states are not determined by Eq. (12) and do not enter into the equation of motion (11). The solutions of Eq. (11) are automatically orthogonal to all redundant states (except for $E=0$).

Assuming H_{coll} in Eq. (11) to be given by the direct part of the RGM Hamiltonian leads to the equation

$$\Lambda (T_{\text{rel}} + V_f) \psi , \quad (13)$$

which is the original version of Saito's orthogonal-

ty condition model (OCM) (Ref. 11), and has been quite a successful approximation of the full RGM. Replacing V_f by a more general effective nucleus-nucleus potential V_{eff} leads to a generalization of Saito's OCM

$$\Lambda(T_{\text{rel}} + V_{\text{eff}})\psi = E\psi, \quad (14)$$

which has been shown to be a very good approximation of the full RGM in a number of cases, even if V_{eff} is assumed to have a very simple local structure.⁸⁻¹⁰ A further improvement of the OCM is given by Schmid's fish bone model,¹² which includes a nonlocal potential with a structure similar to the structure of the exchange contribution of the kinetic energy operator to the nucleus-nucleus interaction. For an RGM based on purely harmonic nucleon-nucleon forces, the simple OCM (13) is exactly equivalent⁶ to the full RGM.

The generalized OCM equation (14) is exactly equivalent to the full RGM equation (3), if the effective potential V_{eff} obeys the factorization equation

$$H = A^{1/2}(T_{\text{rel}} + V_{\text{eff}})A^{1/2}. \quad (15)$$

Resolving Eq. (15) for V_{eff} yields

$$V_{\text{eff}} = A^{-1/2}HA^{-1/2} - T_{\text{rel}}, \quad (16)$$

where the inverse operator $A^{-1/2}$ obeys

$$A^{1/2}A^{-1/2} = \Lambda \quad (17)$$

and is only defined in the space of nonredundant states. Following Ref. 13 the natural decomposition of V_{eff} into direct and exchange parts is (in the space of nonredundant states):

$$V_{\text{eff}} = V_f + V_{\text{ex}}, \quad (18)$$

so that the correct definition of the exchange part V_{ex} of the nucleus-nucleus potential is

$$V_{\text{ex}} = \Lambda(V_{\text{eff}} - V_f)\Lambda, \quad (19)$$

with the effective potential of Eq. (16) and the folding potential of Eq. (1). Strictly speaking, matrix elements of V_{ex} containing redundant states are not determined uniquely; in Eq. (19) they are made to vanish by the projectors Λ .

In the nonorthogonal representation (3) and (8) of the equation of relative motion, the influence of the Pauli principle appears to be clear: Neglecting antisymmetrization leads to the simple local folding potential and the inclusion of exchange effects is responsible for the nonlocal terms in the

nucleus-nucleus interaction; using the fact that $\mathcal{A}^\dagger \mathcal{A} (= \mathcal{A}^2)$ is proportional to \mathcal{A} , the exchange contributions are usually decomposed in a seemingly natural way into a sum of contributions originating from one-nucleon exchange, two nucleon exchange, etc.^{14,15} However, the nonorthogonal representation of the equation of relative motion is a poor basis for physical interpretation.

In the correctly orthogonalized equation (14) [containing the exact effective potential (16)], we can still clearly distinguish between direct and exchange contributions. The folding potential V_f is still the direct part of the nucleus-nucleus potential; the exchange contribution is given by V_{ex} defined by Eq. (19). It is now no longer obvious that the exchange potential (i.e., V_{ex}) must necessarily be highly nonlocal. Also, there is no obvious decomposition of V_{eff} into a sum of contributions from one-nucleon exchange, two nucleon exchange, etc. If only some of the permutations in \mathcal{A} are included in the bra state on the right hand side of Eq. (4), then the same permutations must be included in the ket¹⁶; otherwise the corresponding overlap operator defined by Eqs. (4) and (5) need no longer be positive semidefinite and the whole procedure of renormalization leading to Eq. (14) becomes meaningless. On the other hand, including, e.g., only the permutations exchanging one nucleon between the two nuclei in both bra and ket in Eq. (4) means that the matrix elements will also contain terms conventionally interpreted as two-nucleon exchange contributions and even direct contributions.

It has been shown that the full exchange potential V_{ex} in the properly orthogonalized equation (14) can be approximated quite well by a local potential in a number of examples,^{6,8-10} but so far V_{ex} has never been calculated exactly. In the following section we discuss the various contributions to V_{ex} in the simple and much studied α - α system.

III. EXCHANGE CONTRIBUTIONS TO THE NUCLEUS-NUCLEUS POTENTIAL

The RGM Hamiltonian H defined by Eq. (4) contains contributions T , V_C , and V_N coming from the microscopic many-body operators for the kinetic energy of relative motion, the Coulomb potential energy, and the pure nuclear potential energy, respectively. Correspondingly, the effective potential (16) and the exchange potential (19) can be written as a sum of contributions from kinetic,

Coulomb potential, and nuclear potential energy, i.e.,

$$V_{\text{ex}} = V_{\text{ex}}^T + V_{\text{ex}}^C + V_{\text{ex}}^N. \quad (20)$$

With Eqs. (16) and (19) we have:

$$V_{\text{ex}}^T = \Lambda(A^{-1/2}TA^{-1/2} - T_{\text{rel}})\Lambda, \quad (21)$$

$$V_{\text{ex}}^C = \Lambda(A^{1/2}V_C A^{-1/2} - V_f^C)\Lambda, \quad (22)$$

$$V_{\text{ex}}^N = \Lambda(A^{-1/2}V_N A^{-1/2} - V_f^N)\Lambda, \quad (23)$$

where V_f^C and V_f^N are the Coulomb and nuclear parts of the folding potential (1), respectively.

In the following subsections the various exchange contributions (21)–(23) are discussed for the α - α example. The potentials are calculated exactly using numerical techniques developed by Fiebig and Timm.^{13,17} The internal states of the α particle are described by $(0s)^4$ harmonic oscillator wave functions with $\hbar\omega = 17.83$ MeV. The redundant states are oscillator states of relative motion with principle quantum numbers $N = 2n + l$ less than four.⁸ The nonredundant eigenstates of the RGM overlap operator A defined by Eq. (4) are the other oscillator states; the corresponding eigenvalues μ_N depend only on the principle quantum number N and are given by⁸

$$\mu_N = 1 - 4 \cdot \left(\frac{1}{2}\right)^N + 3\delta_{N,0}. \quad (24)$$

The most natural representation of the potentials (21)–(23) is in the basis of eigenstates of A , i.e., in the harmonic oscillator basis. In order to illustrate how local or nonlocal certain contributions are we also use coordinate representation. In the partial waves $l=0$ and 2 the projectors Λ in Eqs. (21)–(23) introduce a substantial nonlocality, which is spurious in the sense that the structure of the potential is only relevant in the space of nonredundant states. A unique definition of the potential in coordinate representation is possible for the partial waves $l \geq 4$, where there are no redundant states.

A. The “kinetic energy exchange potential”

The matrix elements of the kinetic energy part T of the RGM Hamiltonian have a particularly simple form in the basis of (spherical) oscillator states $\phi_{nlm}(\vec{r})$, which are eigenstates of the RGM overlap operator:

$$\begin{aligned} \langle \phi_{nlm} | T | \phi_{n'l'm'} \rangle \\ = \delta_{l,l'} \delta_{m,m'} \mu_N \langle \phi_{nlm} | T_{\text{rel}} | \phi_{n'l'm'} \rangle, \end{aligned} \quad (25)$$

where N is the smaller of the numbers $2n + l$, $2n' + l$. A simple formula of the form (25) holds¹⁸ for all systems of two SU_3 -scalar clusters if the corresponding microscopic many-body operator (in this case, kinetic energy) separates exactly into contributions acting only on the internal coordinates of the respective nuclei and a relative motion part acting only on the relative distance coordinate \vec{x} [see Eq. (2)]. In Eq. (25), only the diagonal ($n = n'$) and first nondiagonal ($n = n' \pm 1$) matrix elements are different from zero:

$$\langle \phi_{n,l} | T_{\text{rel}} | \phi_{n,l} \rangle = \frac{1}{2} \hbar\omega(2n + l + \frac{3}{2}), \quad (26)$$

$$\begin{aligned} \langle \phi_{n,l} | T_{\text{rel}} | \phi_{n+1,l} \rangle \\ = -\hbar\omega[(n+1)(2n+2l+3)/8]^{1/2}. \end{aligned} \quad (27)$$

(All matrices are independent of and diagonal in the quantum number m which is dropped from now on.)

From Eqs. (25)–(27) the kinetic energy exchange potential (21) can easily be given analytically in the harmonic oscillator basis ($2n + l > 2$):

$$\langle \phi_{n,l} | V_{\text{ex}}^T | \phi_{n,l} \rangle = 0, \quad (28)$$

$$\begin{aligned} \langle \phi_{n,l} | V_{\text{ex}}^T | \phi_{n+1,l} \rangle &= (\sqrt{\mu_{2n+l}/\mu_{2(n+1)+l}} - 1) \\ &\times \langle \phi_{n,l} | T_{\text{rel}} | \phi_{n+1,l} \rangle. \end{aligned} \quad (29)$$

The exchange potential V_{ex}^T has a highly nonlocal structure in coordinate space. However, its magnitude is very much smaller than the direct part T_{rel} . The reason for this lies in the factor

$$F_N = \sqrt{\mu_N/\mu_{N+2}} - 1, \quad (30)$$

which is quite small. From Eq. (24) we have $F_4 = -0.106$, $F_6 = -0.024$, $F_8 = -0.006$, etc., in the α - α case. The sign of the matrix elements of V_{ex}^T is positive in the present case, but this can be changed by redefining the oscillator basis states. As an operator, V_{ex}^T is indefinite and can neither be said to be attractive nor repulsive. The statement that the kinetic energy exchange potential is very weak and nonlocal can be generalized to all systems in which the nonvanishing eigenvalues $\mu_{n,l}$ of the RGM overlap operator A are, for fixed l , slowly varying functions of n . This is in fact a typical property of the eigenvalue spectrum of A , particularly in heavy systems, where the eigenvalues $\mu_{n,l}$ approach the limit unity (from below or above) very gradually.⁸

B. The Coulomb exchange potential

Numerical values of the diagonal matrix elements $\langle \phi_{nl} | V_{\text{ex}}^C | \phi_{nl} \rangle$ are plotted in Fig. 1 against $N = 2n + l$ for partial waves $l = 0$ to $l = 6$ in the α - α system. The nondiagonal matrix elements are smaller in magnitude and have alternating signs.

In comparison with the direct Coulomb potential, which varies between 1 MeV and about 2.5 MeV for α - α separations between 6 and 2 fm, the matrix elements of V_{ex}^C in Fig. 1 are small. The potential is predominantly attractive since all expectation values in oscillator states are negative. For fixed values of $N = 2n + l$ the matrix elements depend weakly on l and their magnitude decreases with increasing l .

In order to illustrate the behavior of V_{ex}^C in coordinate space, we have extracted from the nonlocal expression $V_{\text{ex}}^C(\vec{r}, \vec{r}')$ in each partial wave a "potential function"

$$U_{\text{ex}}^C(r) = \int_{-2r}^{2r} V_{\text{ex}}^C \left[r + \frac{s}{2}, r - \frac{s}{2} \right] ds, \quad (31)$$

and [for $U_{\text{ex}}^C(r) \neq 0$] a "nonlocality function"

$$D_r^C(s) = V_{\text{ex}}^C \left[r + \frac{s}{2}, r - \frac{s}{2} \right] / U_{\text{ex}}^C(r), \quad (32)$$

which is normalized by

$$\int_{-2r}^{+2r} D_r^C(s) ds = 1. \quad (33)$$

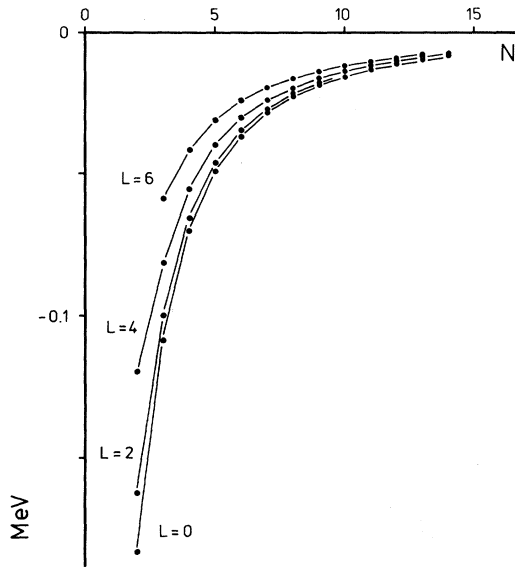


FIG. 1. Diagonal matrix elements of the α - α Coulomb exchange potential V_{ex}^C in the harmonic oscillator basis.

We use the convention in which radial matrix elements are defined by

$$\langle \phi_l | V | \phi_l' \rangle = \int_0^\infty dr \int_0^\infty dr' \phi_l(r) V(r, r') \phi_l'(r').$$

Figure 2 shows the potential function (31) in the partial waves $l = 4$ and $l = 6$. The potential function is predominantly attractive and very much weaker than the direct Coulomb potential V_f^C .

Figure 3 shows the nonlocality function (32) for $l = 4$ and $l = 6$ for various separations r . The simple, short ranged structure of D_r^C indicates that the approximation of the exchange potential by a local potential or by a nonlocal potential of simple structure, such as has been used in simple phenomenological models,¹⁹ can be reasonable in the correctly orthogonalized version (14) of the equation of relative motion. Such a simple ansatz would be entirely inconsistent and wrong if used for the exchange term appearing in the nonorthogonal representation (8) of the equation.

C. The nuclear exchange potential

The form of the nuclear exchange potential V_{ex}^N depends of course on the nucleon-nucleon interac-

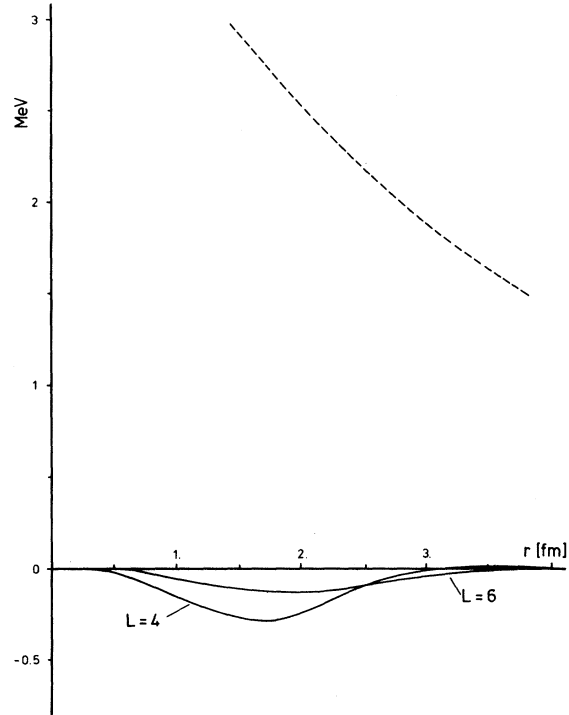


FIG. 2. Potential functions (solid lines) derived from the α - α Coulomb exchange potential in the partial waves $l = 4$ and $l = 6$. The dashed line is the direct Coulomb potential V_f^C .

tion v_{NN} on which the microscopic treatment of the nucleus-nucleus system is based. For a purely harmonic nucleon-nucleon interaction

$$v_{NN}(\vec{r}) = m\omega^2 r^2 / 2(A_1 + A_2), \quad (34)$$

the exchange potential V_{ex}^N precisely cancels the kinetic energy exchange potential V_{ex}^T , so that in this case the simple version (13) of the OCM equation, in which the nucleus-nucleus potential is assumed to be merely the folding potential (1), is exactly equivalent to the full RGM (see Ref. 8 and the Appendix of Ref. 6). This is true for all two-nucleus systems.

In the following we discuss the nuclear exchange potential V_{ex}^N in the α - α system for three different nucleon-nucleon potentials, which have been used extensively in microscopic studies of nuclear structure and nuclear scattering involving light nuclei.

First we assume v_{NN} to be the potential which was used by Thompson *et al.*²⁰ (with the parameter $y=0.92$), which is quite close to a Serber force. In this case the folding potential V_f is a Gaussian

$$V_f(\vec{r}) = V_1 \exp(-r^2/a_1^2) \quad (35)$$

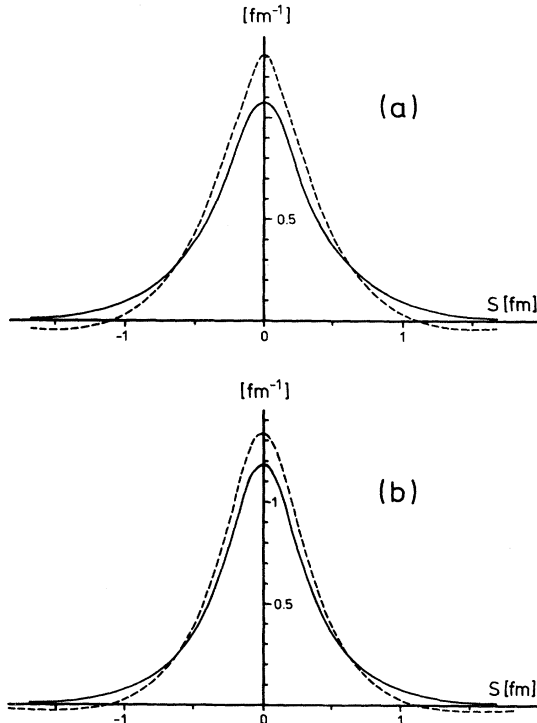


FIG. 3. Nonlocality functions (32) derived from the α - α Coulomb exchange potential in the partial waves $l=4$ [(a) solid line corresponds to $r=1.65$ fm, dashed line to $r=2.56$ fm] and $l=6$ [(b) solid line $r=2$ fm, dashed line $r=3.4$ fm].

with the parameters

$$V_1 = -81.72 \text{ MeV}, \quad a_1 = 2.38 \text{ fm}. \quad (36)$$

The diagonal matrix elements of V_{ex}^N in the oscillator basis are displayed in Fig. 4 for the partial waves $l=0$ to $l=8$. The nondiagonal matrix elements of V_{ex}^N are smaller in magnitude than the diagonal matrix elements of the same row or column and have alternating sign. The angular momentum dependence of V_{ex}^N follows the same pattern as for V_{ex}^C discussed in Sec. III B.

Although the direct potential (35) and (36) is purely attractive, the negative sign of all (diagonal) matrix elements in Fig. 4 shows that the exchange potential V_{ex}^N is also at least predominantly attractive. This shows that an attractive direct potential need not necessarily be associated with a repulsive exchange potential.

In coordinate representation the nuclear exchange potential contributes together with the kinetic energy exchange potential (and the Coulomb exchange potential) to the nonlocal part of the nucleus-nucleus interaction. In the following we discuss the behavior of the sum of kinetic and nuclear potential contributions

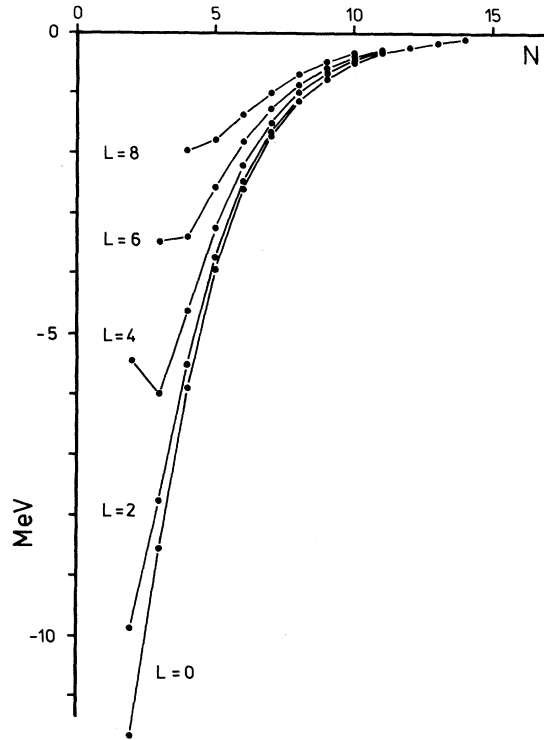


FIG. 4. Diagonal oscillator matrix elements of the nuclear exchange potential V_{ex}^N based on the nucleon-nucleon interaction of Thompson *et al.* (Ref. 20).

$$V_{\text{ex}}^{TN} = V_{\text{ex}}^T + V_{\text{ex}}^N \quad (37)$$

in coordinate space. This quantity vanishes identically for the purely harmonic nucleon-nucleon interaction (34).

Figure 5 shows the potential functions $U_{\text{ex}}^{TN}(r)$ defined in analogy to Eq. (31) in the partial waves $l=4$ and $l=6$. The potential functions are small in magnitude compared with the folding potential V_f^N . The nonlocality functions D_r^{TN} , defined in analogy to Eq. (32), are shown in Fig. 6 for $l=4$ and two different values of r . In the region where the magnitude of the potential function is large ($1 \text{ fm} \lesssim r \lesssim 2 \text{ fm}$) the nuclear exchange potential (including kinetic exchange contributions) appears to be well represented by an almost local potential in which the nonlocal kernel $V_{\text{ex}}^{TN}(r, r')$ is different from zero only in a narrow region of up to about 1 fm from the diagonal. The degree of nonlocality becomes larger for larger α - α separations ($\approx 3 \text{ fm}$) but the magnitude of the potential is already quite small here.

The results summarized in Figs. 4–6 are modified considerably if we replace the Gaussian

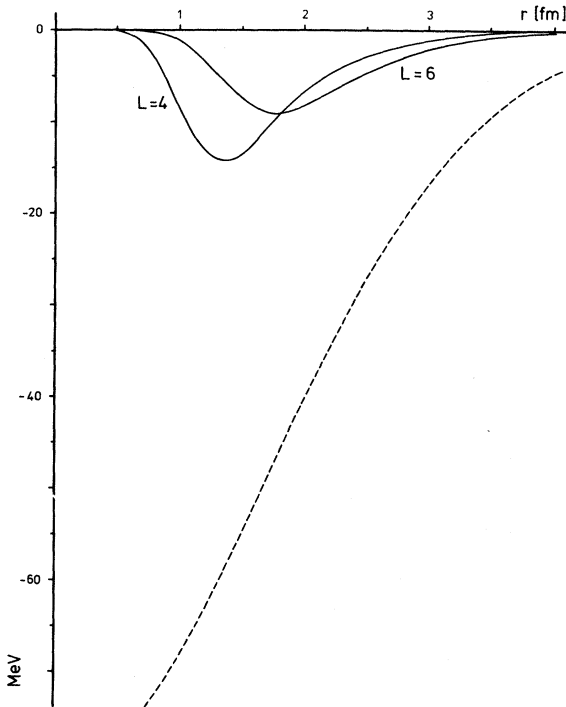


FIG. 5. Potential functions (solid lines) derived from the α - α kinetic plus nuclear exchange potential V_{ex}^{TN} defined by Eq. (37) on the basis of the nucleon-nucleon interaction of Thompson *et al.* (Ref. 20). The dashed line is the corresponding folding potential (1).

nucleon-nucleon interaction of Thompson *et al.*²⁰ by the interaction $V1$ of Volkov²¹ (with a Majorana exchange parameter $M=0.6$). The spatial part of this interaction is a sum of two Gaussians, and hence, the resulting folding potential (1) contains two terms of Gaussian form (35) with the parameters

$$\begin{aligned} V_1 &= -91.25 \text{ MeV}, & V_2 &= +35.69 \text{ MeV}, \\ a_1 &= 2.44 \text{ fm}, & a_2 &= 2.08 \text{ fm}. \end{aligned} \quad (38)$$

The diagonal oscillator matrix elements $\langle \phi_{nl} | V_{\text{ex}}^N | \phi_{nl} \rangle$ derived from the $V1$ force (see Fig. 7) are negative for $2n+l < 8$ and positive for $2n+l > 8$. This indefinite property of the operator V_{ex}^N is reflected in the potential functions $U_{\text{ex}}^{TN}(r)$ (shown in Fig. 8), which are positive at short separations and mainly negative at larger separations. The structure of the nonlocality function (Fig. 9) now depends sensitively on the separation r . In the region of positive (repulsive) potential functions the nonlocality is small, while the structure of the potential is substantially nonlocal in the region where the potential function is negative.

The nucleon-nucleon interaction $B1$ of Brink and Boeker²² also consists of two Gaussian terms. The corresponding folding potential is given by

$$\begin{aligned} V_1 &= +447.52 \text{ MeV}, & V_2 &= -190.27 \text{ MeV}, \\ a_1 &= 1.99 \text{ fm}, & a_2 &= 2.33 \text{ fm}, \end{aligned} \quad (39)$$

and has the peculiarity of being predominantly repulsive. This repulsive direct potential is largely

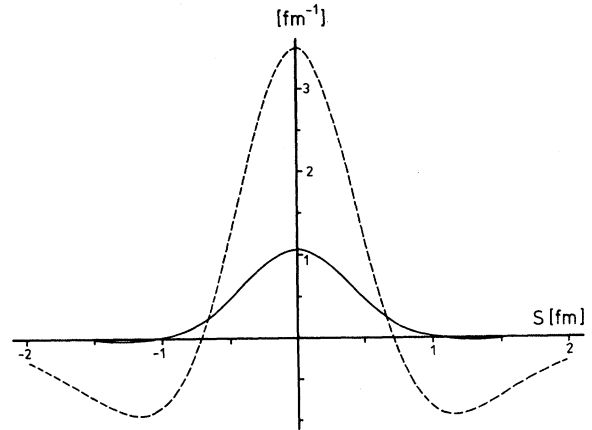


FIG. 6. The nonlocality function derived from the α - α kinetic plus nuclear exchange potential V_{ex}^{TN} on the basis of the nucleon-nucleon interaction of Thompson *et al.* (Ref. 20) for $l=4$ and $r=1.34 \text{ fm}$ (solid line) and $r=2.87 \text{ fm}$ (dashed line).

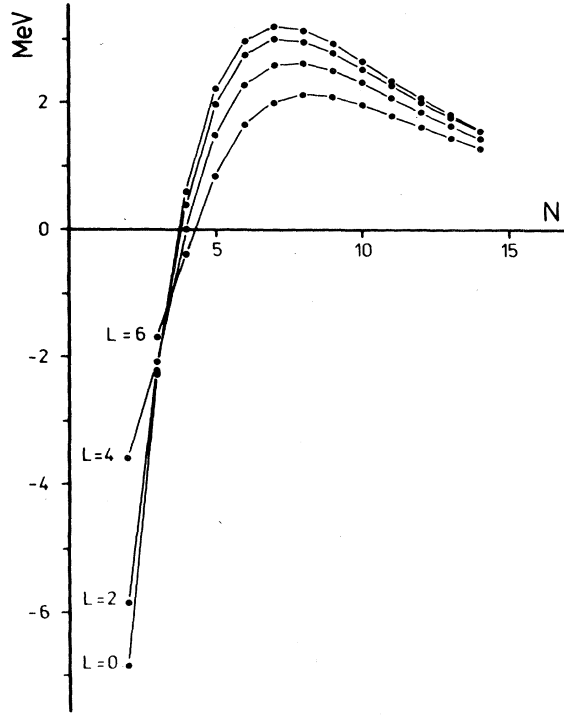


FIG. 7. Diagonal oscillator matrix elements of the α - α nuclear exchange potential based on Volkov's nucleon-nucleon interaction V_1 .

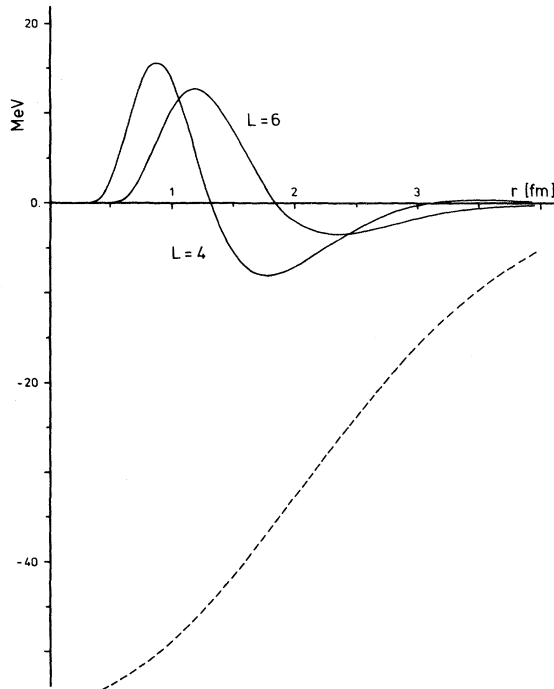


FIG. 8. Potential functions (solid lines) derived from the α - α kinetic plus nuclear exchange potential based on Volkov's V_1 nucleon-nucleon interaction. The dashed line is the corresponding folding potential (1).

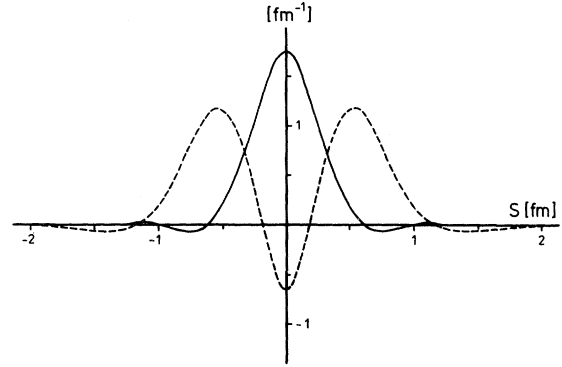


FIG. 9. Nonlocality function derived from the α - α kinetic plus nuclear exchange potential (based on Volkov's V_1 nucleon-nucleon interaction) for $l=4$ and $r=0.88$ fm (solid line) and $r=1.8$ fm (dashed line).

compensated by a very strong attractive exchange potential V_{ex}^N , as is shown in Fig. 10, where the potential function $U_{ex}^{TN}(r)$ derived from the B_1 nucleon-nucleon interaction is plotted for $l=4$ and $l=6$. The degree of nonlocality is quite small in this case, as is illustrated by the nonlocality functions shown in Fig. 11.

Finally, Fig. 12 displays the sum of the direct folding potential and the potential function derived from the nonlocal exchange potential V_{ex}^{TN} for

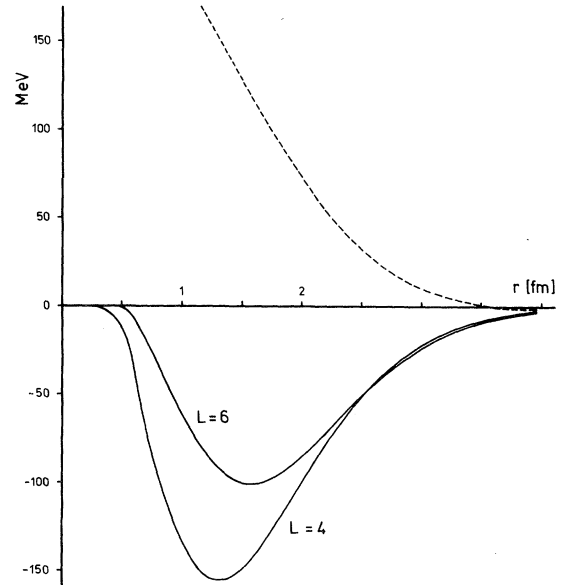


FIG. 10. Potential functions (solid lines) derived from the α - α kinetic plus nuclear exchange potential based on the B_1 nucleon-nucleon interaction of Brink and Boeker. The dashed line is the corresponding folding potential (1).

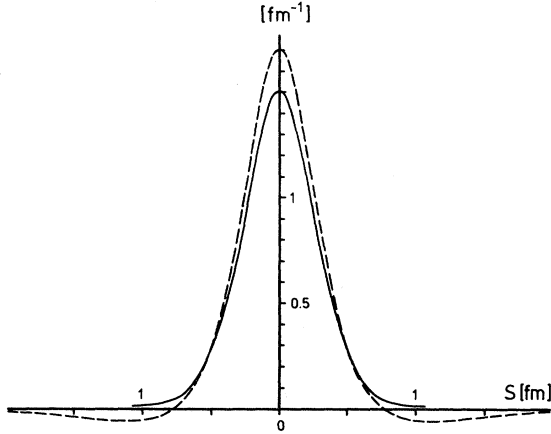


FIG. 11. Nonlocality functions derived from the α - α kinetic plus nuclear exchange potential (based on the $B1$ nucleon-nucleon interaction) for $l=4$ and $r=1.3$ fm (solid line) and $r=2.7$ fm (dashed line).

$l=4$. The three curves correspond to the three different nucleon-nucleon interactions discussed in this section. If the nonlocality function were a delta function, these curves would represent the exact α - α potential (neglecting Coulomb contributions) to be inserted in the orthogonalized representation of the equation of relative motion.

Except for small separations (less than ca. 2 fm) all three potentials are quite similar. This is remarkable considering how different the underlying nucleon-nucleon interactions and the corresponding folding potentials [cf. Eqs. (36), (38), and (39)] are. It is consistent with the observation that all three forces yield a reasonable description of the α - α scattering data and of the resonances forming the ground state band of ^8Be if used in the framework of a completely antisymmetrized theory.^{9,20}

IV. CONCLUSION

The nucleus-nucleus potentials discussed in this paper are potentials in the correctly orthogonalized representations (11) and (14) of the equation of relative motion and can be directly compared with potentials appearing in phenomenological models based on a Schrödinger equation of relative motion of the two nuclei. In contrast, the “potentials” appearing in the nonorthogonal version (8) of the equation of relative motion have no direct physical interpretation. In these unphysical “potentials” the exchange of nucleons between two nuclei (resulting from the requirements of the Pauli principle) leads to highly nonlocal, explicitly energy dependent

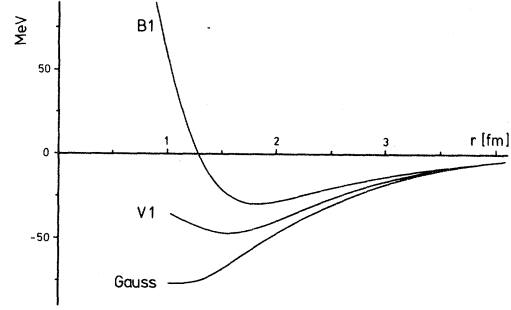


FIG. 12. The respective sums of the α - α folding potential (1) and the potential function $U_{\text{ex}}^{\text{TN}}$ derived from the kinetic plus nuclear exchange potential $V_{\text{ex}}^{\text{TN}}$ in the partial wave $l=4$ for the different nucleon-nucleon interactions: $B1$, $V1$, and the Gaussian $N-N$ interaction of Thompson *et al.* (Ref. 20).

contributions which cannot be consistently approximated by local potentials or by nonlocal potentials of simple structure. Such a general statement cannot be made for the exchange potentials in the correctly orthogonalized equation of relative motion.

The kinetic energy exchange potential is in general an indefinite (i.e., neither attractive nor repulsive) highly nonlocal potential with matrix elements which are small in comparison with the matrix elements of the direct kinetic energy operator. In the α - α example, the Coulomb exchange potential was shown to be weak and predominantly attractive with a short ranged nonlocality of simple structure.

The results of Sec. III C show, however, that a variety of different structures can be obtained in the exchange potentials based on different nucleon-nucleon interactions, even in such a simple system as α - α :

A purely harmonic nucleon-nucleon interaction (34) leads to an exchange potential which exactly cancels the kinetic energy exchange potential.

The Gaussian nucleon-nucleon interaction of Thompson *et al.*²⁰ yields an attractive direct (=folding) potential and a weak, predominantly attractive, and almost local exchange potential.

The $V1$ nucleon-nucleon interaction of Volkov²¹ yields an attractive folding potential and a weak, indefinite exchange potential of considerable nonlocality.

The $B1$ nucleon-nucleon interaction of Brink and Boeker²² yields a repulsive direct potential and a predominantly attractive, almost local exchange potential. In contrast to all other examples, the ex-

change potential is not weak but is of the same order of magnitude as the direct contribution in this case.

From these examples it is clear that the influ-

ence of the Pauli Principle on the structure of nucleus-nucleus potentials is very complex and depends very sensitively on the choice of the nucleon-nucleon interaction.

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