Higher order effects on the magnetic form factor of ^{17}O

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We obtain the magnetic form factor of ^{17}O in a microscopic effective operator approach which includes self-consistency effects, first order core polarization, and second order number conserving sets of effective operator diagrams. The self-consistency effects yield a reduction in the cross section ranging from 10% to 20% through the region of experimental data. Core polarization reduces the $M3$ contribution by a factor of 2 as compared to a factor of 3 reduction obtained by Arima, Horikawa, Hyuga, and Suzuki. In addition, we find the core polarization yields a slight $M1$ enhancement at its peak and a substantial $M5$ reduction so that the overall agreement with experiment is poor. We then explore second order core polarization effects and evaluate the two number conserving sets of diagrams. Their contributions are non-negligible but fail to resolve the discrepancy between theory and experiment. We also include an exchange current amplitude. The lack of agreement between theory and experiment invites theoretical attention to additional corrections and to phenomena which may enhance magnetic scattering.

[NUCLEAR REACTIONS $^{17}O(e,e)$; magnetic form factor, theory including higher order effects. Effective operator within self-consistent framework.]

I. INTRODUCTION

There is considerable experimental[1] and theoretical[2,3] interest in the ground state magnetization density of ^{17}O. Our recent development of a systematic approach to self-consistency effects in core plus valence nucleon systems[4] provides a framework for studying this magnetic form factor in considerable detail. Here, we report the results of an application of our wave functions for ^{17}O to this experiment and we include higher order contributions to the effective operator in the manner they were included in the effective Hamiltonian for ^{17}O. We therefore extend the study of Ref. 2 by incorporating many new effects. First, we treat the valence nucleon in a self-consistent framework so the single particle (sp) insertions are included to all orders. This step results in a $d_{5/2}$ sp wave function for the ground state of ^{17}O which is strikingly similar to a Woods-Saxon wave function found successful in (d,p) analyses.[5] Second, we incorporate the perturbative corrections to the ^{17}O magnetic form factor in the Brandow effective operator expansion[6] utilizing the Brueckner G matrix calculated from the Reid soft core potential.[7,8] This extends the work of Ref. 2, which employed a phenomenological interaction for the core polarization effects. Third, we explore the effects of the next order in perturbation theory by evaluating the two number conserving sets of diagrams. We do not repeat the evaluation of the exchange current contribution which was found to be small and positive in Ref. 2. The primary purpose of the present effort is to determine the extent to which our valence-core self-consistency effects, the use of a realistic two-body interaction, and higher order perturbative corrections can resolve the existing discrepancies between theory and experiment as they have been described in Ref. 1. Our conclusion is that, rather than resolving these discrepancies, these higher order effects enhance the disagreements and indicate a growing puzzle.

II. CALCULATIONS

The experimental data for the ^{17}O transverse magnetic form factor $F_T{}^2$ is plotted in Fig. 1 vs

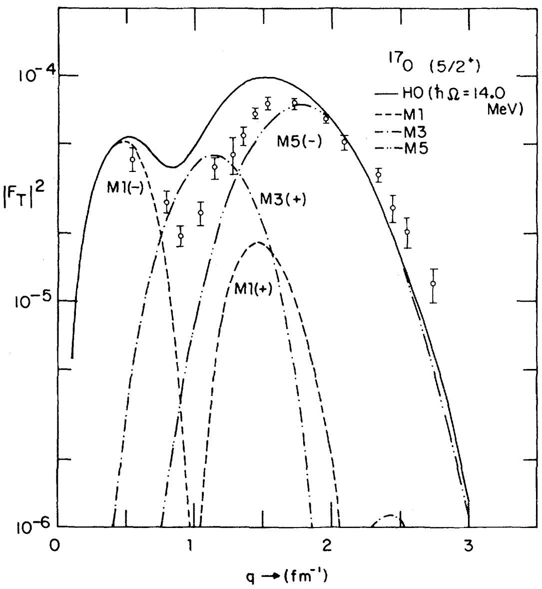

FIG. 1. Transverse magnetic form factor squared $F_T{}^2$ of ^{17}O vs effective momentum transfer q. The experimental data is taken from Ref. 1. The solid line is the sum of the $M1$, $M3$, and $M5$ contributions which are shown separately using their respective phases. These curves were obtained using a simple harmonic oscillator (HO) $d_{5/2}$ wave function.

q_{eff}, where q_{eff} is the effective momentum transfer[1] to be used in a plane wave Born calculation. We use q_{eff} throughout and refer to it simply as q. The data is compared with a calculation of $F_T{}^2$ using a pure harmonic oscillator (HO) $0d_{5/2}$ wave function with oscillator parameter $\hbar\Omega = 14$ MeV ($b = 1.72$ fm.). We also show the squares and phases of the individual $M1$, $M3$, and $M5$ form factors to aid in later discussions. Center of mass and finite nucleon size corrections have been included in all calculations by multiplying the single particle (sp) operator matrix elements by

$$\exp[-\tfrac{1}{4}(a_p{}^2 - b^2/17)q^2]$$

with $a_p = 0.657$ fm. The HO calculation does not reproduce the minimum around $q = 0.9$ fm^{-1} nor does it fit the large q data.

In our earlier treatment of the ^{17}O nucleus we found a self-consistent sp Hamiltonian which incorporated all multiparticle-multihole effects in perturbation theory up to second order in the reaction matrix G. This Hamiltonian was diagonalized in a very large valence space to yield sp energies in fair agreement with experiment and a self-consistent $d_{5/2}$ wave function quite similar to a Woods-Saxon wave function obtained in a (d,p) analysis[5] of ^{17}O. The $d_{5/2}$ wave function thus obtained should be a reasonable starting point for calculating matrix elements of other one-body operators in the $A = 17$ system since all perturbation diagrams involving Hartree-Fock insertions vanish. Thus, in zeroth and first order perturbation theory we must only consider the diagrams shown in Figs. 2(a) and 2(b). Diagrams such as that shown in Fig. 2(c) are automatically included in zeroth order.

Arima *et al.*[2] have studied the effects of Fig. 2(b) using HO wave functions and a phenomenological interaction and found a substantial reduction of the $M3$ multipole but little effect on the $M1$ and $M5$. This vastly improves the agreement with data in the region of the minimum but does not effect the rest of the curve. In particular, the $M5$ multipole is still underestimated.

The contributions of diagrams 2(a) and 2 (b) to the doubly reduced (spin and isospin) transition matrix element are given by

$$F_0^{\mathcal{J}\mathcal{T}} = \langle v \,|||\, \mathcal{O}^{\mathcal{J}\mathcal{T}} \,|||\, v \rangle \tag{1}$$

and

$$F_1^{\mathcal{J}\mathcal{T}} = 2\sum_{\substack{aA\\JT}} (-1)^{j_v + j_a + J + T + 1} \times \hat{J}\hat{T}(1+\delta_{av})^{1/2} \begin{Bmatrix} \frac{1}{2} & \frac{1}{2} & \mathcal{T} \\ \frac{1}{2} & \frac{1}{2} & T \end{Bmatrix} \times \begin{Bmatrix} j_v & j_v & \mathcal{J} \\ j_A & j_a & J \end{Bmatrix} \frac{\langle vAJT \,|\, G(\omega_A) \,|\, vaJT \rangle \langle a \,|||\, \mathcal{O}^{\mathcal{J}\mathcal{T}} \,|||\, A \rangle}{E_A - E_a} \,. \tag{2}$$

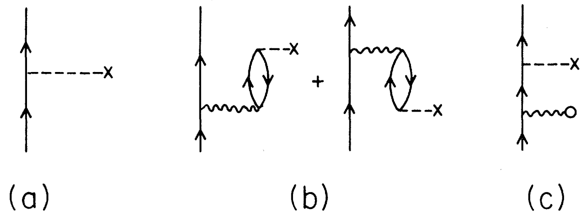

FIG. 2. Lowest order diagrams in the effective operator expansion for a single valence nucleon interacting with an external field. Diagram (c) is automatically included in our zeroth order self-consistent calculation.

Here v represents the $d_{5/2}$ sp state, $\hat{J}$ is $(2J+1)$, capital letters A are summed over hole (core) states, and small letters are summed over valence and unoccupied states. The reaction matrix G is based on the Reid soft core potential[7] and is evaluated[8] at the starting energy $\omega_A = E_A + E_v$.

A. Self consistent sp wave functions

The effect of replacing the HO $0d_{5/2}$ wave function by the self-consistent wave function (SCWF) in evaluating the zeroth order term is shown by the solid curve in Fig. 3. We find a uniform reduction of 10% to 20% in all three multipoles. This reduction improves the fit to the data at low q, where the data lies below the HO values but increases the discrepancy in the $M5$ region where the data already lies above the HO calculation. This reduction can be understood qualitatively by noting that the SCWF is more spread out than the HO wave function. Thus, the use of a SCWF yields results similar to those we would obtain using an HO wave function with a larger size parameter b. This reduction of the $M5$ multipole with the use of "realistic" sp wave functions was also shown in Ref. 1, where a calculation of $F_T{}^2$ was carried out using a Woods-Saxon wave function generated from a potential chosen to fit the energy levels in the $A=15$ and 17 nuclei and the ^{16}O elastic electron scattering data. Thus, it seems clear that improvements in the sp $d_{5/2}$ wave function will not lead to substantially better agreement with the data and we must examine more complicated structure effects.

B. First order core polarization

In the calculation of Ref. 2, HO wave functions and energies were used to describe the sp states and $G(\omega_A)$ in Eq. (2) was replaced by a phenomenological (Gaussian with Rosenfeld) exchange interaction. We refer to the phenomenological potential as a Rosenfeld interaction. Our calculation of F_1

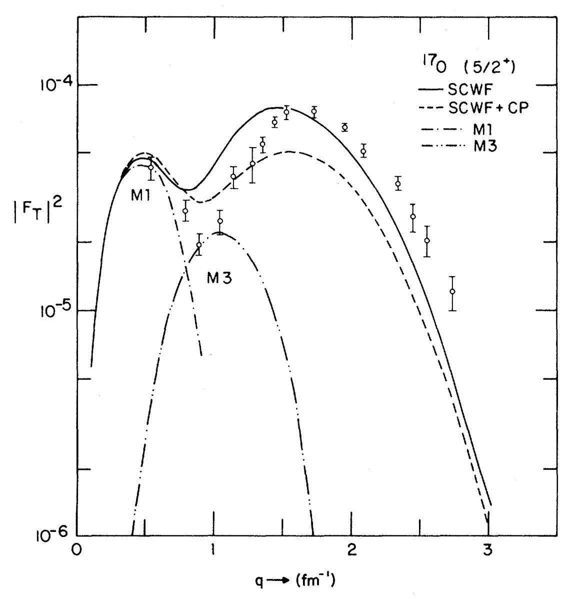

FIG. 3. Transverse magnetic form factor squared $F_T{}^2$ vs effective momentum transfer q. The first peaks of the $M1$ and $M3$ amplitudes squared are shown for the SCWF plus CP calculation. Contrast these with the results in Fig. 1.

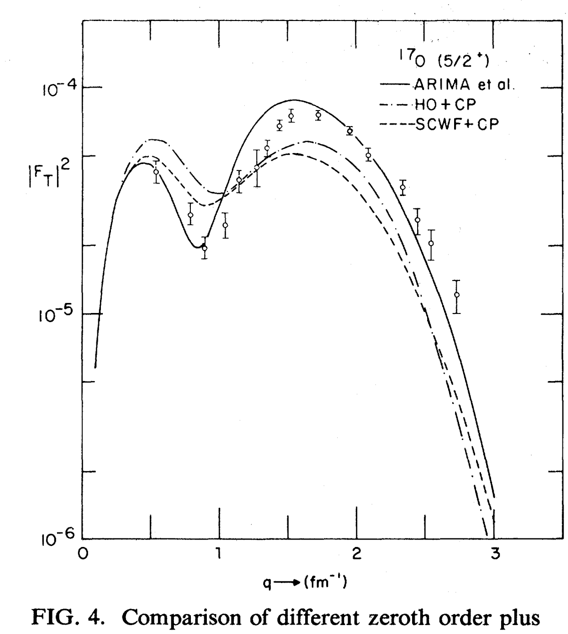

FIG. 4. Comparison of different zeroth order plus first order core polarization calculations for the transverse magnetic form factor squared $F_T{}^2$ vs effective momentum transfer q. The solid curve corresponds to the Rosenfeld force calculation of Ref. 2 and does not include the exchange current contribution.

was checked by first reproducing the results obtained in Ref. 2. We then improved the calculations in a number of ways. We first repeated the calculations using HO wave functions and energies and a G matrix based on the Reid interaction. Intermediate state summations were carried through 10 unoccupied major shells but were well converged after four major shells. The results we obtained are substantially different from those obtained with the Rosenfeld interaction, as can be seen in Fig. 4. We do find a substantial reduction of the $M3$, though not as large as for the Rosenfeld force. However, the Reid potential also gives a substantial enhancement and broadening of the $M1$, filling in the minimum in F_T^2 around $q=0.8-1.2$ fm. Moreover, we find significant reduction in the $M5$ and the second peak of the $M1$ so as to substantially reduce the cross section at the second maximum and beyond.

A third calculation of F_T^2 was then carried out using the G matrix, the self-consistent $d_{5/2}$ wave function, and the self-consistent sp spectrum obtained in our earlier work. It should be noted that only the sp energies of the intermediate states employed in this study are self-consistent, not their wave functions. The results of this calculation are shown in both Figs. 3 and 4 and it is apparent by comparing these figures that the core polarization contribution is approximately the same, whether or not self-consistency is included in the $d_{5/2}$ wave function. However, the core polarization is reduced somewhat when the self-consistent spectrum is used due to larger absolute values of E_A-E_a and smaller values of $\omega_A=E_A+E_v$. Since both self-consistency effects and core polarization tend to reduce the $M5$ multipole, the curve falls well below the data at large q. It is perhaps interesting to note that the slopes of the total form factor curve at large q is somewhat improved by the use of the SCWF even though the absolute magnitude disagrees.

C. Higher order core polarization

Since the contribution of Fig. 2(b) is large it is important to consider still higher terms in the perturbation expansion for the effective operator. One then must face the usual question of which diagrams to include and which ones to ignore. We have calculated the contribution of the second order (in G) number conserving sets of diagrams shown in Fig. 5. These particular sets of diagrams are important for a number of reasons. First of all, they generally exhibit a substantial degree of cancellation and thus should not lead to an overestimate of second order effects. They cancel exactly if the single particle operator is replaced by the momentum independent number operator. Thus, it is interesting to study the degree to which they cancel as a function of momentum transfer. Finally, since we have seen that the first order core polarization term is large, we must have a sizable $2p$-$1h$ component in the nuclear wave function and the zeroth order results should be renormalized. The inclusion of the second order number conserving sets accounts for this renormalization.

Expanding briefly on this latter point we can write the ^{17}O wave function schematically as

$$\psi=\alpha\,|\,1p\,\rangle+\beta\,|\,2p,1h\,\rangle+\gamma\,|\,3p,2h\,\rangle\ ,$$

where we will choose all amplitudes to be real. Then for any Hermitian single particle operator $\hat{O}$,

$$\langle\psi\,|\,\hat{O}\,|\,\psi\rangle=\alpha^2\langle 1\mathrm{p}\,|\,\hat{O}\,|\,1\mathrm{p}\rangle+\beta^2\langle 2\mathrm{p},1h\,|\,\hat{O}\,|\,2\mathrm{p},1\mathrm{h}\rangle+\gamma^2\langle 3\mathrm{p},2\mathrm{h}\,|\,\hat{O}\,|\,3\mathrm{p},2\mathrm{h}\rangle+2\alpha\beta\langle 1\mathrm{p}\,|\,\hat{O}\,|\,2\mathrm{p},1\mathrm{h}\rangle$$
$$+2\beta\gamma\langle 2\mathrm{p},1\mathrm{h}\,|\,\hat{O}\,|\,3\mathrm{p},2\mathrm{h}\rangle\ .$$

There is no term involving $\alpha\gamma$ since a single particle operator cannot connect the $|1\mathrm{p}\rangle$ and $|3\mathrm{p},2\mathrm{h}\rangle$ states. The term with $2\alpha\beta$ contains the first order core polarization results. Figures 5(a), 5(d), and 5(e) combine to reduce the coefficient of the zeroth order term to α^2, Figs. 5(b) and 5(c) contribute to the β^2 term, while Figs. 5(f) and 5(g) contribute to the γ^2 term. The number conserving sets do not give a completely correct normalization since they do not include diagrams coupling the $|2\mathrm{p},1\mathrm{h}\rangle$ component to the $|3\mathrm{p},2\mathrm{h}\rangle$ component; i.e., we are neglecting the $2\beta\gamma$ term. We would expect this term to be small since both β and γ are small and there is no obvious coherence. The expressions for Figs. 5(a)$-$5(g) can be found in Ref. 9. These diagrams were calculated in the oscillator basis but with the self-consistent set of single particle energies.

The results obtained by adding the second order contribution to the self-consistent zeroth and first order results are shown in Fig. 6. We find the second order contribution to be much smaller than

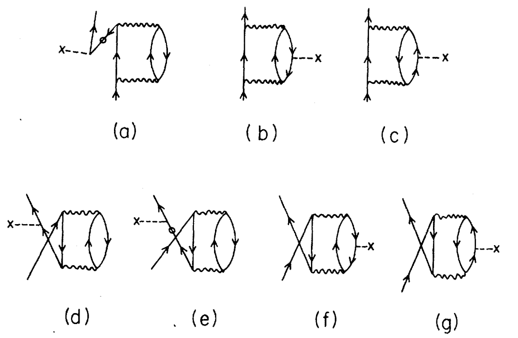

FIG. 5. Second order number conserving diagrams in the effective operator expansion for a single valence nucleon interacting with an external field.

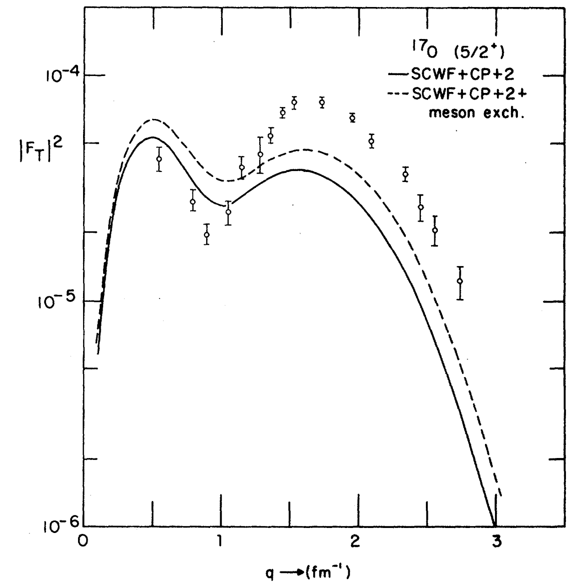

FIG. 6. Transverse magnetic form factor squared $F_T{}^2$ vs effective momentum transfer q. Both curves include the second order number conserving sets of diagrams which is signified by the " + 2" in the label.

the first order but definitely important. The normalization diagrams [Figs. 5(a), 5(d), and 5(e)] combine to reduce the zeroth order term by approximately 15% and the intermediate sums have converged fairly well after the *s-d-g* shell is included. The remaining number conserving diagrams cancel the normalization diagrams when the number operator is evaluated. (This was used as a test of our codes.) However, we find the remaining diagrams do not cancel the normalization diagrams for the magnetic multipole operators. Instead, we find a slight enhancement of the $M1$ for low q and a reduction for large q. The $M3$ is slightly reduced and the $M5$ substantially reduced over the entire range of momentum transfer. Including more states in the intermediate summations does not affect the results appreciably. We also show in Fig. 6 the complete results through second order in G along with a meson exchange contribution taken from the work of Arima *et al.*[2] The meson exchange contribution helps at large q but the overall results do not compare favorably with experiment.

III. DISCUSSION

The large discrepancy between our results and those reported in Ref. 2 arises predominantly from two effects. Our use of a SCWF resulted in a 10% to 20% reduction in $F_T{}^2$ for all q. The remaining difference arises mainly from our use of a reaction matrix based on the Reid soft core potential while a phenomenological Rosenfeld potential was used in Ref. 2.

It might be argued that the Rosenfeld interaction is a better "effective" interaction, somehow taking into account higher order processes which we omit. We would argue, however, that structure calculations should not be done with purely phenomenological interactions, since systematic corrections (such as the evaluation of higher order diagrams) are then meaningless. It is, of course, not obvious that the Reid potential is a sufficiently accurate nucleon-nucleon interaction and we plan to repeat our calculations with other realistic interactions.

The discrepancy between theory and experiment is not easily understood. The ^{16}O nucleus is not a good closed core and its wave function is known to contain appreciable 2p,2h and 4p,4h admixtures. However, we have included at least a part of the 2p,2h states in perturbation theory and the effects go in the wrong direction. We would expect a more complete calculation to reinforce our results, unless the many-particle, many-hole states are the dominant effect in the cross section. In that case, our perturbation treatment would need modifications to specifically include such many-particle, many-hole states directly.

We have included in Fig. 6 a meson exchange contribution taken from Ref. 2. The size of this contribution is significant and suggests that further work on meson exchange effects, baryon resonances, relativistic effects, or the inclusion of three-body forces might be fruitful. In particular, these effects may influence the form factor at large q where the discrepancy between theory and experiment is greatest.

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