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Beta decay properties using a statistical model

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Beta decay properties ($t_{1/2}$, $\bar{E}_\beta$, $\bar{E}_\gamma$, $\bar{E}_n$, and P_n) are calculated for fission product nuclei by a statistical model and are found to be in good agreement with experiment.

[RADIOACTIVITY Rb, Sr, I, Xe; calculated $T_{1/2}$, $\bar{E}_\beta$, $\bar{E}_\gamma$, $\bar{E}_n$, P_n; compared to experiment.]

In a recent Brief Report,[1] Klapdor points out e evidence for structure in the beta strength function and notes its importance to neutrino oscillation experiments. In other articles,[2] he and his co-workers have shown that the inclusion of this structure is crucial in obtaining reliable results for β-decay half-lives and the spectra of β-delayed particles. However, in this paper we will show that indeed it is possible to accurately predict beta decay properties with a statistical model having only one free global parameter.

The beta decay properties of fission products are important in the design and operation of fission reactors, affecting the length of the reactor (or asymptotic) period and the amount of decay heat after reactor shutdown. The beta decay half-life also affects the path of the r process in stellar nucleosynthesis. In addition, the interpretation of reactor neutrino oscillation experiments depends on the assumed spectra of antineutrinos emitted in the the fission product beta decay.

The present work assumes that any intrinsic structure is not important. The beta strength function S_β^λ for a transition to a state in the daughter at energy E via multipole λ is given by

$$S_\beta^\lambda(E)dE=\sum_{J,\pi}\rho(E,J,\pi)\beta_\lambda(E)dE/D,$$

where $\rho(E,J,\pi)$ is the density of levels with spin J and parity π at excitation energy E, β_λ is the average reduced transition probability per level for moment λ in the interval $(E,E+dE)$, and D is the vector coupling constant ($\sim$ 6250 s). The half-life is given by

$$t_{1/2}{}^{-1}=\sum_\lambda\int_0^{Q_\beta}S_\beta^\lambda(E)f_\lambda(Z,Q_\beta-E)dE,$$

where f contains kinematic factors and the Fermi function. Average beta and gamma energies depend on similar integrals. Delayed neutron emission is calculated by assuming that the daughter is a compound nucleus which then statistically decays as in the Hauser-Feshbach approach.[3]

Using the ENDF/B-V fission product file which contains 877 nuclei,[4] energy-dependent β_λ's were found for allowed $0^+\rightarrow1^+$ transitions (50 cases) and for other allowed transitions (over 600 cases), corresponding to logft values of 4.3 and 5.6, respectively. No dependence on either transition energy or on mass was found. First forbidden transitions use a β_λ corresponding to logft of 7.1.

Energies, spins, and parities for low lying states were taken from the Table of Isotopes.[5] Where these were unknown, the quantities were taken from neighboring nuclei, assuming that paired nucleons could be ignored. Although logft values (and hence, β_λ's) are known for some cases, in the calculations described below, only the derived β_λ's were used. Above the discrete states, a continuum description based on the back shifted Fermi gas formula

$$\rho\propto\exp[2a(E-\Delta)]$$

is used whose parameters are based on the work of Holmes *et al.*[6] In the calculation of beta rates, the level density parameter a was multiplied by $N/(N+Z)$, where N is the number of neutrons and Z is the number of protons in the daughter nucleus. This modification is the only free parameter in this model and is taken as fixed for all the calculations. The value chosen was based on calculations of half-lives of Rb isotopes and is not unique.

The physical basis for this modification is unclear. An (N/A) modification is equivalent to considering only levels in the daughter nuclide based on neutron single particle states. However, in β decay the states based on proton single particle states would be the ones of importance if any are. Models using the Gamow-Teller resonance[2] predict a slower increase in the relevant level density than in the normal level density. Whether the N/A modification ($\sim$ 0.6 for the nuclei studied here) is mocking this effect is not known.

Figure 1 presents the predicted half-lives of Rb isotopes for which results have been published for the gross theory[7] and the microscopic calculations of Klapdor.[2] As can be seen, the present statistical approach predicts the measured values[5] quite well and significantly better than the gross theory. Similar agreement is seen in Table I between the present work and measured half-lives for Sr, I, and Xe isotopes, implying that the modification of the level density parameter has some significance.

Increasing the modification factor from N/A to $N/A + 0.1$ ($\sim$ 0.6 to $\sim$ 0.7) decreases the predicted half-lives by $\sim$30% to $\sim$50% for $Q=5$ and 10 MeV, respectively, and results in poorer agreement with experiment half-lives than the N/A modification. Using unmodified nuclear level densities, the predicted half-lives decrease by 90% (for $Q_\beta=5$ MeV) and 97% (for $Q_\beta=10$ MeV) from the N/A modified results. Thus without a modification, the predictions for half-life are in strong disagreement with experiment.

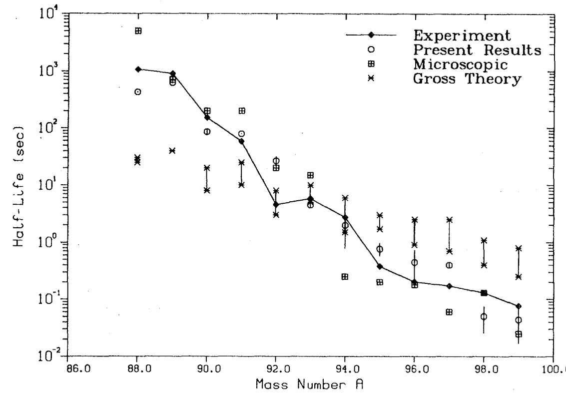

FIG. 1. Half-lives of Rb isotopes.

TABLE I. Beta decay half-lives (s); $Q_\beta > 2.5$ MeV.

Isotope	Experimental	Present model[a,b]	Gross theory[a]
^{93}Sr	444.0	440.0 ±350.0 (0.99±0.79)	25.0 (0.06)
^{94}Sr	75.0	52.0 ± 11.0 (0.69±0.15)	80.0 (1.1)
^{95}Sr	24.4	31.0 ± 21.0 (1.3 ±0.9)	8.0 (0.33)
^{96}Sr	1.1	0.95± 0.50 (0.86±0.45)	10.0 (9.1)
^{97}Sr	0.40	1.7 ± 1.0 (4.3 ±2.5)	2.5 (6.3)
^{98}Sr	0.7	0.35± 0.18 (0.50±0.26)	3.0 (4.3)
^{99}Sr	0.6	2.3 ± 1.6 (3.8 ±2.7)	1.0 (1.7)
^{136}I	46.0	17.5 ± 1.9 (0.38±0.04)	50.0 (1.1)
^{137}I	24.5	19.3 ± 4.7 (0.79±0.19)	4.0 (0.16)
^{138}I	6.5	1.9 ± 1.2 (0.29±0.18)	3.0 (0.46)
^{139}I	2.3	1.6 ± 1.1 (0.70±0.48)	2.0 (0.87)
^{140}I	0.8	2.7 ± 0.6 (3.4 ±0.8)	1.0 (1.3)
^{141}I	0.5	0.25± 0.17 (0.50±0.34)	1.0 (2.0)
^{137}Xe	229.0	81.0 ± 3.0 (0.35±0.01)	12.0 (0.05)
^{138}Xe	846.0	1420. ±130 (1.7 ±0.2)	20.0 (0.02)
^{139}Xe	39.7	18.4 ± 10.7 (0.46±0.27)	8.0 (0.20)
^{140}Xe	14.0	95.6 ± 8.2 (6.8 ±0.6)	8.0 (0.57)
^{141}Xe	1.73	7.3 ± 0.8 (4.2 ±0.5)	3.0 (1.7)
^{142}Xe	1.2	0.84± 0.07 (0.70±0.06)	2.5 (2.1)
^{143}Xe	0.96	1.08± 0.64 (1.1 ±0.68)	1.5 (1.6)
^{144}Xe	1.2	1.34± 0.81 (1.1 ±0.67)	1.2 (1.0)
^{145}Xe	0.9	0.81± 0.48 (0.90±0.53)	1.0 (1.1)

[a]The ratio to experiment is given in parenthesis.
[b]Uncertainty is due to uncertainty in Q_β value.

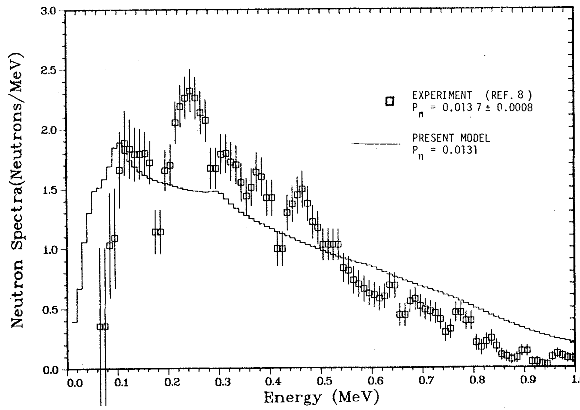

FIG. 2. Delayed neutron spectrum of ^{93}Rb.

Figure 2 compares the measured delayed neutron spectrum[8] with the model results for ^{93}Rb. Since the low-lying discrete states of the granddaughter are the only structure of significance in the present model, the fine structure observed in delayed neutron experiments is not predicted. However, the overall shape is well described, as is the probability of delayed neutron emission[9] ($P_n - \exp = 0.0137 \pm 0.0008$, P_n — model — 0.0131).

Other beta decay properties such as average beta and average gamma energies are also well predicted. However, such average quantities are experimentally well known only near the line of beta stability, where few transitions are involved. Far from the line of stability where a statistical approach should work best, the experimental data is uncertain[10] or unavailable.

It should be noted that although the approach seems useful when many levels are involved, the discrepancies are larger when only a few levels are involved; for example, for the low mass iodines, where only 3 allowed transitions are known, the use of an averaged logft value leads to half-life predictions off by a factor of 10. Whether such discrepancies are a result only of statistical fluctuations or point to important nuclear structure awaits further investigation. However, average energies still seem to be relatively well predicted even in these cases.

In conclusion, for some important fission product nuclei, half-lives, and other beta decay properties can be accurately calculated using a simple statistical approach, which has only one free global parameter.

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