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Model for the spin, isospin, and energy dependence of pion-nucleus annihilation

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The pion-nucleus optical potential is extended to include terms describing the effect of pion annihilation on *pn* and *pp*/*nn* pairs with accompanying possibility of spin flip. An isobar model is generalized and fit to pion production data in all charge channels, and then the strengths for different spin-isospin absorption channels are projected out as a function of energy.

[NUCLEAR REACTIONS Pion annihilation, absorption, function of energy, isospin, spin, optical potential.]

I. INTRODUCTION

Since the pion is a boson it can be annihilated in a nucleus and convert its mass into energy. By removing particles from the elastic channel and sometimes emitting back into it, annihilation influences both the absorption and the dispersion of pions by nuclei—the first process contributing to the imaginary part of the optical potential, the second process contributing to the real part. While it has been known for quite some time that annihilation is the dominant contributor to the imaginary part of the potential near zero energy,[1–3] the recent experiments of Johnson *et al.*[4] made it clear that annihilation is also of major importance at low energies, while the experiments of Belotti *et al.*[5] and Navon *et al.*[6] have proven its importance at intermediate energies.

Formally, one cannot add in the annihilation channel to a multiple scattering theory without accounting for the role of annihilation in generating both the pion-nucleon (πN) force [Figs. 1(a) and (b)] and the nucleon-nucleon (NN) force [Fig. 1(c)]. Although this problem has not yet been solved for nuclei[7,8] (except formally, as by Mizutani and Koltun[9]), a variety of studies of the simplier πNN system[9] have shown that the elastic scattering amplitude can be separated into a piece "MS," which has multiple scattering but *no* annihilation, and another piece, "ABS," which contains all the annihilation contributions (plus some MS):

$$\langle \pi NN \mid T \mid \pi NN \rangle = T^{\rm MS} + T^{\rm ABS} . \quad (1)$$

In most of the works of Ref. 9 the condition necessary to avoid double counting in $T^{\rm MS}$ is that the πN P_{11} interaction not contain the nucleon pole term [Fig. 1(a)], since this is the origin for absorption. Although belated, these developments are actually the theoretical basis for earlier phenomenological studies such as those by Brueckner, Thouless, and Beder.[10]

In contrast to the πd system where the annihilation makes a minor contribution to the total cross section, it can be the *dominating* process in the π-nucleus system. In this case, Mizutani and Koltun,[9] Rinat,[7] and Kowalski *et al.*[8] have emphasized that a separation such as Eq. (1) *may* hold for the πA T matrix and after further approximation for the optical potential. This would provide the

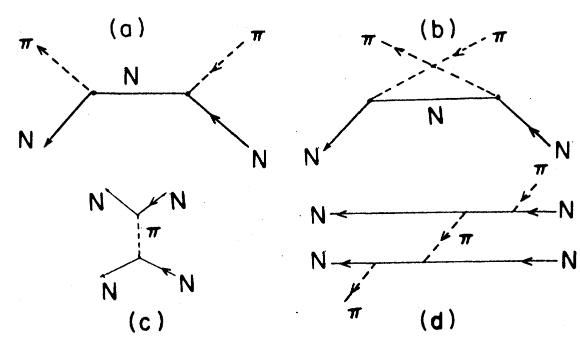

FIG. 1. Scattering: (a) and (b) πN, (c) NN, (d) π-NN via absorption or double πN scattering.

theoretical basis for adding an absorption term to the optical potential as done originally by Ericson and Ericson,[2] and recently updated.[3,11,12] A key assumption of this model, which is still without firm theoretical basis, is that energy-momentum conservation biases pion annihilation towards occurring on two nucleons within the nucleus, and consequently it is represented by an optical potential proportional to the nuclear density squared. The imaginary "rho-squared" terms describe S-wave (B_0) and P-wave (C_0) πNN absorption, and the real terms describe dispersion ($\pi NN \to NN \to \pi NN$):

$$V^{\mathrm{ABS}}(r) = \frac{-4\pi A^2}{2\mu_{\pi A}} \times [B_0 \rho^2(r) + C_0 \vec{\nabla} \cdot \rho^2(r) \vec{\nabla}]. \tag{2}$$

More recently, Landau and Thomas[13] have introduced a $\vec{p}$-space version of (2) which incorporates the finite range of the πNN interaction [via $g(p)$] and can be extended to higher partial waves:

$$V^{\mathrm{ABS}}(\vec{k}',\vec{k};E) = -4\pi[2\mu_{\pi A}(2\pi)^3]^{-1} A(A-1) \times \left[B_0(E) \frac{g_0(Q')g_0(Q)}{g_0{}^2(Q_E)} + C_0(E) \frac{g_1(Q')g_1(Q)}{g_1{}^2(Q_E)} \vec{k}' \cdot \vec{k} + D_0(E) \frac{g_2(Q')g_2(Q)}{g_2{}^2(Q_E)} k_E{}^4 P_2(\cos\theta_{k'k}) + \cdots \right] \hat{\rho}^2(\vec{k}' - \vec{k}). \tag{3}$$

Here, Q is the pion momentum in the πNN c.m. and ρ^2 is the Fourier transform of the isoscalar density squared. In general, V^{ABS} with phenomenological values of B_0 and C_0 is crucial in fitting pionic widths and shift,[2,3] and in reproducing the 30–50 MeV scattering.[4,11–13]

As shown by Ericson and Ericson[2,3] and others,[14] somewhat more than half of the phenomenological value of ImB_0 can be accounted for by using the experimental $\pi d \to pp$ cross section and the optical theorem (for the contribution to the elastic amplitude due to absorption). More recent works have estimated the strength of the ρ^2 terms via field theoretic models. Miller[15] investigated the importance of absorption on one nucleon and found $< 10\%$ effects at 50 MeV. Hachenberg *et al.*[16] used an effective Lagrangian field theory and claim that S wave absorption is sensitive to the long range part of the NN force (the final 2 N's should be a P state which vanishes at $r=0$). Their result is of the right order of magnitude (within $\sim 30\%$), but also very sensitive to the assumed off-shell behavior of the πN t matrix (a π scattering on one nucleon preceeds its absorption on the other).

In related and more extensive work, the Michigan State Group[17] has used the phenomenological, zero-range Hamiltonian of Hamilton and Woodruff, and Koltun and Reitan to calculate the threshold value and energy dependence of both the real and imaginary parts of B_0 and C_0 by treating them as terms in the second order optical potential. Here the NN force is treated carefully, ρ and π exchanges are included, and the model is successfully tested on the $\pi d \to pp$ problem before the quasideuterons are inserted into the nuclear environment. By using a Fermi gas model for the nucleon-nucleon wave function, and inserting form factors at the vertices to provide high momentum cutoffs, the latest, more "accurate" work of Chai and Riska determined $0.20\ \mu^{-4} < \mathrm{Im}B_0 < 0.27\ \mu^{-4}$ (vs 0.42 μ^{-4} for the phenomenological value) and $\mathrm{Im}C_0 \simeq 0.043\mu^{-6}$ (vs $0.036-0.07\mu$ as phenomenological values and vs 0.1μ as found in the earlier work of Ko and Riska). Here $\mu = m_\pi$, the pion mass. The real parts, which derive from a principal value integral, are very sensitive to the short range NN correlation and appear to be less reliable. Likewise, the energy dependence of ImC_0, while resonant in shape, is obtained only after imposing the condition that Im$C > 0$; probably it is only reliable at the lower energies.

Hofmann[18] and Oset and Weise[19] have also calculated absorption via the modificiation of the isobar hole theory—at the microscopic level—arising from absorptive effects on the Δ within nuclei. Of particular interest is their finding that up to ~ 50 MeV the 2p-2h excitations have similar behavior to the ρ^2 terms in the optical potential.

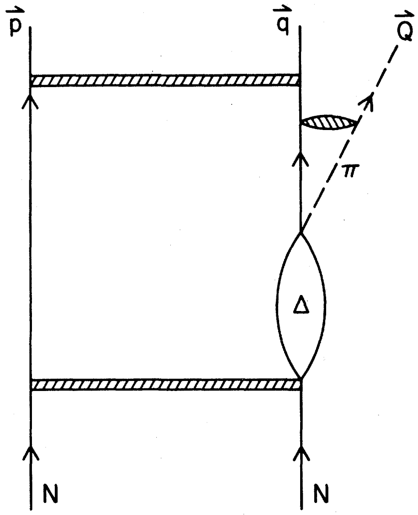

FIG. 2 Schematic representation of isobar model for $NN \rightarrow \pi NN$ showing formation of the isobar and the two body interactions included.

In the work presented here we lay the ground work for an extension of the rho-squared potential Eqs. (2) and (3) to a wider class of nuclei. Explicitly, we update and extend Mandelstam's isobar model of pion *production* in NN collisions,[20,21] to obtain the spin, isospin, and energy dependence of B, C, and D. The basis of the model, summarized in Fig. 2, is a pion being produced via the decay of a P_{33} resonant state formed from one of the incident nucleons. After the decay the final state NN and πN interactions are included.

We find that even with energy-independent production matrix elements this model is capable of producing excellent fits to the latest total cross section data in all charge channels from threshold to $T_\pi = 400$ MeV ($T_p \simeq 1.2$ GeV). By projecting out the channel dependence of the absorption amplitude, we obtain the energy, spin, and isospin dependences needed in a general optical potential. At present we are applying these results in a study of pion interactions with the three nucleon system.

A limitation in most models of nuclear absorption is the insertion of the two nucleon mechanism into the many body medium. We use the same procedure adapted by Ref. 2, which essentially assumes the absorption amplitude is of zero range and that the NN short range correlation function is similar in all nuclei. Furthermore, we do not address the question of the size of the real parts of the annihilation parameters, or if a more detailed dependence on the nuclear structure than ρ^2 is derivable. These are important concerns since they question the validity of all two nucleon absorption mechanisms, and of our phenomenological model. For example, a recent letter by Riska and Sarafian[32] states that the nuclear medium produces large effects on the pion propagator and the πNN vertex which—even after a strong cancellation—still produce a factor of 2 increase in the threshold, s-wave absorption strength for infinite nuclear matter. Possibly, after a more realistic model is developed,

TABLE I. Decomposition of the low partial wave pion production states in terms of the initial nucleons (col. 1), the intermediate ΔN state (col. 2), or the final (NN,π) state (col. 3). Column 3 is also decomposed into the isospin of the initial and final two nucleons.

NN Initial state	$(\pi N,N)$ Final state	(NN,π) Final state		
$^{2S+1}L_{J\pi}$	$[L(\pi N),\ L(\Delta N)_S]_{J\pi}$	$[^{2S+1}L(NN)_j,\ l(\pi,NN)]_J$		
$J = J_\pi + J_N + J_n$	$S = S_\Delta + S_N$	$j = j_N + j_N$ $I^{NN} = 1 \rightarrow 0[\sigma_{10}, \sigma_{10}(d)]$	σ_{11}	σ_{01}
$^3S_{1+}, {}^3D_{1+}$				$(^1S_0P)_1$
$^1S_{0+}$		$(^3D_1p)_0$	$(^3P_0s)_0$	
$^1D_{2+}$	$(\Delta S_2)_{2+}$	$(^3D_1p)_2$	$(^3P_2s)_2$	
$^3P_{0-}$	$(\Delta P_1)_{0-}$	$(^1P_1p)_0$	$(^1S_0s)_0, (^3P_1p)_0$	
$^3P_{1-}$	$(\Delta P_1)_{1-}, (\Delta P_2)_{1-}$	$(^3S_1s)_1, (^1P_1p)_1$	$(^3P_{0,1,2}p)_1$	
$^3P_{2-}, {}^3F_{2-}$	$(\Delta P_1)_{2-}, (\Delta P_2)_{2-}$	$(^1P_,p)_2$	$(^3P_{1,2}p)_2$	
$^3F_{3-}$	$(\Delta P_2)_{3-}$		$(^3P_2p)_3$	

it would also be valuable to deduce its spin, isospin, and energy dependence.

For the above reasons we think the relative size and energy dependence of the terms we calculate are more interesting than their absolute magnitudes. In Sec. II we describe the model and present our fits to the total cross sections. In Sec. III we relate the production amplitude to the optical potential parameters and determine their energy dependence. In Sec. III C we compare our results with those of some other recent works.

II. MODEL FOR PION PRODUCTION AND ABSORPTION CROSS SECTIONS

The conservation of angular momentum, parity, and isospin, combined with the generalized Pauli principle, constrain pion production to specific eigenchannels. For the energies under consideration, the important eigenchannels are given in Table I as states of the initial NN system (column 1), as states of the final $(\pi N,N)$ system (column 2), or as states of the equivalent (NN,π) system (column 3). There are nine possible NN states leading to a nucleon in an S- or P-wave state relative to the Δ (S- and P-wave "production" in Mandelstam's terminology), and with the pion in an s- or p-wave state relative to the two nucleons. As found a number of times,[9,22] the $^1D_2$ (NN) channel (S-wave production) dominates absorption at low energies (~ 50 MeV) since this is the only channel in which the Δ and other N is in a relative S state.

Isospin conservation relates 11 possible reactions to four independent isospin cross sections:

$$\sigma(pp\to\pi^+d)=\sigma(nn\to\pi^-d)=2\sigma(pn\to\pi^0d)=\sigma_{10}(d)\equiv\sigma_\gamma, \quad (4)$$

$$\sigma(pp\to\pi^+np)=\sigma(nn\to\pi^-pn)=2\sigma(pn\to\pi^0pn)=\sigma_{10}+\sigma_{11}\equiv\sigma_\alpha, \quad (5)$$

$$\sigma(pp\to\pi^0pp)=\sigma(nn\to\pi^0nn)=\sigma_{11}\equiv\sigma_\beta, \quad (6)$$

$$\sigma(np\to\pi^+nn)=\sigma(np\to\pi^-pp)=\tfrac{1}{2}(\sigma_{11}+\sigma_{01}), \quad (7)$$

$$\sigma(np\to\pi^0np)=\tfrac{1}{2}(\sigma_{10}+\sigma_{01}). \quad (8)$$

We note in (4) and (5) that there are different amplitudes if the np pair is bound as a deuteron (γ) or free (α). The corresponding absorption cross sectons are related via the principle of detailed balance; e.g., after spin summation,

$$\sigma(\pi^+d\to pp)=\frac{\frac{1}{2}(2S_p+1)^2P_p{}^2}{(2S_d+1)(2S_\pi+1)p_\pi{}^2}\times\sigma(pp\to\pi^+d)=(2P_p{}^2/3p_\pi{}^2)\sigma(pp\to\pi^+d), \quad (9)$$

$$\sigma(\pi^+np\to pp)=\frac{\frac{1}{2}(2S_p+1)^2P_p{}^2}{(2S_p+1)(2S_n+1)(2S_\pi+1)p_\pi{}^2}\times\sigma(pp\to\pi^+np)=(P_p{}^2/2p_\pi{}^2)\sigma(pp\to\pi^+np), \quad (10)$$

$$\sigma(\pi^0pp\to pp)=\frac{P_p{}^2}{p_\pi{}^2}\sigma(pp\to\pi^0pp), \quad (11)$$

$$\sigma(\pi^+nn\to np)=(2P_p{}^2/p_\pi{}^2)\sigma(np\to\pi^+nn). \quad (12)$$

The πN interaction part of Fig. 2 derives from the elastic scattering amplitude with the Chew-Low formula used for the πN phase-shift:

$$f(q)=\frac{e^{i\delta}\sin\delta}{q^2}=\frac{q}{\omega(1-\omega r)-\frac{4}{3}\left[\frac{f}{\mu}\right]q^3i}\frac{4}{3}\left[\frac{f}{\mu}\right]^2, \quad (13)$$

where $r=0.518m_\pi{}^{-1}$ represents the P_{33} effective range, f^2 is taken as 0.08, ω is the pion's total energy, and the extra q factor in the denominator arises from the removal of one of the πN vertices from the elastic amplitude. The independent kinematic variables for the reaction are either the pion momentum $\vec{Q}$ and the final NN relative momentum $\vec{P}$ (see Fig. 2), or the individual nucleon's momenta $\vec{q}=-\vec{P}-\vec{Q}/2$ and $\vec{P}=\vec{p}-\vec{Q}/2$.

The expressions for the production amplitudes depend upon the degree of integration over the momenta of the final state nucleons. As Mandelstam shows in his Eqs. (4.20)–(4.25), it is possible to separate the model's dependence on the angle θ between the produced pion and the incident nucleon, and then integrate over all other variables. In this way, as we do in the next section, the conventional $L_{\pi,D}=0$, 1, 2 absorption amplitudes are obtained.

Since the π moves out of the interaction region faster than the nucleons, and the NN force has a longer range than the πN one, it is assumed that the NN interaction continues after the πN interaction has ceased. We then "correct" for the (S wave) final state NN interaction by multiplying the S wave part of the elementary production amplitude by $\Omega(P)$,[23] the increase in the probability of finding two nucleons at their hard core radius caused by the attractive part of the appropriate NN interaction:

$$\Omega(P)+1=\frac{\left[-P_B+\frac{1}{2}r_0'c+iP\right]\left[c(1-r_0'P_B+\frac{1}{4}r_0'^2c)+a+bP^2\right]^{1/2}}{c(1-r_0'P_B+\frac{1}{4}r_0'^2c)}. \tag{14}$$

Here, r_0' is the effective range of the attractive part of the NN potential, $P_B=\sqrt{mE_B}$, a and b are related to the scattering length and effective range, and $c=P^2+P_B{}^2$.

Although the diagram in Fig. 2 appears simple, there is a number of kinematic integrations and spin-isospin-angular momentum decompositions needed to obtain the cross sections. For example, if we start with an initial pp state, there will be one amplitude for S state (Δ with respect to N) π production—the $^1D_2$ eigenchannel, and five possible P state amplitudes (if we ignore NN F states and conserve parity). These match the transitions indicated in Table I, and in that notation the P state amplitudes are $b_{JS}=b_{01}$, b_{11}, b_{12}, b_{21}, b_{22}. Some typical production amplitudes F_J^L, with all but the Q and θ dependence integrated numerically, are given in terms of these parameters by

$$F_{02}^t(Q,\theta)=a^2I_{01}^t(Q,\theta), \tag{15}$$

$$F_{10}^s(Q,\theta)=\tfrac{2}{27}\,|\,b_{01}\,|^2I_{10}^s, \tag{16a}$$

$$F_{10}^t(Q,\theta)=\tfrac{1}{27}\,|\,b_{01}\,|^3I_{11}^t\ , \tag{16b}$$

$$F_{11}^s(Q,\theta)=\tfrac{2}{9}\,|\,b_{11}\,|^2I_{11}^s. \tag{17a}$$

$$F_{12}^s(Q,\theta)=\tfrac{10}{27}\,|\,b_{21}\,|^2I_{12}^s, \tag{17b}$$

$$F_{11}^t(Q,\theta)=\tfrac{1}{27}\,|-b_{11}+\sqrt{5}b_{12}\,|^2I_{10}^t+\tfrac{1}{36}\,|\,b_{11}+\sqrt{5}b_{12}\,|^2I_{11}^t+\tfrac{1}{540}\,|\,5b_{11}+\sqrt{5}b_{12}\,|^2I_{12}^t(Q,\theta), \tag{18}$$

$$F_{12}^t(Q,\theta)=\tfrac{5}{108}\,|-b_{21}+3b_{22}\,|^2I_{11}^t+\tfrac{5}{36}\,|\,b_{21}+b_{22}\,|^2I_{12}^t, \tag{19}$$

where the superscript (s,t) indicates singlet or triplet outgoing nucleons. The integrals I's contain the Q and θ dependence of the amplitudes, the final state NN interaction [Eq. (14)], phase space factors $S(Q)$ [Eq. (20)], the Chew-Low factors $f(q)$ [Eq. (13)], and integrations over angles; they are given explicitly by Mandelstam's Eqs. (4.9)–(4.13) except for our more accurate phase space factor:

$$S(Q)=\frac{1}{\pi^2\Omega}\frac{m+T/2}{[(m+T/2)^2-m^2]^{1/2}}\frac{E_n(p)[T+2m-\Omega-E_n(p)]P^2Q^2}{(T+2m-\Omega)P+\frac{1}{2}[2E_n(p)-T-2m+\Omega]Q\cos\theta}, \tag{20}$$

and with an extra $P_B/4$ for a deuteron final state.

The formulas for the cross sections are now

$$\sigma_{pp\to\pi^+np}(\theta,Q)=\sum_\alpha\left[\tfrac{20}{9}(F_\alpha^s+F_\alpha^t+\tfrac{4}{3}(-F_\alpha^{s'}+F_\alpha^{t'})\right], \tag{21}$$

$$\sigma_{pp\to\pi^0pp}(\theta,Q)=\sum_\alpha\left[\tfrac{4}{9}(F_\alpha^s+F_\alpha^t+\tfrac{4}{9}(F_\alpha^{s'}-F_\alpha^{t'})\right], \tag{22}$$

with this same form for a deuteron in the final state but with different values of the I's and only triplet NN states contributing. The primes in Eqs. (21) and (22) denote production from the interchanged nucleon.

In the present version of the model, the imposition of unitarity amounts to the inclusion of the πNN final state interaction via multiple, on-energy-shell pion production and annihilation. We thus consider Fig. 2 as the lowest order K matrix and calculate T via

$$T=\frac{K}{1+i\pi K\delta(E_f-E_i)}. \quad (23a)$$

If the nucleon-nucleon elastic interaction in channel α is characterized by the phase shift δ_α, the new amplitudes T_α are given in terms of the K_α by

$$T_\alpha=\frac{K_\alpha e^{i\delta_\alpha}\cos\delta_\alpha}{1+\rho e^{i\delta_\alpha}\cos\delta_\alpha}. \quad (23b)$$

Here, $\rho=\sigma/[(2J+1)4\pi/Q^2]$ is the ratio of uncorrected total cross section (for this channel) to maximum possible total cross section. In our calculation we took δ_α from the recent compilation of NN phases by Bystricky *et al.*,[24] and found Eq. (24) to be an especially important correlation in the $pp\to\pi^+d$ channel.

In order to simplify his final analysis, Mandelstam[20] assumed production with deuteron formation is just a function of one S wave amplitude "a" and two P wave amplitudes:

$$b_a=(\sqrt{5}b_{11}+b_{12})/\sqrt{6}$$
$$=b_{01}=(-b_{21}+b_{22})/\sqrt{2}\,, \quad (24a)$$
$$b_b=(-b_{11}+\sqrt{5}b_{12})/\sqrt{6}$$
$$=(b_{21}+b_{22})/\sqrt{2}. \quad (24b)$$

The S-state production amplitude was fixed at $|a|^2=0.030$ via $\sigma(pp\to\pi^+d)$ near threshold, b_b was found to be $\simeq 0$ via the angular dependence of pi production, and $|b_a|^2=0.134$ was found from the reaction $pp\to\pi^+pn$ at 660 MeV.

We have treated the five parameters a, b_a, b_b, $b_c=b_{01}$, and d_d (the coupling between the πnp and πd final state) as adjustable and fit the reactions

$$pp\to\begin{cases} pn\pi^+ & (25a)\\ pp\pi^0 & (25b)\\ d\pi^+ & (25c)\end{cases}$$

for proton kinetic energies less than 1 GeV. The data were taken from the compilation of Bystricky and Lehar[25] with 20, 24, and 24 points for (25a), (25b) and (25c), respectively. In our final search all of the data were fit simultaneously. Since Δ dominance is not accurate at very low energies, the low energy (S wave) cross sections were included in our results via phenomenological fits. We used Spuller and Measday's[26] fit, $\sigma(pp\to\pi^+d)\to 0.247\eta$, and Dunaitsev and Prokoshkin's fit,[27] $\sigma(pp\to\pi^0pp)\to 0.032\eta^2$, for the π^0pp and π^+np channels ($\eta=Q_\pi/m_\pi$). We obtained $\chi^2=2.3$/degree of freedom and the values

$$|a|^2=0.0132,\quad |b_a|^2=0.1236,$$
$$|b_b|^2=0.0002,\quad |b_c|^2=0.4662, \quad (26)$$
$$|b_d|^2=0.9735.$$

The value for b_a is close to Mandelstam's value and is determined mainly by the large $pp\to\pi^+np$ cross section. The fits obtained to the experimental *production* data are displayed in Figs. 3–5. The dashed curves show the corresponding pion *absorption* cross sections, with the divergent threshold behavior of $\pi^+d\to pp$.

III. CONNECTION WITH THE OPTICAL POTENTIAL

A. The $\pi NN\to NN\to\pi NN$ amplitude

The optical theorem $(4\pi/Q)\,\mathrm{Im}F^{\mathrm{el}}(0°)=\sigma^{\mathrm{tot}}$ tells us that part of the πNN elastic scattering ampli-

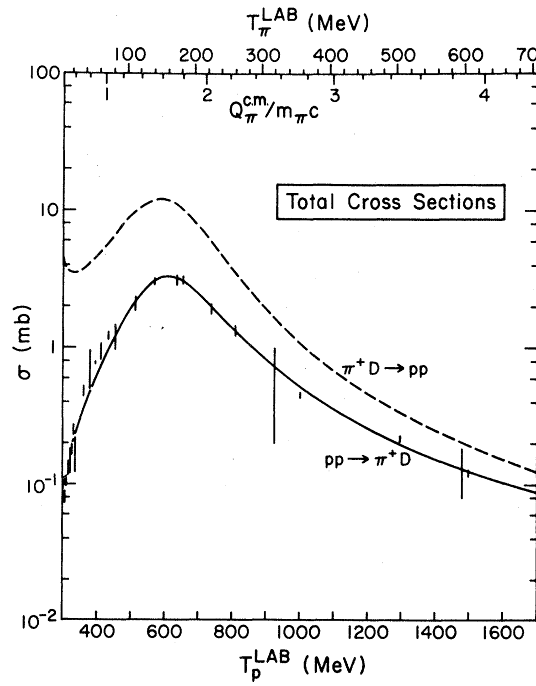

FIG. 3. Theoretical cross section for single pion production $pp\to\pi^+d$ (solid curve), and absorption $\pi^+d\to pp$ (dashed curve). T_p^{lab} is the laboratory energy of the proton beam for production, T_π^{lab} is the laboratory energy of the pion beam for absorption, and $Q_\pi^{\mathrm{c.m.}}$ is the pion momentum in the π-NN center of mass. The data are from Refs. 25–27.

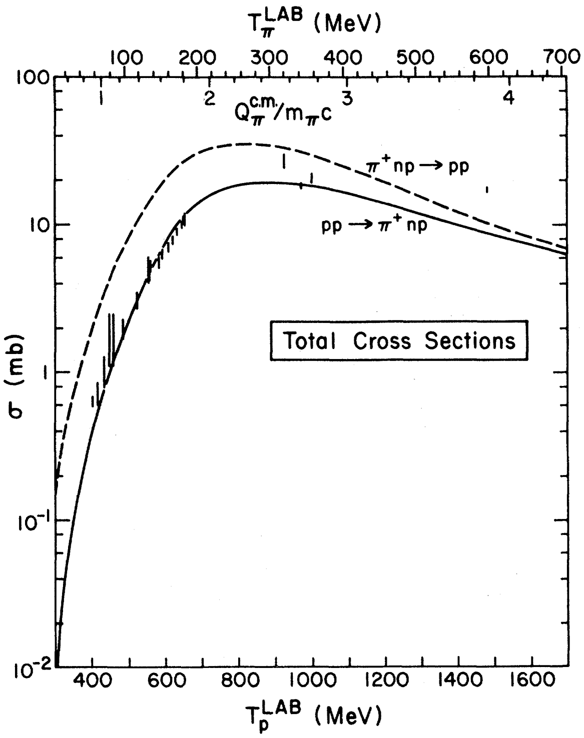

FIG. 4. Same as Fig. 3, but now for the reactions $pp \rightarrow \pi^+ np$ (free) and $\pi^+ np \rightarrow pp$.

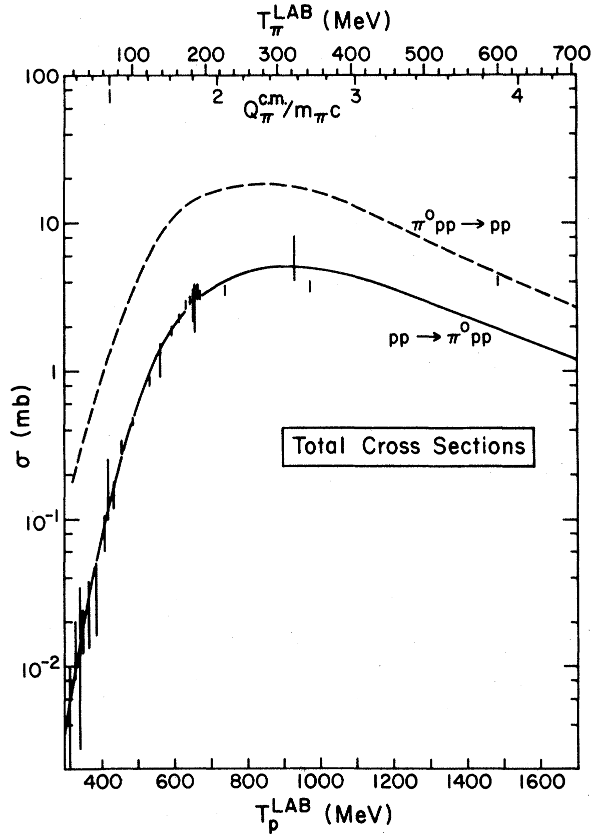

FIG. 5. Same as Fig. 3, but now for the reactions $pp \rightarrow \pi^0 pp \rightarrow pp$.

tude must arise from the absorption process since the latter contributes to σ^{tot}, i.e.,

$$\mathrm{Im}F^{el}_{ABS}(0^\circ) \equiv \mathrm{Im}F(\pi NN \rightleftarrows NN)\,|_{0^\circ} = \frac{Q}{4\pi}\sigma^{tot}(\pi NN \rightarrow NN). \tag{27}$$

Since our model contains the contribution of each eigenchannel to $\sigma^{tot}(\pi NN \rightarrow NN)$, by making a 1:1 correspondence between the terms in F and σ^{tot}, we determine the eigenchannel decomposition of $F(\pi NN \rightleftarrows NN)$.

To place the $\pi NN \rightleftarrows NN$ process in the nuclear medium, we first assume that the short ranged nucleon-nucleon correlation present in F effectively will be included by using the isobar model which fits the $NN \rightarrow \pi NN$ data. Next we note that F can be expressed as the expectation value between pion and deuteron states of the pion-two nucleon operator $\hat{F}$;

$$F(\pi NN \rightleftarrows NN) = \langle \phi_d, \vec{Q}' \,|\, \hat{F} \,|\, \phi_d, \vec{Q} \rangle. \tag{28}$$

We next assume that $\hat{F}$ can be written as an operator of zero range in the two nucleons's coordinates[28],

$$\hat{F}(\pi NN \rightleftarrows NN) = \delta(\vec{r}_1 - \vec{r}_2) f_{\pi NN}(\vec{Q}', \vec{Q}), \tag{29}$$

and then are able to make the identifications

$$F(\pi NN \rightleftarrows NN) = |\,\phi_d(0)\,|^2 f_{\pi NN}(\vec{Q}', \vec{Q}), \tag{30}$$

$$\mathrm{Im} f_{\pi NN}(\vec{Q}, \vec{Q}) = \frac{Q\sigma^{tot}(\pi D \rightarrow NN)}{4\pi\,|\,\phi_d(0)\,|^2}. \tag{31}$$

The above two assumptions are thus equivalent to assuming that nuclear pion absorption is proportional to deuteron absorption, and to including in f (in some "effective ", phenomenological way) NN correlations and the various NN (and πNN) channels present in $\sigma(\pi d \rightarrow NN)$. However, this inclusion is indirect, and Eqs. (28)—(31) should be replaced by a more fundamental theory—such as those in Ref. 9—in which the nucleon-nucleon interaction is treated in a more elementary manner and then extended to include nuclear medium effects. Yet both extensions may be needed simultaneously. For example, while the deuteron D state is important for absorption on free deuterium, (since this state contains high relative momentum components) it may be less important for nuclear absorption after the high momentum components introduced by the short ranged NN correlations are included.

If we now assume that the pion-nucleus "true" absorption process is affected only weakly by the nuclear environment we can identify the on-shell, forward scattering $\mathrm{Im} f_{\pi NN}$ with the (true absorption) amplitude which multiplies the rho-squared terms in the optical potential, Eq. (3):

$$\mathrm{Im} f_{\pi N}(\vec{Q},\vec{Q}) = \mathrm{Im}[\widetilde{B}(Q)\ \widetilde{C}(Q)Q^2 + \widetilde{D}(Q)Q^4 + \cdots] \equiv \mathrm{Im}[f_s(\vec{Q}'\cdot\vec{Q}) + f_p(\vec{Q}'\cdot\vec{Q}) + \cdots]_{\vec{Q}=\vec{Q}'}$$
$$= \frac{1}{\gamma}\mathrm{Im}[B + C\vec{k}'\cdot\vec{k} + Dk^4 P_2(\cos\theta_{kk'}) + \cdots]_{\vec{k}'=\vec{k}}. \quad (32)$$

Here, $\widetilde{B}$, $\widetilde{C}$, $\widetilde{D}$, . . . contain spin and isospin dependences still to be specified, and the γ factor and relation of of $\vec{Q}$ to $\vec{k}$ is given by Eqs. (52)–(58). If we express the B and C parameters in term of absorption on nn, pp, or pn pairs, the optical potential, Eq. (3), generalizes to

$$V^{\mathrm{ABS}}(\vec{k}',\vec{k};E) = -4\pi[2\mu_{\pi A}(2\pi)^3]^{-1}\{U^S_{\mathrm{ABS}} + U^P_{\mathrm{ABS}}\vec{k}'\cdot\vec{k}\}$$

$$U^{S,P}_{\mathrm{ABS}} = 4\left[N(N-1)\begin{Bmatrix}B\\C\end{Bmatrix}_{nn}(Q_E)\hat{\rho}_n{}^2(\vec{k}'-\vec{k})\delta_{\pi,\pi^+} + ZN\begin{Bmatrix}B\\C\end{Bmatrix}_{pn}(Q_E)\rho_p\rho_n(\vec{k}'-\vec{k})\right. \quad (33a)$$

$$\left. + Z(Z-1)\begin{Bmatrix}B\\C\end{Bmatrix}_{pp}(Q_E)\hat{\rho}_p{}^2(\vec{k}'-\vec{k})\delta_{\pi,\pi^-}\right]\frac{g_{0,1}(Q')g_{0,1}(Q)}{g^2_{0,1}(Q_E)}\ , \quad (33b)$$

where each density $\rho_i(r)$ is normalized to one, and our isobar model for the B's and C's includes absorption on the interchanged pair. If the nucleus had $\rho_p = \rho_n = \rho$ and $Z = N = A/2$, the B_0 of Eq. (3) would be related to the above B's by

$$B_0 = \frac{A}{A-1}B_{pn} + \left[\frac{A-\frac{1}{2}}{A-1}\right]B_{nn}$$
$$\simeq B_{pn} + B_{nn}, \quad (34)$$

i.e., the usual quasideuteron model of absorption supplemented by a correction (B_{nn}) for absorption on like pairs.

In Eq. (31) the deuteron wave function at the origin $\phi_d(0)$ enters as an overall normalization factor. Since this is a difficult quantity to determine with certainty,[3] we have more confidence in the energy dependence and relative sizes of B, C, and D than in their absolute magnitudes. Furthermore, in applying Eq. (32), we assume Eq. (31) is valid—with the same value of $|\phi_d(0)|^2$—for absorption on any NN pair. This assumption, which is equivalent to assuming the relative wave function for any pair of nucleons in a nucleus is similar for small $\vec{r}$, also introduces some uncertainty into the present calculation.

The above identification depends upon a relatively weak coupling of the absorptive and elastic channels in the πNN and pion-nucleus systems. Although Afnan and Blankleider[22] have shown that there is weak coupling in the πNN system, annihilation is relatively stronger in the nucleus,[7,14] and a number of current studies on the coupling of the channels are now underway.[29] In our case, we again expect the relative sizes and energy dependences to be more reliable than the absolute magnitudes.

To proceed, we write $f_{\pi NN}$ as a sum over the allowed eigenchannel amplitudes β_α with explicit projection operators $\hat{P}_\alpha$:

$$f_{\pi NN} = \hat{P}^{(I)}_{1,1}\sum_{\substack{LJ\\ij}}\hat{P}_{L,J}\hat{P}_i^{(I_{NN})}\hat{P}_j^{(S_{NN})}\beta^{(L)}_{JI}(S,T)\ . \quad (35)$$

Since we use an isobar model, only total isospin $I=1$ states are allowed ($I_\Delta + I_{N_f} = 2,1$; yet $I_{N_i} + I_{N_i} = 0,1$ so $I=1$ overlaps). When the complete form for these projection operators are inserted in Eq. (35), a stunning variety of combinations of $\vec{t}_1$, $\vec{t}_2$, $\vec{T} = \vec{t}_1 + \vec{t}_2$, $\vec{I}$, $\vec{\sigma}_1$, $\vec{\sigma}_2$, $\vec{S}$, $\vec{L}$, and β's occur. To simplify the problem, we have examined only those terms which may contribute to pion scattering from a spin $\frac{1}{2}$ nucleus. The πNN s-

wave amplitude, which accepts contributions from S and P wave NN states, has the form

$$f_s = B_0 + B_1 \vec{\sigma}_1 \cdot \vec{\sigma}_2 + B_2 \vec{\tau}_1 \cdot \vec{\tau}_2 + B_3 (\vec{\sigma}_1 \cdot \vec{\sigma}_2)(\vec{\tau}_1 \cdot \vec{\tau}_2), \quad (36)$$

where the explicit formulas for the B's in terms of the β's are given in Appendix A. The p wave amplitude is more complicated and its evaluation requires careful consideration of the allowed parity and symmetry of the states. We remove some of the complications by forming a weighted average of the probabilities for each total angular momentum state of the nucleons. f_p then has the form

$$f_p = \vec{Q}\cdot\vec{Q}'\{C_0 + C_1 \vec{\sigma}_1 \cdot \vec{\sigma}_2 + C_2 \vec{\tau}_1 \cdot \vec{\tau}_2 + C_3 (\vec{\sigma}_1 \cdot \vec{\sigma}_2)(\vec{\tau}_1 \cdot \vec{\tau}_2) + [C_4 + C_5 \sigma_1 \cdot \vec{\sigma}_2 + C_6 \vec{\tau}_1 \cdot \vec{\tau}_2 + C_7 (\vec{\sigma}_1 \cdot \vec{\sigma}_2)(\vec{\tau}_1 \cdot \vec{\tau}_2)] \vec{S}\cdot\vec{L} + [C_8 + C_9 \vec{\sigma}_1 \cdot \vec{\sigma}_2 + C_{10} \vec{\tau}_1 \cdot \vec{\tau}_2 + C_{11} (\vec{\sigma}_1 \cdot \vec{\sigma}_2)(\vec{\tau}_1 \cdot \vec{\tau}_2)](\vec{S}\cdot\vec{L})^2\}, \quad (37)$$

where the expressions for the C's are also in Appendix A. The d wave amplitude has the same form as Eq. (37), except with $\vec{Q}\cdot\vec{Q}'$ replaced by $Q^4 P_2(\cos\theta)$, and the C's replaced by D's (see Appendix A).

We next remove the isospin operators from these amplitudes by evaluating them for pion absorption on the different possible two nucleon pairs. Since the NN wave function must be antisymmetric under space-spin-isospin interchange, the isospin states possible are related to the angular momentum states possible and we get different answers when the two nucleons are in relative S or P state. If we let λ be the probability for the two (absorbing) nucleons to be in a P state ($1-\lambda$ for S state), the πNN amplitudes take the forms for $\pi^- p \rightleftarrows pn$ ($\pi^+ nn \rightleftarrows pn$):

$$f_s = [B_0 + B_2 - 3(B_1 + B_3)](1-\lambda), \quad (38)$$

$$f_p = Q^2 P_1(\cos\theta)\{\lambda[C_0 + C_1 + C_2 + C_3 + (C_4 + C_5 + C_6 + C_7)\vec{S}\cdot\vec{L} + (C_8 + C_9 + C_{10} + C_{11})(\vec{S}\cdot\vec{L})^2] + (1-\lambda)[C_0' - 3C_1' + C_2' - 3C_3' + (C_4' - 3C_5' + C_6' - 3C_7')\vec{S}\cdot\vec{L} + (C_8' - 3C_9' + C_{10}' - 3C_{11}')(\vec{S}\cdot\vec{L})^2]\}, \quad (39)$$

$$f_D = Q^4 P_2(\cos\theta)(1-\lambda)\{D_0 + D_2 - 3(D_1 + D_3) + [D_4 + D_6 - 3(D_5 + D_7)]\vec{S}\cdot\vec{L} + [D_8 + D_{10} - 3(D_9 + D_{11})](\vec{S}\cdot\vec{L})^2\}, \quad (40)$$

and for $\pi^+ np \rightleftarrows pp$ ($\pi^- np \rightleftarrows nn$):

$$f_s = (B_0 - 2B_2 - 3B_3)(1-\lambda), \quad (41)$$

$$f_p = Q^2 P_1(\cos\theta)\{\lambda[C_0 + 3C_3 + (C_4 + 3C_7)\vec{S}\cdot\vec{L} + (C_8 + 3C_{11})(\vec{S}\cdot\vec{L})^2] + (1-\lambda)[C_0' - 2C_2' - 3C_3' + (C_4' - 2C_6' - 3C_7')\vec{S}\cdot\vec{L} + (C_8' - 2C_{10}' - 3C_{11}')(\vec{S}\cdot\vec{L})^2]\}, \quad (42)$$

$$f_D = Q^4 P_2(\cos\theta)(1-\lambda)[D_0 - 2D_2 - 3D_3 + (D_4 - 2D_6 - 3D_7)\vec{S}\cdot\vec{L} + (D_8 - 2D_{10} - 3D_{11})(\vec{S}\cdot\vec{L})^2]. \quad (43)$$

In the final calculation of the absorption strengths, the relative probability of finding two nucleons in a P state (λ) and the distribution of the nuclear spin $\vec{S}(\equiv \frac{1}{2})$ depend upon the detailed structure of the nucleus in which the NN pair is embedded.

Next we decompose each $(\vec{S}\cdot\vec{L})^2$ term into central, spin-flip (of two nucleons), and double spin-flip (of one nucleon) terms. We drop the double spin-flip terms, since they are of the same order as the ignored terms in the second order optical potential. Our final expressions for the $\pi NN \rightleftarrows NN$ amplitude in terms of the β's and λ are then, for $\pi^- pp \rightleftarrows pn$ ($\pi^+ nn \rightleftarrows pn$),

$$f_s = \beta_{01}^{(0)}(01)(1-\lambda), \quad (44)$$

$$f_p = \lambda\{\tfrac{1}{6}\beta_{01}^{(1)}(11) + \tfrac{2}{5}\beta_{11}^{1}(11) + \tfrac{4}{3}\beta_{21}^{(1)}(11) + [-\tfrac{1}{12}\beta_{01}^{(1)}(11) - \tfrac{3}{10}\beta_{11}^{(1)}(11)]\vec{S}\cdot\vec{L}\} Q^2 P_1(\cos\theta), \quad (45)$$

$$f_d = 5\beta_{21}^{(2)}(01)(1-\lambda)Q^4 P_2(\cos\theta) , \tag{46}$$

and, for $\pi^+ np \rightleftarrows pp$ ($\pi^- pn \rightleftarrows nn$),

$$f_s = \tfrac{1}{4}(1-\lambda)[3\beta_{11}^{(0)}(10)+\beta_{01}^{(0)}(01)], \tag{47}$$

$$f_p = Q^2 P_1(\cos\theta)\big(\tfrac{9}{8}\beta_{21}^{(1)}(10)(1-\lambda)+\lambda[\tfrac{1}{4}\beta_{11}^{(1)}(00)+\tfrac{1}{8}\beta_{01}^{(1)}(11)+\tfrac{3}{10}\beta_{11}^{(1)}(11)+\beta_{21}^{(1)}(11)]$$
$$+\vec{S}\cdot\vec{L}\{\tfrac{15}{16}\beta_{21}^{(1)}(10)(1-\lambda)-\lambda[\tfrac{1}{16}\beta_{01}^{(1)}(11)+\tfrac{9}{40}\beta_{11}^{(1)}(11)]\}\big), \tag{48}$$

$$f_d = Q^4 P_2(\cos\theta)\frac{(1-\lambda)}{4}\{5\beta_{21}^{(2)}(01)+\tfrac{3}{2}\beta_{11}^{(2)}(10)+\tfrac{15}{2}\beta_{21}^{(2)}(10)-\vec{S}\cdot\vec{L}[\tfrac{9}{4}\beta_{11}^{(2)}(10)+\tfrac{5}{4}\beta_{21}^{(2)}(10)]\}. \tag{49}$$

We determine the β parameters by eliminating the channels not present in the isobar model, and then matching to the corresponding term in the isobar model. The three reactions examined were $pp \rightleftarrows$ ($\pi^+ np$, $\pi^0 pp$, $\pi^+ D$) which we label (α,β,γ). The results of this step are given in Appendix B. To undertake the second step, we solved for the Imβ's in terms of appropriately weighted contributions to the reaction cross sections. In addition, we averaged the free and bound *np* channels into a single effective one. Thus, as is evident in Figs. 3–5, at low energy the *NN* absorption in nuclei is mainly on *D*'s, whereas at higher energy it passes over to free *np* pairs. Our final results for the $\beta_{JI}^{(L)}(S,T)$'s in terms of the isobar model's corresponding contribution to the total cross section follow from Eq. (31):

$$|\phi_d(0)|^2\frac{4\pi}{Q}\,\mathrm{Im}\begin{bmatrix}\beta_{01}^{(0)}(01)\\ \beta_{11}^{(0)}(10)\\ \beta_{01}^{(1)}(11)\\ \beta_{11}^{(1)}(00)\\ \beta_{11}^{(1)}(11)\\ \beta_{21}^{(1)}(10)\\ \beta_{21}^{(1)}(11)\\ \beta_{11}^{(2)}(10)\\ \beta_{21}^{(2)}(01)\\ \beta_{21}^{(2)}(10)\end{bmatrix} = \begin{bmatrix}(2\sigma_\alpha+\frac{1}{2}\sigma_\beta)_{01}(01)\\ (\frac{4}{3}\sigma_\alpha+\sigma_\gamma)_{11}(10)\\ 27(\frac{2}{3}\sigma_\alpha+\frac{1}{2}\sigma_\beta)_{01}(11)\\ 12(\sigma_\alpha)_{11}(00)\\ 3(\frac{2}{3}\sigma_\alpha+\frac{1}{2}\sigma_\beta)_{11}(11)\\ \frac{9}{5}(\frac{4}{3}\sigma_\alpha+\sigma_\gamma)_{21}(10)\\ \frac{27}{10}(\frac{2}{3}\sigma_\alpha+\frac{1}{2}\sigma_\beta)_{21}(11)\\ 5(\frac{4}{3}\sigma_\alpha+\sigma_\gamma)_{11}(10)\\ (2\sigma_\alpha+\frac{1}{2}\sigma_\beta)_{21}(01)\\ 3(\frac{4}{3}\sigma_\alpha+\alpha_\gamma)_{21}(10)\end{bmatrix} . \quad \begin{matrix}(50a)\\(50b)\\(50c)\\(50d)\\(50e)\\(50f)\\(50g)\\(50h)\\(50i)\\(50j)\end{matrix}$$

We used a Hulthén wave function for the deuteron, with parameters $(a,b)=(0.232,\ 1.202)\ \mathrm{fm}^{-1}$. This produces $|\phi_d(0)|^2=0.0641\ \mathrm{fm}^{-3}$, a density $\sim 30\%$ lower than that of the nucleons in the surface of carbon, or $\sim 60\%$ lower than at the center of carbon.

B. Numerical results

In Fig. 6 we present our numerical result for the imaginary part of the *s* wave ($L_\pi, NN=0$) amplitudes Im$\widetilde{B}$ ($\pi^- pp \to pn \to \pi^- pp$) and Im$\widetilde{B}$ (π^+ "pn" $\rightleftarrows pp$), and the full *p* wave amplitude Q^2 Im$\widetilde{C}$ ($\pi^+ np$), all evaluated in the π-*NN* c.m. These results are for $\lambda=0$, i.e., the initial two nucleons in a pure $L_{N,N}=0$ (*S*) state. [As Eqs. (44) and (47) show, $\widetilde{B}$ is proportional to $(1-\lambda)$ so these results can be uniformly reduced for other values of λ .] An important aspect to note in Fig. 6 is that for $T_\pi^{\mathrm{lab}} \lesssim 100$ MeV the $\pi^+ pn \rightleftarrows pp$ absorption dominates and is finite for $T_\pi \to 0$, whereas at higher energies *s* wave absorption on like pairs is more likely. We also note that the peak value for *s* wave π^+ absorption on an *nn* pair is ~four times that for absorpton or an *np* pair. This is a simple consequence of resonance absorption occurring predominately in the spin-singlet *NN* state, a state which we weigh by $\frac{1}{4}$ for the *np* system and by 1 for the *nn* system. For these *s* wave amplitudes there is no spin flip.

The nonvanishing *threshold* values we find from our calculations are

$$\mathrm{Im}\widetilde{B}(\pi^+ \text{“}np\text{”} \leftrightarrow pp) \to 0.017 m_\pi^{-4},$$

$$\mathrm{Im}\widetilde{C}(\pi^+ \text{“}np\text{”} \leftrightarrow pp) \to 0.037 m_\pi^{-6}.$$

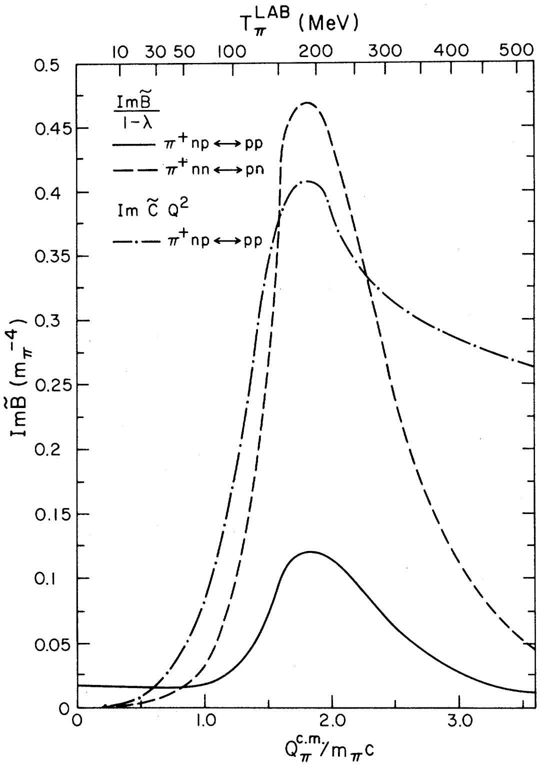

FIG. 6. The imaginary part of the π-NN s wave absorption strength $\tilde{B}$ (in the π-NN center of mass) as a function of pion c.m. momentum/lab energy. The solid curve is for absorption on an np pair, the dashed curve for absorption on an nn/pp pair, and the dotted-dashed curve for p wave absorption on an np pair (without the Q^2 factor removed).

These lie slightly below the phenomenological values $0.03 \lesssim \text{Im}B_0(0) \lesssim 0.06 m_\pi^{-4}$ and $0.04 \lesssim \text{Im}C_0(0) \lesssim 0.08 m_\pi^{-6}$, and are essentially unchanged by the "angle transformation," Eqs. (52)–(58). These are similar, however, to the threshold values found with other two nucleon

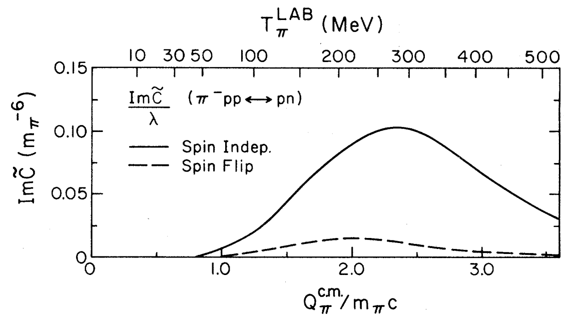

FIG. 7. The imaginary part of the π-NN p wave absorptive strength $\tilde{C}$ for absorption on a "like" pair of nucleons in a relative P state ($\lambda=1$). The dashed curve gives the strength of the spin flip (spin orbit) term.

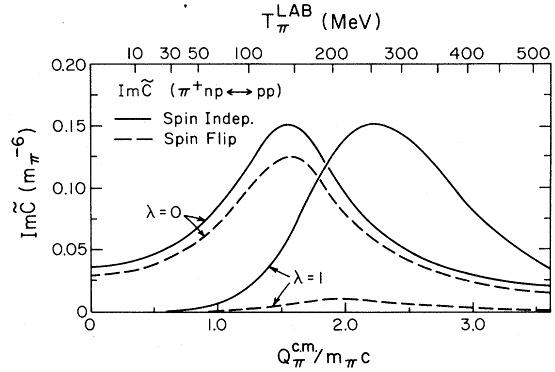

FIG. 8. Same as Fig. 7 except now for absorption on an "unlike" pair (pn). The $\lambda=0$ curves are for the absorbing nucleons in a relative S state.

models; e.g., Chai and Riska[17] using a pion rescattering model find $\text{Im}(B_0, C_0) = (0.027, 0.043)$. As indicated earlier,[13,14,32] we do not expect to reproduce the phenomenological strengths since we expect many body effects to be important and the ρ^2 prescription to be overly simple.

In Fig. 7 we present the p wave amplitude $\text{Im}\tilde{C}$ $(\pi^+ nn \rightleftarrows np) \equiv \text{Im}\tilde{C}$ $(\pi^- pp \rightleftarrows nn)$. Here the solid curve is the spin independent part and the dashed curve is the negative of the spin flip part. The results shown are for $\lambda=1$. Note that the spin flip amplitude is small, $\sim 15\%$ of the nonflip part, and that both vanish as $T_\pi \to 0$. Since the isobar model contains rather limited parts of the d wave amplitude, we do not display those results here.

In Fig. 8 we present the p wave amplitudes for $\pi^\pm$ "np"$\rightleftarrows\{{pp \atop nn}\}$, Eq. (48), summed over "free" and "bound" np pairs. The solid curves are the spin-independent parts and the dashed curves the negative of spin-flip ones. Since there is now no simple λ proportionality, we present the results for $\lambda=0$ and $\lambda=1$ separately. Note that if $\lambda=0$ the spin-flip amplitude is comparable to the non-spin-flip one and they are both nonvanishing for $T^\pi \to 0$. Thus for pion -^{3}He/^{3}H scattering, we expect large spin flip contributions from the pn pairs, but much smaller spin-flip contributions from the nn or pp pairs (which are coupled to total spin$=0$).

In order to compare the s and p wave strengths, in Fig. 6 we also plot $Q_\pi^2 \tilde{C}$ as a function of Q_π. For $T_\pi^{\text{lab}} \leq 30$ MeV, where π-np absorption in nuclei is most important, the s wave term dominates, yet for higher energies the $\tilde{C}$ term is much larger with the ratio of peak values of $\gtrsim 3$. As expected, the energy dependence of our $\text{Im}\tilde{C}(\pi^+ np)$ is similar to that of other workers—all having a characteristic resonance peak. Yet the isobar model's $\mathrm{Im}\tilde{B}(\pi^+ np)$ also shows a peak not present, e.g., in the pion rescattering model of Ref. 17 (the energy dependence of $\tilde{B}_0$ in that model arises predominately from "the energy dependence of the πN amplitude").

C. Angle transformation

Since the preceding absorption amplitudes have been determined in the π-NN c.m., it is still necessary to relate these to the $B_0, C_0 \ldots$ parameters in the π-nucleus optical potential, Eq. (32), and thus in the π-A c.m reference frame. This procedure, which needs be applied to off-energy-shell scattering, mixes P and higher waves into the "S wave" parameters and is crucial for the linear terms in the pion optical potential.[30,14] It is now commonly called the "angle transformation" or mapping of scattering amplitudes. Explicitly, there is an overall multiplicative factor γ,

$$B+C\vec{k}'\cdot\vec{k}+Dk^4P_2(\cos\theta_{k'k})+\ldots$$
$$=\gamma[\tilde{B}+\tilde{C}\vec{Q}'\cdot\vec{Q}+\cdots], \quad (52)$$

$$\gamma=\left[\frac{E_\pi(Q)E_\pi(Q')E_D(Q)E_D(Q')}{E_\pi(k)E_\pi(k')E_D(p_0)E_D(p_0')}\right]^{1/2}, \quad (53)$$

and then the relation between $\vec{k}$ and $\vec{Q}$. In our application of the present absorption model to pi-nucleus scattering, we employ an "optimal" choice of the momentum of the two nucleon pair, $\vec{p}_0$ in Eq. (53),

$$\vec{p}_0=-2\vec{k}/A+\frac{A-2}{2A}(\vec{k}'-\vec{k}), \quad (54)$$

and then use the invariant Aaron, Amado, and Young (AAY) procedure[14] for relating the off-shell momentum variables. Although this procedure is unambiguous and fairly rigorous for a potential theory, the results are not simple since $B, C, \ldots$ will now depend on the off-shell scattering angle as well as energy. Therefore, we show the qualitative effect of the angle transformation by using the more approximate prescription[14]

$$\gamma=\frac{E_\pi(Q)E_D(Q)}{E_\pi(P^{\mathrm{lab}})m_D},$$
$$\vec{Q}'\cdot\vec{Q}=\alpha\vec{k}'\cdot\vec{k}-\beta k^2, \quad (55)$$

$$\alpha=\frac{1+\epsilon}{1+2\epsilon+m_\pi^2/m_D^2},$$
$$\beta=\frac{\epsilon}{1+2\epsilon+m_\pi^2/m_D^2}, \quad (56)$$
$$\epsilon=E_\pi(k)/m_D.$$

With this simple mapping, the (on-shell) optical potential parameters are

$$B_0=\gamma[\tilde{B}_0-\beta(k/m_\pi)^2\tilde{C}_0], \quad (57)$$

$$C_0=\alpha\gamma\tilde{C}_0, \quad (58)$$

where Eq. (57) uses the standard convention of m_π units.

In Figs. 9 and 10 we present the result of this approximate transformation on S- and P-wave amplitudes, respectively. The P wave amplitudes are reduced only slightly (no π-NN D waves were included). In contrast, the subtraction in Eq. (57) greatly reduces the effective S wave parameter B_0 for $T_\pi^{lab} \gtrsim 50$ MeV, and eventually it goes negative. Since $\mathrm{Im}\tilde{B}$ and $\mathrm{Im}\tilde{C}$ are always positive in the π-NN c.m., this behavior does not violate any π-NN unitarity principle. However, it does indicate that at high energy the s wave part of the ρ^2 term in the optical potential could change sign—

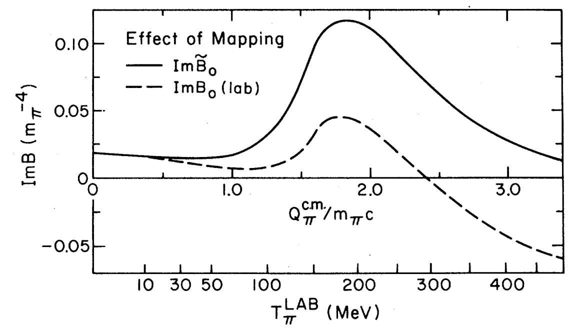

FIG. 9. The effective π-np s wave absorption strength B_0 in the *laboratory reference frame,* obtained by the approximate transformation [Eqs. (57) and (58)] of the s- and p-wave c.m. strengths $\tilde{B}_0$ and $\tilde{C}_0$.

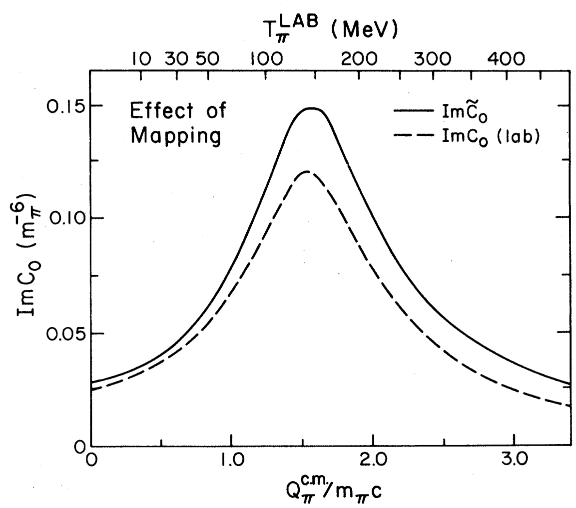

FIG. 10. Same as Fig. 9, except now for the effective p wave strength C_0 and just Eq. (58).

although the s and p wave sum would still be absorptive. Consequently, we predict that the phenomenological, effective values of B_0 obtained from "fitting" pion scattering data for low and medium energies should decrease strongly with energy—as found by Liu[31] in his 28–260 MeV phenomenological fits to π-^{4}He and π-^{12}C data.

IV. CONCLUSION AND SUMMARY

We have constructed a model which permits the pion optical potential to be extended so as to include terms describing the effect of pion annihilation on pn and pp/nn pairs with the accompanying possibility of spin flip from a spin $\frac{1}{2}$ nucleus. To do this, we extended and updated Mandelstam's isobar model of pion production in NN collisions and generalized the quasideuteron model of Eckstein, and Ericson and Ericson. After fitting the lastest two nucleon pion production cross sections in all charge channels from threshold to $T_\pi \simeq 400$ MeV, we extracted the above spin and isospin dependences as well as the energy dependence of the second order optical potential parameters B, C, and D.

Our results for the threshold values of the isoscalar B and C amplitudes are similar to those of other theoretical studies and somewhat smaller than the average phenomenological values. Naturally, our isobar model's predictions for the energy dependences of the microscopic Im$\widetilde{B}$ and Im$\widetilde{C}$ is of resonance shape. We find, however, that the s wave strength B is greatly reduced for $T_\pi > 30$ MeV by the angle transformation and loses much of its resonance shape—in agreement with some phenomenological studies.

The most important results of our work are the new determinations of the strengths, relative to the standard isoscalar (pn) term, of the potential terms describing annihilation on like (nn/pp) nucleon pairs and of pion spin flip scattering arising from the annihilation process (they can both be large). These should be particularly useful in studies of the nuclear structure of light nuclei deduced from pion scattering.

Our model is by no means a complete or rigorous study of pion absorption by nuclei. In particular, a more accurate description of the πN vertex, the NN force in nuclei, NN correlations, the real parts of the amplitude, and the influence of nuclear structure on the absorption process should be included (the assumed ρ^2 dependence is known to be crude). For these reasons we think the relative size and energy dependence of the terms we calculate are more interesting than their absolute magnitudes.

ACKNOWLEDGMENTS

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APPENDIX A

The general $\pi NN \rightarrow NN \rightarrow \pi NN$ amplitude is given in spin-isospin space by Eqs. (36)–(43) in terms of parameters B, C, and D. In this Appendix we express the B's, C's, and D's in terms of the eigenchannel amplitudes $\beta_{JI}^{(L)}(S,T)$'s, where, for example, $\beta_{11}(10)$ refers to the deuteron final state.

$$B_0 = \tfrac{3}{16}[\beta_{11}^{(0)}(10) + \beta_{01}^{(0)}(01)] = -3B_3, \qquad \text{(A1)}$$

$$B_1 = \tfrac{1}{16}[\beta_{11}^{(0)}(10) - 3\beta_{01}^{(0)}(01)], \qquad \text{(A2)}$$

$$B_2 = \tfrac{1}{16}[\beta_{01}^{(0)}(01) - 3\beta_{11}^{(0)}(10)], \qquad \text{(A3)}$$

$$C_0 = \tfrac{1}{16}[\tfrac{1}{3}\beta_{01}^{(1)}(00) + \beta_{11}^{(1)}(00) + \tfrac{5}{3}\beta_{21}^{(1)}(00) + 3\beta_{01}^{(1)}(11) + \tfrac{9}{5}\beta_{11}^{(1)}(11) + 18\beta_{21}^{(1)}(11)], \qquad \text{(A4)}$$

$$C_1 = -\tfrac{1}{16}[\tfrac{1}{3}\beta_{01}^{(1)}(00) + \beta_{11}^{(1)}(00) + \tfrac{5}{3}\beta_{21}^{(1)}(00) - \beta_{01}^{(1)}(11) - \tfrac{3}{5}\beta_{11}^{(1)}(11) - 6\beta_{21}^{(1)}(11)], \qquad \text{(A5)}$$

$$C_2 = C_1, \qquad \text{(A6)}$$

$$C_3 = \tfrac{1}{16}[\tfrac{1}{3}\beta_{01}^{(1)}(00) + \beta_{11}^{(1)}(00) + \tfrac{5}{3}\beta_{21}^{(1)}(00) + \tfrac{1}{3}\beta_{01}^{(1)}(11) + \tfrac{1}{5}\beta_{11}^{(1)}(11) + 2\beta_{21}^{(1)}(11)], \qquad \text{(A7)}$$

$$C_4 = 3C_5 = 3C_6 = 9C_7 = -\tfrac{3}{16}[\tfrac{1}{2}\beta_{01}^{(1)}(11) + \tfrac{3}{5}\beta_{(11)}^{(1)} + \beta_{21}^{(1)}(11)], \qquad \text{(A8)}$$

$$C_8=3C_9=3C_{10}=9C_{11}=-\tfrac{3}{16}[\tfrac{1}{2}\beta_{01}^{(1)}(11)-\tfrac{3}{5}\beta_{11}^{(1)}(11)+2\beta_{21}^{(1)}(11)], \tag{A9}$$

$$C_0'=3C_1'=-C_2'=-3C_3'=\tfrac{3}{16}\beta_{21}^{(1)}(10), \tag{A10}$$

$$C_4'=3C_5'=-C_6'=-3C_7'=\tfrac{9}{32}\beta_{21}^{(1)}(10), \tag{A11}$$

$$C_8'=3C_9'=-C_{10}'=-3C_{11}'=\tfrac{3}{32}\beta_{21}^{(1)}(10), \tag{A12}$$

$$D_0=-3D_3=\tfrac{15}{16}[\beta_{21}^{(2)}(10)-\tfrac{1}{5}\beta_{11}^{(2)}(10)+\beta_{21}^{(2)}(01)], \tag{A13}$$

$$D_1=\tfrac{5}{16}[\beta_{21}^{(2)}(10)-\tfrac{1}{5}\beta_{11}^{(2)}(10)-3\beta_{21}^{(2)}(01)], \tag{A14}$$

$$D_2=-\tfrac{15}{16}[\beta_{21}^{(2)}(10)-\tfrac{1}{5}\beta_{11}^{(2)}(10)-\tfrac{1}{3}\beta_{21}^{(2)}(01)], \tag{A15}$$

$$D_4=3D_5=-D_6=-3D_7=-\tfrac{15}{16}[\tfrac{1}{6}\beta_{21}^{(2)}(10)+\tfrac{1}{10}\beta_{11}^{(2)}(10)], \tag{A16}$$

$$D_8=3D_9=-D_{10}=-3D_{11}=\tfrac{15}{16}[\tfrac{1}{10}\beta_{11}^{(2)}(10)-\tfrac{1}{6}\beta_{21}^{(2)}(10)]. \tag{A17}$$

APPENDIX B

The general $\pi NN \to NN \to \pi NN$ amplitudes, Eqs. (44)–(49), contain contributions from more channels than exist in our simple isobar model. If we denote the three reactions $pp \rightleftarrows (\pi^+ np, \pi^0 pp, \pi^+ d)$ with the labels (α,β,γ), respectively, then the appropriate identifications for our model are

$$f_\alpha^s=\tfrac{1}{4}[3\beta_{11}^{(0)}(10)+\beta_{01}^{(0)}(01)], \tag{B1}$$

$$f_\alpha^p=3Q^2\tfrac{1}{4}[\tfrac{1}{3}(\beta_{01}^{(1)}(10)+3\beta_{11}^{(1)}(10)+5\beta_{21}^{(1)}(10)+\beta_{11}^{(1)}(01)], \tag{B2}$$

$$f_\alpha^d=5Q^4\tfrac{1}{4}[\tfrac{3}{5}\beta_{11}^{(2)}(10)+\beta_{21}^{(2)}(01)+\beta_{21}^{(2)}(10)], \tag{B3}$$

$$f_\alpha^{p'}=3Q^2\tfrac{1}{36}[\beta_{01}^{(1)}(00)+3\beta_{11}^{(1)}(00)+5\beta_{21}^{(1)}(00)+\beta_{01}^{(1)}(11)+9\beta_{11}^{(1)}(11)+10\beta_{21}^{(1)}(11)+7\beta_{31}^{(1)}(11)], \tag{B4}$$

$$f_\beta^s=\beta_{01}^{(0)}, \tag{B5}$$

$$f_\beta^p=3Q^2\beta_{11}^{(1)}(01), \tag{B6}$$

$$f_\beta^d=5Q^4\beta_{21}^{(2)}(01), \tag{B7}$$

$$f_\beta^{p'}=3Q^2\tfrac{1}{27}[\beta_{01}^{(1)}(11)+9\beta_{11}^{(1)}+10\beta_{21}^{(1)}+7\beta_{31}^{(1)}(11)], \tag{B8}$$

$$f_\gamma^s=\beta_{11}^{(0)}(10), \tag{B9}$$

$$f_\gamma^p=3Q^2\tfrac{1}{9}[\beta_{01}^{(1)}(10)+3\beta_{11}^{(1)}(10)+5\beta_{21}^{(1)}(10)], \tag{B10}$$

$$f_\gamma^d=5Q^4\tfrac{1}{15}[3\beta_{11}^{(2)}(10)+5\beta_{21}^{(2)}(10)+7\beta_{31}^{(2)}(10)]. \tag{B11}$$

In these expressions, the primed amplitudes refer to the NN system in a relative P state and the unprimed amplitudes are for relative S state.

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