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Unified theory of rearrangement scattering

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We present a unified microscopic theory of rearrangement scattering which satisfies the Pauli principle. The equations for the transition amplitudes and wave functions are reduced to effective two-body equations with the complexity put into the distortion potentials and the transition potentials. The central theme in the development of this theory is the importance of elastic scattering in the initial and the final channel, although this is not a prerequisite for its validity as the theory is exact. The elastic degrees of freedom are absorbed into the distorted waves using projection techniques in combination with the Faddeev formulation. The resulting transition amplitude can be expanded in the transition potential, leading to a modified coupled channel series. The transition potential, which has a constructive nature (i.e., no counter terms), can also be expanded. We provide arguments for the convergence of both expansions. Various contributions to the transition potential are discussed, and we put special emphasis on a rescattering term which has not been considered before in the context of coupled channel equations. Various inconsistencies which may hamper standard coupled channel calculations are discussed as well.

[NUCLEAR REACTIONS Microscopic theory of rearrangement scattering. DWBA. Coupled channel effects. Rescattering contribution. Two-nucleon mechanism. (p,d) reaction.]

I. INTRODUCTION

We give a unified microscopic description of rearrangement scattering in terms of its three basic ingredients: nuclear structure, direct elastic scattering, and pure rearrangement effects. Of these three, nuclear structure information has always been treated as basic physical input for rearrangement calculations. We propose to extend this physical input to the (direct) elastic optical potential operator. Admittedly, the idea to use the optical potential as input has been applied for years in DWBA calculations; however, we provide a microscopic framework which accomplishes this in a natural and rigorous way. The major portion of this paper is devoted to the rearrangement aspects of the problem: the relation of distorted waves to elastic wave functions, the expansion of the transition amplitude, and the expansion of the transition potential. The equations are developed so as to optimize their practicality. In its final form the theory is expressed in familiar terms, such as distorted waves, coupled channel effects, etc.; however, there are also some new elements (in particular, a phase-equivalent transformation) whose practical implications have to be studied further. The final structure of the equations is sufficiently rich enough that its practicality does not depend on overall *ad hoc* approximations which affect every aspect of the problem. Instead, its practicality depends on the convergence of the expansions and on more specific approximations which can be checked independently. In this paper we only give exact relationships, postponing the discussion of various approximations to a subsequent paper. We will, however, comment on the convergence properties of the expansions, and discuss the lowest order contributions (DWBA, coupled channel terms, etc.) to the rearrangement amplitudes.

The development of this microscopic theory of reactions was motivated both by a desire to resolve certain shortcomings in some of the current reaction theories, and by the need for a practical, but consistent, theory to analyze (p,d) reactions at intermediate energies. Its guidelines were derived from our physical understanding of transfer reaction data (in particular, in terms of DWBA); however, its development was only made possible by a

suitable synthesis of various theoretical techniques: Faddeev equations,[1] Feshbach theory,[2] many-body scattering theory,[3] and, to a minor extent, multiple scattering theory.[4,5]

Let me first discuss the drawbacks which plague some of the current theories of rearrangement scattering. This discussion only serves to emphasize these shortcomings, and should not be seen as an exhaustive review. Many descriptions of rearrangement reactions are based on three-body models,[6–10] the three particles being the projectile (ejectile), the transferred particle, and the core. In this approach, effective potentials between the clusters are given from the onset and their dynamical origin and form never enter explicitly in the formulation. These effective potentials both have to describe scattering and binding phenomena, so that they should be energy-dependent and complex (at least above the first inelastic threshold). However, since such potentials are very hard to use consistently, one either ignores the constraints set by the three-body model,[10] or one maintains consistency by allowing only those potentials to be complex which do not have to support a bound state.[9] However, even in the latter case inconsistencies of a slightly different nature may arise (such as different neutron and proton-nucleus effective potentials[9]). A further drawback of these three-body models is that one usually makes no attempt to relate the effective potentials to more microscopic entities, such as the microscopic otpical potential. In summary, three-body models usually lack a firm microscopic basis, consistency, and/or flexibility (in choosing the effective potentials), all of which are properties which seem indispensable for a practical, microscopic theory.

Slightly different in philosophy, although in many cases equivalent, are the theories[11] which start from the exact Schrödinger equation, and subsequently limit the Hilbert space to the asymptotic channel configurations. Depending on when and how this truncation is done, one gets different theories, the main exponent being the distorted wave Born approximation (DWBA).[12] This approximation results if one truncates the space *after* the introduction of distorting potentials in the asymptotic channels. Formally, any distorting potential can be used, and possible microscopic criteria, such as the minimization of the omitted terms, have proven to be quite ineffective in practice.[11] Therefore, one usually chooses the distorting potential on intuitive grounds, and takes it to be equal to the elastic optical potential. This procedure is quite unsatisfactory from a microscopic point of view since it offers no guarantee that the remaining terms in the transition operator are small. Nonetheless, the DWBA has proved quite successful in describing the experimental data, which clearly tells us something about the physics of the reaction, since it is unlikely that this success was all accidental. We therefore want to exploit our experience with the DWBA to give us some guidance in developing our microscopic theory.

In order to define such a guiding principle we first want to rephrase the DWBA in more microscopic language. The DWBA amplitude usually employed in fits to experimental data is the distorted wave matrix element of a simple transition potential [such as the neutron-proton interaction in the (p,d) reaction]. Since the plane wave matrix element of the same transition potential is much less successful in explaining the data, it seems that much of the strength of the higher-order terms in the exact T matrix can be absorbed into the distorted waves. In more physical terms: The process takes place in three stages: elastic scattering in the initial channel, the actual transition, and elastic scattering in the final channel. This picture stresses the importance of the asymptotic channel configurations and deemphasizes the *external* role of the more complicated many-body degrees of freedom. These latter degrees of freedom, however, play an essential role in determining the (microscopic) optical potential, as all higher-order contributions to this potential derive from the very existence of these nonelastic degrees of freedom. Since the potential determines the distorted waves, these many-body degrees of freedom are still essential for a reasonable description of the process. This conclusion is supported by a recent model study of the (p,d) reaction, where we showed[8] that continuum contributions to the scattering amplitude become increasingly important when the energy goes up, but still can be represented reasonably well by the distorted waves. In the same publication we show that a rigorous cutoff of the continuum degrees of freedom (the bound-state approximation) leads to much poorer results. Another study[9] also emphasizes the importance of the optical potential, and thus implicitly the importance of many-body degrees of freedom.

The fundamental lesson to be learned is the following: Despite the fact that many-body degrees of freedom play a very important role in transfer reactions, their main effect can be absorbed into the distorted waves, i.e., in a wave function with a

simple two-body structure. Or to rephrase this in a useful guiding principle for our microscopic theory: The initial and final channel configurations should be treated explicitly and in a rigorous way, whereas the more complicated many-body degrees of freedom should be absorbed into potentials and treated in an implicit—possibly perturbative—way. This also leads to a practical formulation: effective two-body equations with the complexity buried in the potentials.

The guiding principle is very similar to the one applied in the multiple scattering theory of elastic scattering.[5] Therefore, it should come as no surprise that our theory will accommodate the microscopic elastic potential in a very natural way. In fact, one of the main original purposes of our work was to make the role of the microscopic optical potential in rearrangement reactions more explicit. We felt that a duplication of the enormous amount of work on microscopic theories of elastic scattering in the context of rearrangement theory would be inappropriate (Refs. 4 and 13–18 give some idea of this effort). Instead, the theory should be flexible enough to accommodate the microscopic optical potential. The present theory accomplishes this goal in a natural way, and reserves a very important role for this potential. As it turns out, this also increases the practicality and accuracy of the theory, as we can now take the best potential and their associated wave functions from the literature, and use them to evaluate the distorting potential and distorted waves in the present framework. Perhaps we should stress the nontrivial nature of the guiding principle; neither the three-body models, which ignore the role of the many-body degrees of freedom, nor the majority of many-body scattering formulations, which only become practical after a rigorous cutoff in the Hilbert space, cherish this principle.

The development of the present theory is made possible by an enormous simplification (*not* approximation) at the very start. By concentrating ourselves on one particular amplitude, itself part of the total physical amplitude, we reduce the amount of two-body channels which has to be considered explicitly from $2^{N-1}-1$ in N-body scattering theory to a mere three or four. In this way we give proper account of the dynamics of the process, and put the burden of dealing with many-body singularities on the effective potentials. These effective potentials still have a very simple form and do not have the nested structure typical of some many-body scattering theories.[19] In this paper we limit ourselves to those amplitudes which only necessitate the explicit treatment of three channels. This enables us to use standard Faddeev techniques, which, as we will see, are extremely useful. The limitation is not very serious in practice; all proton-induced transfer reactions are covered by the present theory, whereas the direct and exchange amplitudes (defined later) for any rearrangement process are covered as well.

The outline of this paper is as follows. In Sec. II we start the formal separation of two-body and many-body degrees of freedom in the effective Faddeev equations for the amplitude under consideration. This separation is accomplished by using standard projection techniques,[20] which do not give rise to so-called nonorthogonality problems, as the Faddeev equations account properly for the channel dynamics.[21] The use of projectors also automatically solves the problem of the antisymmetrization among identical nucleons. In Sec. III we discuss the connection between the distorting potentials and the microscopic optical potentials. This relation is still obscured by the presence of a correction term, whose task it is to eliminate the rearrangement contribution from the optical potential. In order to treat this correction term and the associated recoupling term in the transition operator at the same footing, we analyze the transition amplitude in more detail in Sec. IV. This leads to the introduction of a new distorting potential which no longer contains this correction term and is phase equivalent with the microscopic optical potential. The resulting transition operator contains the same transition potential as before; however, the expansion in recoupling terms has changed in a simple but drastic way. Some implications of these changes for standard coupled channel calculations are discussed. In Sec. V we discuss some explicit expressions for the distorting potential and distorted wave function in terms of the elastic input. A rigorous calculation of these objects requires breakup-matrix elements of the microscopic optical potential operator; however, we hint at some approximations which will reduce this to pure elastic information. In Sec. VI we discuss the expansion of the transition potential. This potential has a purely constructive character (i.e., no counter terms). We discuss various first order terms, stressing in particular a rescattering amplitude, which is never included in coupled channel calculations. Finally, in Sec. VII we give a brief summary, and discuss various aspects and possible extensions of the theory.

II. MICROSCOPIC THEORY OF REARRANGEMENT SCATTERING

We consider a projectile consisting of n nucleons incident on a target containing A nucleons. The whole system will be governed by a nuclear many-body Hamiltonian of the form

$$H=\sum_{i=1}^{A+n} T_i+\sum_{i<j}^{A+n} V_{ij} \ , \tag{2.1}$$

so that we neglect many-body forces and do not treat pion and isobar degrees of freedom explicitly. Given the scattering process $A(n,m)A+n-m$, with $m \geq n$, one can distinguish $(n+1)$ different types of amplitudes (assuming $A>n$) depending on how many particles of the projectile are contained in the final ejectile $0,1,\ldots,n$). Of course, we cannot discriminate the different asymptotic states physically; however, the microscopic description is quite different.

These $(n+1)$ different amplitudes can be divided into those with an intrinsic two-, three-, and four-body structure. Those with a two-body structure describe direct elastic scattering (requires $n=m$) and are not discussed here. Those with a three- and four-body structure represent true rearrangement processes. The three-body amplitudes correspond to the case where the ejectile contains all (direct) or none (exchange) of the projectile nucleons. The remaining $(n-1)$ amplitudes have a four-body structure. The dynamics of these latter processes is such that the final state cannot be reached in a single-step transfer process, so that a complex multistep mechanism is required. All proton induced $(n=1)$ and direct single-step processes have the simpler intrinsic three-body structure, which will be investigated in this paper. Let us give an example to illustrate this classification: The physical deuteron elastic scattering amplitude has three parts[22]: one representing elastic scattering without exchange (considered as input in this theory); one representing the exchange of one particle, which cannot be dealt with by the present theory, as it has an intrinsic four-body structure; and one in which both particles are exchanged, which *can* be dealt with by the present theory. For illustrative purposes, we will often specialize our discussion to the direct (p,d) amplitude; however, this does in no way limit the generality of the theory. In the (p,d) reaction we distinguish two amplitudes: the direct and the exchange amplitude, both shown in Fig. 1. The direct amplitude contains the important single-step pickup process,

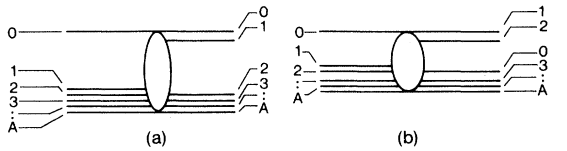

FIG. 1. Direct and exchange amplitude in the (p,d) reaction.

whose dominance is usually assumed at forward angles, and is essential for obtaining simple spectroscopic information. The exchange amplitude contains the heavy particle exchange diagram, important at backward angles. In order to obtain the physical (properly symmetrized) amplitude one multiplies the direct amplitude with A, the exchange amplitude with $A(A-1)/2$, and adds them up. Both the direct and exchange processes involve at least three clusters of particles: the projectile (proton), the transferred particle(s) (neutron deuteron), and the core ($A-1$ or $A-2$). Obviously, this three-body structure has to be taken into account in an exact description of these amplitudes. The enormous simplification of our theory over the usual many-body scattering theories is due to the fact that we only treat three of the 2^A-1 two-body channels explicitly. This reduction is possible because we treat different amplitudes by different equations.

The three two-body channels are uniquely determined by the cluster structure of the amplitude under consideration. For definiteness we will discuss the direct amplitude shown in Fig. 1(a). The total Hamiltonian is written as the sum of the core Hamiltonian H_0, which contains all the interactions among the $A-1$ core nucleons plus all the kinetic energy operators:

$$H_0=K+\sum_{\substack{i<j \\ i>1}}^{A} V_{ij} \ . \tag{2.2}$$

Using standard three-body language we define the intercluster interactions V_i:

$$V_1=\sum_{i>1}^{A} V_{i1} \ , \tag{2.3}$$

$$V_2=\sum_{i>1}^{A} V_{0i} \ , \tag{2.4}$$

$$V_3=V_{01} \ . \tag{2.5}$$

As usual, the channel Hamiltonians are defined by

$$H_i=H_0+V_i, \quad i=1,2,3 \ . \tag{2.6}$$

Asymptotic channel wave functions will satisfy

$$H_i\phi_i = E\phi_i \ . \tag{2.7}$$

In analogy to Faddeev's theory[1] we introduce channel components which correspond to the three arrangement channels defined by Eq. (2.6):

$$\psi = \psi_1 + \psi_2 + \psi_3 \ , \tag{2.8}$$

with

$$(E - H_i)\psi_i = V_i \sum_{j \neq i} \psi_j, \quad i,j = 1,2,3 \ . \tag{2.9}$$

If we sum these equations we recover the original Schrödinger equation for the full wave function. In channel coupling array[23] theory one can prove that a channel component contributes exclusively to the asymptotic two-cluster channel specified by the label of the channel component. In the present case we only made a partial decomposition of the wave function, and only treat three of the $2^A - 1$ two-body channels explicitly. Under these circumstances ψ_i will contribute to many asymptotic channels. However, only ψ_i will contribute to the asymptotic wave in two-body channel i. This property of the Faddeev components $\psi_i = (E - H_0 + i0)^{-1} V_i \psi = G_0 V_i \psi$ is well known[1] and was recently reconsidered in the context of many-body scattering theory.[24] The property follows from the fact that the other components $\psi_j = G_0 V_j \psi$ *cannot* generate the scattering singularity (in momentum space) in channel i. This is due to the fact that $G_0 V_j$ does not contain all the interactions internal to channel i (which are necessary for generating the channel eigenstates and the typical scattering singularity) but does contain interactions external to channel i (in the form of V_j), which prevent the creation of the channel eigenstate.[25] The correctness of this property is confirmed explicitly in Appendix C, where we prove that the channel components carry the physical amplitude.

The channel components are extremely useful in the present context as they account in a natural way for the dynamics of the reaction process. As a consequence we can easily impose the boundary conditions and locate the relevant asymptotic information in the different components. Also, it enables us to use projection techniques without facing the problem of channel nonorthogonality. In our (p,d) example the incident wave is carried by ψ_1, so that

$$\psi_1 \sim e^{i\vec{k}_p \vec{r}_1} \eta_1(0) \Psi_1(1, \ldots, A) + \text{outgoing waves} \quad (r_1 \text{ large}) \ , \tag{2.10}$$

which obviously satisfies Eq. (2.7). The other components ψ_2 and ψ_3 do not contain outgoing or incident waves in channel 1. The target wave function Ψ_1 in (2.10) is antisymmetric under interchange of two nucleons. The physical wave function should also be antisymmetric among the projectile (0) and the target nucleons; however, this antisymmetrization does not affect the determination of the present amplitude, and just fixes the statistical factor which multiplies the direct amplitude when it is added together with the exchange amplitude. The (p,d) direct amplitude is now carried by ψ_3:

$$\psi_3 \sim \frac{e^{ik_d r_3}}{r_3} \eta_3(0,1) \Psi_3(2, \ldots, A) \times T_{31}^{\text{direct}}(\vec{k}_d, \vec{k}_p, E) \quad (r_3 \text{ large}) \ , \tag{2.11}$$

whereas ψ_2 will carry information on the (p,n) amplitude (assuming the two-body neutron-nucleus channel exists).

As pointed out in the Introduction, our aim is to treat the asymptotic channel degrees of freedom explicitly and the more complicated many-body degrees of freedom implicitly. To accomplish this we use standard projection operator techniques.[26] Each of the three channels has its own projector, e.g., the proton channel projector is given by

$$P_1 = \int d\vec{k} \, | \, \vec{k} \eta_1(0) \Psi_1(1, \ldots, A) \rangle \times \langle \Psi_1(1, \ldots, A) \eta_1(0) \vec{k} \, | \ . \tag{2.12}$$

This definition is easily generalized to include excited states by writing P_1 as a sum over the excited states $\Psi_1^{(\alpha)}$. We can also set $P_i = 0$ if we do not want to treat a channel explicitly. This is usually the case for the third channel which neither corresponds to the initial, nor to the final channel [such as the neutron channel in the (p,d) case]. The complement of P_i is called Q_i:

$$\psi_i = P_i \psi_i + Q_i \psi_i = \psi_i^P + \psi_i^Q \ . \tag{2.13}$$

In defining Q_i this way we have implicitly assumed that the Hilbert space is well defined. Since the original incoming wave is fully antisymmetric in the target nucleon coordinates $1,2, \ldots, A$, and since the Hamiltonian is manifestly symmetric in all nucleon coordinates, the exact wave function also displays this symmetry. The channel components, however, are defined in a nonsymmetric fashion and will not display the full symmetry, although they still will be antisymmetric among the

core nucleons. The Hilbert space is therefore limited to wave functions which are completely antisymmetric in the particle coordinates $2,3,\ldots,A$, but not further restricted in the symmetry properties of particles 1 and 0. A further discussion of symmetry properties is given at the end of this section.

We now eliminate ψ_i^Q everywhere in favor of ψ_i^P. The resulting equations are a straightforward matrix generalization of the usual Feshbach equations[2]:

$$(\underline{\underline{E}}-\underline{\underline{P}}\,\underline{\underline{H}}\,\underline{\underline{P}}-\underline{\underline{P}}\,\underline{\underline{U}}\,\underline{\underline{P}})\underline{\psi}^P=\underline{0}\ , \tag{2.14}$$

where

$$\underline{\underline{U}}=\underline{\underline{V}}\,\underline{\underline{Q}}\,\underline{\underline{G}}_{QQ}\,\underline{\underline{Q}}\,\underline{\underline{V}}\ , \tag{2.15}$$

$$(\underline{\underline{H}})_{ij}=\delta_{ij}H_i+\bar{\delta}_{ij}V_i\ , \tag{2.16}$$

$$(\underline{\underline{V}})_{ij}=V_i\bar{\delta}_{ij}\ , \tag{2.17}$$

$$\underline{\underline{G}}_{QQ}=(\underline{\underline{E}}-\underline{\underline{Q}}\,\underline{\underline{H}}\,\underline{\underline{Q}}+i0)^{-1}\ , \tag{2.18}$$

and

$$(\underline{\psi}^P)_i=\psi_i^P=P_i\psi_i=|\,\eta_i\Psi_i\chi_i\,\rangle\ . \tag{2.19}$$

Projection and energy operators are diagonal [e.g., $(\underline{\underline{P}})_{ij}=\delta_{ij}P_i$], and $\bar{\delta}_{ij}=1-\delta_{ij}$. Naturally, $\underline{\psi}^P$ is subject to the usual incoming boundary condition [e.g., Eq. (2.10)]. If we now project with $\langle\Psi_i\eta_i|$ from the left one obtains the following set of effective two-body equations for the relative motion wave functions:

$$(E+E_i-K_i-\langle U_{ii}\rangle)\chi_i$$

$$=\sum_{j\neq i}\langle V_i+U_{ij}\rangle\chi_j,\quad i,j=1,2,3, \tag{2.20}$$

where E_i is the total binding energy and K_i is the relative kinetic energy in channel i. In deriving (2.20) we employed the property $[H_i,P_i]=[H_i,Q_i]=0$. We also introduced a shorthand notation for the distorting potentials,

$$\langle U_{ii}\rangle=\langle\eta_i\Psi_i\,|\,U_{ii}\,|\,\eta_i\Psi_i\rangle\ , \tag{2.21}$$

and the transition potentials,

$$\langle V_i+U_{ij}\rangle=\langle\eta_i\Psi_i\,|\,V_i+U_{ij}\,|\,\eta_j\Psi_j\rangle\ . \tag{2.22}$$

Equation (2.20) is our first basic result. By making a partial wave expansion of the operators we can reduce it to a set of one variable integrodifferential equations for the relative wave functions χ_i. The relative wave function for the deuteron channel contains the asymptotic information about the direct (p,d) transfer amplitude.

We can also take into account the coupling to excited states explicitly by including these states in the projection operator P_i. Labeling the nuclear states by Greek indices α or β we obtain, instead of (2.20),

$$(E+E_i^{(\alpha)}-K_i-\langle\alpha\,|\,U_{ii}\,|\,\alpha\rangle)\chi_i^\alpha$$

$$=\sum_{\beta\neq\alpha}\langle\alpha\,|\,U_{ii}\,|\,\beta\rangle\chi_i^\beta+\sum_{\substack{j\neq i\\ \beta}}\langle\alpha\,|\,V_i+U_{ij}\,|\,\beta\rangle\chi_j^\beta\ , \tag{2.23}$$

where U_{ij} is now defined in terms of a reduced $\underline{\underline{Q}}$ operator which is complementary to the new $\underline{\underline{P}}$ operator. These equations show a striking resemblance to the well-known coupled channel Born approximation (CCBA) equations.[30] Hence, our formulation may also give a microscopic basis for these latter equations.

Let us conclude this section by discussing some interesting aspects of the theory developed so far. The final equations (2.14) or (2.20) do not contain any nonorthogonality terms (terms of the form P_iP_j with $i\neq j$). This is due to the effective separation of different channel spaces by the Faddeev decomposition. The only way P_i and P_j $(j\neq i)$ can communicate is through the transition potentials V_i+U_{ij}. This property also guarantees a simple intuitive interpretation of the operators in the problem. The property would be lost if we had used distorted Faddeev equations,[31] where an optical potential $P_iV_{\rm opt}P_i$ is added to the left of Eq. (2.9). To compensate for such a term, one has to subtract terms of the same form on the right, leading to nonorthogonality products. The distorted Faddeev equations also lead to nonconstructive transition potentials, a property shared by other approaches which introduce (arbitrary) potentials in the formulation.[9–11]

We now pursue our discussion of symmetry properties. In the scattering region, ψ_1 will not be antisymmetric for the interchange of nucleons between clusters because of the nonsymmetric nature of the Faddeev equations. However, since ψ_1 carries the asymptotic wave function in the elastic channel, we expect that asymptotically the antisymmetry among target nucleons is restored. This is exactly what happens: The lack of symmetry is caused by the operator G_0V_i in $\psi_i=G_0V_i\psi$, and this operator becomes unity in the asymptotic region, thereby removing this lack of symmetry ($\psi=i\epsilon G\phi_{\rm inc}$ itself preserves the symmetry of the in-

incident wave). The lack of antisymmetry among the nucleons 0 and 1 in the asymptotic deuteron channel remains, but is not of serious nature. By completely antisymmetrizing the incident wave (2.10), one would, of course, guarantee that ψ_3 displays the correct symmetry properties for $r_3 \to \infty$, so that unphysical components would be eliminated. The same elimination takes place, when we operate with P_3 on ψ_3, where P_3 is naturally defined in terms of physical (i.e., fully antisymmetric) cluster states. It is therefore easy to show that ψ_3^P carries the correct (direct) physical amplitude.[25] Without any extra effort we have resolved the problem of antisymmetrization by using the projection techniques. As mentioned earlier we will even show explicitly that the amplitudes carried by the channel components are on-shell-equivalent with the exact ones.

III. RELATION BETWEEN DISTORTING AND OPTICAL POTENTIAL

The distorting potentials $\langle U_{ii} \rangle$ are defined as the diagonal matrix elements (2.21) of the operator $\underline{U}$, defined in Eq. (2.15):

$$\underline{\underline{U}} = \underline{\underline{V}}\,\underline{\underline{Q}}\,\underline{\underline{G}}_{QQ}\,\underline{\underline{Q}}\,\underline{\underline{V}} \; . \tag{3.1}$$

In this section we want to relate this distorting potential to the usual microscopic optical potential. The formal definition of the elastic optical potential operator in our notation is[15]

$$U_i^F = \sum_{m \neq i} V_m + \left[\sum_{m \neq i} V_m \right] Q_i (E - Q_i H Q_i + i0)^{-1} Q_i \times \sum_{m \neq i} V_m \; , \tag{3.2}$$

where H is the full Hamiltonian of the $(A+1)$-body system. The (Feshbach) optical potential $\langle U_i^F \rangle$ is the ground-state matrix element of this operator. Notice that the Feshbach optical potential by itself does not give the complete elastic amplitude since it does not account for exchange terms. In order to connect the distorting potential $\langle U_{ii} \rangle$ and the elastic potential $\langle U_i^F \rangle$ we first define generalized scattering operators corresponding to these potentials. One can easily show that U_i^F satisfies the following Lippmann-Schwinger (LS) equations:

$$U_i^F = \sum_{m \neq i} V_m + \sum_{m \neq i} V_m Q_i (E - H_i + i0)^{-1} Q_i U_i^F = \sum_{m \neq i} V_m + U_i^F Q_i (E - H_i + i0)^{-1} \sum_{m \neq i} V_m \; . \tag{3.3}$$

The operator $\underline{U}$ also satisfies a LS equation:

$$\underline{\underline{U}} = \underline{\underline{V}}\,\underline{\underline{Q}}\,\underline{\underline{G}}\,\underline{\underline{V}} + \underline{\underline{V}}\,\underline{\underline{Q}}\,\underline{\underline{G}}\,\underline{\underline{U}} = \underline{\underline{V}}\,\underline{\underline{Q}}\,\underline{\underline{G}}\,\underline{\underline{V}} + \underline{\underline{U}}\,\underline{\underline{Q}}\,\underline{\underline{G}}\,\underline{\underline{V}} \; , \tag{3.4}$$

where

$$(\underline{\underline{G}})_{ij} = \delta_{ij} (E - H_i + i0)^{-1} = \delta_{ij} G_i (E + i0) \; . \tag{3.5}$$

Notice that $[\underline{Q}, \underline{G}] = 0$. One obstacle in relating $\underline{U}$ and U_i^F is the occurrence of the three projection operators Q_i in $\underline{U}$. Therefore, we introduce the scattering operator $\hat{\underline{A}}$ which is free of these projection operators:

$$\hat{\underline{\underline{A}}} = \underline{\underline{V}}\,\underline{\underline{G}}\,\underline{\underline{V}} + \underline{\underline{V}}\,\underline{\underline{G}}\,\hat{\underline{\underline{A}}} = \underline{\underline{V}}\,\underline{\underline{G}}\,\underline{\underline{V}} + \hat{\underline{\underline{A}}}\,\underline{\underline{G}}\,\underline{\underline{V}} \; . \tag{3.6}$$

Obviously, $(\hat{\underline{A}} + \underline{V})$ satisfies a conventional Lippmann-Schwinger equation with driving term $\underline{V}$, so that we can use a standard two-potential formula to connect $\underline{U}$ and $\hat{\underline{A}}$:

$$\underline{\underline{U}} - \hat{\underline{\underline{A}}} = -(\underline{\underline{U}} + \underline{\underline{V}})\underline{\underline{P}}\,\underline{\underline{G}}(\underline{\underline{A}} + \underline{\underline{V}}) = -(\hat{\underline{\underline{A}}} + \underline{\underline{V}})\underline{\underline{P}}\,\underline{\underline{G}}(\underline{\underline{U}} + \underline{\underline{V}}) \; . \tag{3.7}$$

Because of the nondiagonal term, $\langle \hat{A}_{ii} \rangle$ is not the true scattering amplitude corresponding to $\langle U_{ii} \rangle$. One can show that $\langle \hat{A}_{ii} \rangle$ is on shell equivalent with the scattering amplitude belonging to $\langle U_i^F \rangle$: defining

$$\hat{A}_{ij} = V_i G_0 A_{ij}^F \; , \tag{3.8}$$

one derives the following equation from (3.6):

$$A_{ii}^F = \left[\sum_{m \neq i} V_m \right] + \left[\sum_{m \neq i} V_m \right] (E - H + i0)^{-1} \left[\sum_{m \neq i} V_m \right] \; , \tag{3.9}$$

so that

$$A_{ii}^F = U_i^F + U_i^F P_i G_i A_{ii}^F = U_i^F + A_{ii}^F P_i G_i U_i^F \; . \tag{3.10}$$

This shows that $\langle A_{ii}^F \rangle$ is the scattering amplitude corresponding to $\langle U_i^F \rangle$. Since $V_i G_0$ is unity if it operates to the left on an on-shell state, we have proved that $\langle \hat{A}_{ii} \rangle$ is on shell equivalent with the exact elastic scattering amplitude $\langle A_{ii}^F \rangle$. One can now express $\hat{A}_{ii}$ directly in terms of U_i^F:

$$\hat{A}_{ii} = V_i G_0 U_i^F + \hat{A}_{ii} P_i G_i U_i^F \; . \tag{3.11}$$

Using (3.7) we now establish a relationship between U_i^F and U_{ii}. Defining the operator

$$\Delta_i = \sum_{m \neq i} (\hat{A}_{im} + V_i) P_m G_m G_m (U_{mi} + V_m) , \quad (3.12)$$

and its transpose

$$\tilde{\Delta}_i = \sum_{m \neq i} (U_{im} + V_i) P_m G_m (\hat{A}_{mi} + V_m) , \quad (3.13)$$

and eliminating $\hat{A}_{ii}$ from (3.7) and (3.11), one finally obtains

$$U_{ii} = V_i G_0 U_i^F + U_{ii} P_i G_0 U_i^F - \tilde{\Delta}_i (1 - P_i G_i U_i^F) . \quad (3.14)$$

Although this equation could be used to evaluate $\langle U_{ii} \rangle$ in terms of U_i^F, the presence of the nondiagonal term, which requires information on rearrangement amplitudes, is fairly inconvenient. Let us therefore analyze this term in some more detail. It is quite evident that $\tilde{\Delta}_i$ represents the process of rearrangement elastic scattering as illustrated in Fig. 2. Since such rearrangement effects are included to all orders in the coupled channel equation (2.20), they have to be subtracted from the optical potential, and Eq. (3.14) tells us how to do this in the present exact framework. Because of the similarity of these recoupling terms in Eq. (2.20), and the correction terms generated in Eq. (3.14), it would seem rather inconsistent to include these effects in one case but not in the other. Instead, one would like to treat them at the same footing, and exploit possible cancellations between them. This goal can be accomplished by redefining the distorted wave basis in terms of an optical potential $\hat{U}_{ii}$ which is free of the correction term. The correction term is then forced into the transition amplitude and we will see that it can be nicely combined with the recoupling terms already present in this operator.

However, before moving on with this analysis of the transition operator, we want to make some remarks on the magnitude of $\tilde{\Delta}_i$, and more, in particular, on the magnitude of the recoupling effects in the coupled channel equation (2.20). Although these effects proceed via the important two-body channel degrees of freedom, their effect has been shown to be quite small in calculations performed so far.[9,10] Despite the fact that these calculations only included the first part of the transition potential $\langle V_i + U_{im} \rangle$, they should give a fair impression of the magnitude of these effects (since we expect that U_{im} is small with respect to V_i). An important theoretical reason which weighs in favor of the smallness of Δ_i is that this rearrangement process only involves a single nucleon (in the proton example of Fig. 2), so that it does not represent a coherent process, as do the other contributions to the microscopic optical potential. Naturally, this argument does not apply for the deuteron distorting potential, as only two nucleons are involved here.

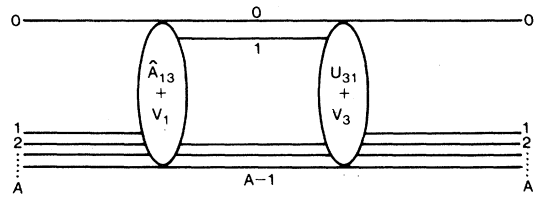

FIG. 2. Rearrangement contribution Δ_1 in the proton optical potential for $P_2 = 0$. $\langle \hat{A}_{im} + V_i \rangle$ is on-shell equivalent with the exact transition amplitude, whereas $\langle U_{mi} + V_m \rangle$ is the transition potential.

IV. REARRANGEMENT OPERATORS IN VARIOUS DISTORTED WAVE BASES

The coupled equation (2.20) defines transition operators in the distorted wave basis $\chi^{(0)}_{i,\vec{k}_i}$. These distorted waves satisfy the ordinary Schrödinger equation

$$(E + E_i - K_i - \langle U_{ii} \rangle) \chi^{(0)}_{i,\vec{k}_i} = 0 , \quad (4.1)$$

where $\vec{k}_i$ specifies the momentum of the incoming wave ($E = k_i^2/2M_i - E_i$). The formal solution of (4.1) is given by one of the following equations:

$$\chi^{(0)}_{i,\vec{k}_i} = |\vec{k}_i\rangle + P_i G_i (E + i0) \langle U_{ii} \rangle \chi^{(0)}_{i,\vec{k}_i} , \quad (4.2)$$

$$\chi^{(0)}_{i,\vec{k}_i} = |\vec{k}_i\rangle + P_i \tilde{G}_i (E + i0) \langle U_{ii} \rangle |\vec{k}_i\rangle , \quad (4.3)$$

$$\chi^{(0)}_{i,\vec{k}_i} = |\vec{k}_i\rangle + P_i G_i (E + i0) \langle A_{ii} \rangle |\vec{k}_i\rangle , \quad (4.4)$$

where

$$\tilde{G}_i (E + i0) = (E - H_i - \langle U_{ii} \rangle + i0)^{-1} \quad (4.5)$$

and

$$\langle A_{ii} \rangle = \langle U_{ii} \rangle + \langle A_{ii} \rangle P_i G_i \langle U_{ii} \rangle . \quad (4.6)$$

Notice that $\langle A_{ii} \rangle$ is the scattering operator corresponding to $\langle U_{ii} \rangle$. The rearrangement amplitude in the basis (4.1) follows directly from Eq. (2.20):

$$T_{ji} = \langle V_j + U_{ji} \rangle \bar{\delta}_{ji} + \langle V_j + U_{jm} \rangle \bar{\delta}_{jm} P_m \tilde{G}_m T_{mi} . \quad (4.7)$$

Higher-order terms in Eq. (4.7) represent repeti-

tions of rearrangement scattering, e.g., pickup leading from the proton to the deuteron channel, followed by scattering back into the elastic channel, and finally a return to the deuteron channel. We now want to combine these recoupling terms with the corresponding subtraction terms in U_{ii} and U_{jj}. To simplify the following considerations we will not treat the uninteresting third channel explicitly and put $P_m=0$, so that Q_m is the unit operator [in the (p,d) example, m is the neutron channel]. This choice also simplifies the calculation of the transition potential $\langle V_j+U_{ji}\rangle$. Equation (4.7) can now be solved for T_{ji} $(i\neq j)$:

$$T_{ji}=\langle V_j+U_{ji}\rangle+\langle V_j+U_{ji}\rangle P_i\tilde{G}_i\langle V_i+U_{ij}\rangle P_j\tilde{G}_j T_{ji}\ , \tag{4.8}$$

or its right-handed partner:

$$T_{ji}=\langle V_j+U_{ji}\rangle+T_{ji}P_i\tilde{G}_i\langle V_i+U_{ij}\rangle P_j\tilde{G}_j\langle V_j+U_{ji}\rangle\ . \tag{4.9}$$

These are linear integral equations in one continuous variable. Since the distorted waves $\chi^{(0)}_{i,\vec{k}_i}$ are eigenfunctions of the Hamiltonian contained in $\tilde{G}_i$ this equation is also equivalent to the coupled channel equations (2.20). Using the off-diagonal form of Eq. (3.7) and the definitions of $\tilde{G}_j$ [Eq. (4.5)] and Δ_i [Eq. (3.12)], one can derive the following identity:

$$\langle V_i+U_{ij}\rangle P_j\tilde{G}_j\langle V_j+U_{ji}\rangle = P_i(1+\hat{A}_{ii}P_iG_i)^{-1}P_i\Delta_iP_i\ . \tag{4.10}$$

We can now express the second term of (4.9) in terms of Δ_i:

$$T_{ji}=\langle V_j+U_{ji}\rangle+T_{ji}P_i\tilde{G}_i(1+\hat{A}_{ii}P_iG_i)^{-1}P_i\Delta_iP_i,\quad i\neq j\ . \tag{4.11}$$

Next we redefine T_{ji} in a basis not involving the subtraction term Δ_i. In analogy with (3.7) we thus define an optical potential $\langle\hat{U}_{ii}\rangle$ satisfying

$$\hat{U}_{ii}=\hat{A}_{ii}-\hat{A}_{ii}P_iG_i\hat{U}_{ii}\ . \tag{4.12}$$

Using (3.11) we can express $\hat{U}_{ii}$ in terms of U_i^F:

$$\hat{U}_{ii}=V_iG_0U_i^F+\hat{U}_{ii}P_iG_0U_i^F\ . \tag{4.13}$$

Hence, we have succeeded in eliminating the nondiagonal term in Eq. (3.14) through the introduction of $\hat{U}_{ii}$. Obviously, Eq. (4.13) will be easier to use in practical calculations than Eq. (3.14). Notice that according to Eqs. (3.8) and (3.10), $\langle\hat{U}_{ii}\rangle$ and $\langle U_i^F\rangle$ are phase equivalent (i.e., they give the same on-shell scattering amplitude and phase shifts). Despite the fact that $\hat{U}_{ii}$ and U_i^F are phase equivalent we expect that the corresponding elastic wave functions in the interior differ more than those corresponding to $\hat{U}_{ii}$ and U_{ii}, which are not phase equivalent, but whose difference is proportional to Δ_i. We have summarized the different potentials and transition operators with their relationships schematically in Fig. 3.

In order to obtain the transition operator in the new distorted wave basis determined by the potential operator defined in Eq. (4.13) we compute the "ratio" of corresponding Möller wave operators:

$$\chi^{(0)}_{i,\vec{k}_i}=[1+P_iG_i(E+i0)\langle A_{ii}\rangle]\,|\,\vec{k}_i\rangle = [1+P_iG_i(E+i0)\langle A_{ii}\rangle]\times[1+P_iG_i(E+i0)\langle\hat{A}_{ii}\rangle]^{-1}\,|\,\hat{\chi}_{i,\vec{k}_i}\rangle\ , \tag{4.14}$$

where $\hat{\chi}_{i,\vec{k}_i}$ is the distorted wave corresponding to $\hat{U}_{ii}$:

$$\hat{\chi}_{i,\vec{k}_i}=|\,\vec{k}_i\rangle+P_iG_i(E+i0)\langle\hat{U}_{ii}\rangle\hat{\chi}_{i,\vec{k}_i}\ . \tag{4.15}$$

In Appendix A we show that Eq. (4.14) can be

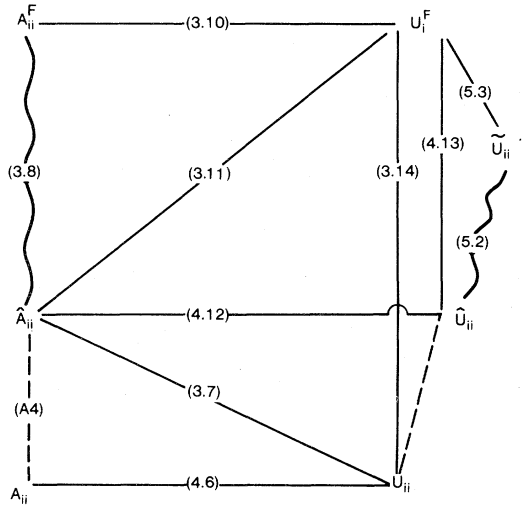

FIG. 3. Summary of different potentials (right) and scattering operators (left) in the theory. Ordinary Lippmann-Schwinger equations between potential and scattering operator are indicated by horizontal lines. Wiggly lines connect on-shell equivalent operators, whereas broken lines connect operators whose difference is first order in Δ_i. The central operators in our theory are the optical potential operator U_i^F and the distorting potential operator $\hat{U}_{ii}$.

rewritten as follows:

$$\chi^{(0)}_{i,\vec{k}_i}=[1+P_iG_i(E+i0)\langle\Delta_i\rangle]^{-1}\hat{\chi}_{i,\vec{k}_i}\ , \tag{4.16}$$

which explicitly shows that the difference between $\chi^{(0)}$ and $\hat{\chi}$ is of order Δ. We have already expressed the recoupling term in T_{ji} in terms of $\langle\Delta_i\rangle$, and now have to combine Eqs. (4.16) and (4.11) to obtain an expression for the transition operator in the basis $\hat{\chi}_{i,\vec{k}_i}$. Converting Eq. (4.11) by using the identity (see Appendix B)

$$(1+P_iG_i\langle\Delta_i\rangle)[1-P_i\tilde{G}_i(1+\hat{A}_{ii}P_iG_i)^{-1}\langle\Delta_i\rangle] =1\ , \tag{4.17}$$

we obtain

$$T_{ji}=\langle V_j+U_{ji}\rangle(1+P_iG_i\langle\Delta_i\rangle)\ , \tag{4.18}$$

so that we find with the help of Eq. (4.16) that

$$\langle\chi^{(0)}_{j,\vec{k}_j}\mid T_{ji}\mid\chi^{(0)}_{i,\vec{k}_i}\rangle=\langle\chi^{(0)}_{j,\vec{k}_j}\mid\langle V_j+U_{ji}\rangle\mid\hat{\chi}_{i,\vec{k}_i}\rangle\ . \tag{4.19a}$$

We can also replace the distorted wave state on the left, using the adjoint of (4.16). Defining the rearrangement operator $\hat{T}_{ji}$ in the new distorted wave basis by

$$\langle\chi^{(0)}_{j,\vec{k}_j}\mid T_{ji}\mid\chi^{(0)}_{j,\vec{k}_j}\rangle=\langle\hat{\chi}_{j,\vec{k}_j}\mid\hat{T}_{ji}\mid\hat{\chi}_{i,\vec{k}_i}\rangle\ , \tag{4.19b}$$

we find that with the help of Eqs. (4.17) and (4.10)

$$\hat{T}_{ji}=P_j(V_j+U_{ji})P_i -P_j(V_j+U_{ji})P_i\tilde{G}_i\langle V_i+U_{ij}\rangle P_j\tilde{G}_j\langle V_j+U_{ji}\rangle\ . \tag{4.20}$$

This expression still refers to the potential U_{ii} and U_{jj} via the Green's functions $\tilde{G}_i$ and $\tilde{G}_j$. In order to obtain an equation expressed exclusively in the new potentials, we introduce operators $\hat{G}_i$ in analogy to $\tilde{G}_i$ [Eq. (4.5)]:

$$\hat{G}_i(E+i0)=(E-H_i-\langle\hat{U}_{ii}\rangle+i0)^{-1}\ . \tag{4.21}$$

One can show that $\hat{G}_i$ and $\tilde{G}_i$ are related by

$$P_i\hat{G}_i=P_i\tilde{G}_i+P_i\tilde{G}_i\langle V_i+U_{ij}\rangle\tilde{G}_j\langle V_j+U_{ji}\rangle\hat{G}_iP_i\ , \tag{4.22}$$

where we made use of Eqs. (3.7), (4.10), and (4.12). This allows us to rewrite (4.20) as follows:

$$\hat{T}_{ji}=P_j(V_j+U_{ji})P_i -P_j(V_j+U_{ji})P_i\tilde{G}_i\langle V_i+U_{ij}\rangle P_j\hat{G}_j\hat{T}_{ji} \tag{4.23a}$$

$$=P_j(V_j+U_{ji})P_i -\hat{T}_{ji}P_i\hat{G}_i\langle V_i+U_{ij}\rangle P_j\hat{G}_j\hat{T}_{ji}\ . \tag{4.23b}$$

This is our basic result for the transition operator. One easily shows that the distorted wave matrix element of $\hat{T}_{ji}$ is identical to the exact plane wave T matrix (see Appendix C), confirming the validity of our discussion so far. Since the distorted waves are eigenfunctions of the Hamiltonian contained in $\hat{G}_i$, one can solve Eq. (4.23) in this basis, and also can use coupled channel methods. Because of the nonlinear character it cannot be solved by the usual Lippmann-Schwinger techniques; however, if the recoupling term is small then we can solve it by iteration. One easily verifies that Eq. (4.23) represents a series expansion in the operator

$$\hat{Z}=P_i\hat{G}_i\langle V_i+U_{ij}\rangle P_j\hat{G}_j\langle V_j+U_{ji}\rangle \tag{4.24}$$

with the formal solution:

$$\hat{T}_{ji}=P_j(V_j+U_{ji})P_i[-\tfrac{1}{2}\hat{Z}^{-1}+\tfrac{1}{2}\hat{Z}^{-1}(1+4\hat{Z})^{1/2}]\ , \tag{4.25}$$

where the square root should, of course, be understood in the sense of a series expansion. This should be compared to T_{ji} in (4.8), whose series expansion can be represented by

$$T_{ji}=P_j(V_j+U_{ji})P_i\{\,1-\tilde{Z}\,\}^{-1}\ , \tag{4.26}$$

where $\tilde{Z}$ is defined by Eq. (4.24) with $\hat{G}_m$ replaced by $\tilde{G}_m$. Notice that the lowest order recoupling terms in $\hat{T}_{ji}$ and T_{ji} have opposite signs, whereas higher order terms will also have different magnitude. This shows that conventional coupled channel equations[32,33] give incorrect results for the recoupling terms if unsubtracted potentials $\langle\hat{U}_{ii}\rangle$ are used. Whether this statement is also true for phenomenological coupled channel calculations using $\langle U_i^F\rangle$ instead of $\langle\hat{U}_{ii}\rangle$ cannot be stated with absolute certainty; however, it is hard to see how the off-shell transformation could affect it. As mentioned before, coupled channel codes can still be used to calculate $\hat{T}_{ji}$; one only has to readjust the coefficient of the coupling potential after every iteration step and keep track of the changes in the wave function after every iteration. One can expect that the radius of convergence of (4.25) is smaller than that of (4.26). In fact, if we could consider $\hat{Z}$ and $\tilde{Z}$ as pure numbers, then the convergence radii would differ by a factor of 4. Despite this expected weaker convergence, the effect of all the recoupling effects in $\hat{T}_{ji}$ will be smaller than in T_{ji} if we have a constructive recoupling effect (i.e., a recoupling

term $\hat{Z}$ or $\tilde{Z}$ which schematically speaking is positive and increases the magnitude of T_{ji}). The reason is that (4.25) alternates in sign for positive $\hat{Z}$. If such a decrease in coupled channel effects would be confirmed by actual calculations, then it would show that the distorted waves $\hat{\chi}_{i,\vec{k}_i}$ are not only more convenient than $\chi^{(0)}_{i,\vec{k}_i}$, but also lead to better first order estimates as more of the higher order effects are contained in $\hat{\chi}_{i,\vec{k}_i}$ than in $\chi^{(0)}_{i,\vec{k}_i}$.

V. EXPLICIT EXPRESSIONS FOR DISTORTING POTENTIALS AND DISTORTED WAVES

Although we have been able to rephrase the exact N-body problem of rearrangement scattering in more familiar intuitive terms, such as distorted waves, coupled channel equations, and transition amplitudes, we have introduced some elements which are not usually present in phenomenological theories. These new elements are the off-shell operator V_iG_0 (unity on shell) and the free three-cluster Green's function G_0. Both of these operators occur in Eq. (4.13), which relates $\hat{U}_{ii}$ to U_i^F. The complexity of V_iG_0 and G_0 is due to the fact that they do not commute with the two-body channel projector P_i. To study them it is sufficient to consider the operator H_0, since both V_i and G_0 can be expressed in H_0 plus operators which commute with P_i. For example, V_i always operates on the channel cluster Ψ_i, so that $V_i\Psi_i=(H_i-H_0)\Psi_i$ $=(K_i-\epsilon_i-\epsilon_{jk}-H_0)\Psi_i$, where K_i is the relative kinetic energy of cluster i and particle i and commutes with P_i, whereas $\epsilon_i+\epsilon_{jk}$ is the sum of the binding energy of particle i (ϵ_i) and the total binding energy of cluster (j,k). Since H_0 contains all three internal cluster Hamiltonians it does not change the cluster eigenstates. Therefore, it is very convenient to expand Ψ_i in these states:

$$\Psi_i=\sum_{\mu,\nu}\!\!\int f^i_{\mu\nu}\eta^\mu_j\eta^\nu_k,\quad i\neq j\neq k\neq i\ . \tag{5.1}$$

Therefore, we will be able to express all relevant matrix elements in terms of matrix elements between those cluster states and η_i. The necessity of knowing the cluster-structure (5.1) of Ψ_i is, of course, nothing new: It is exactly this expansion which one needs for computing the DWBA matrix element $\langle V_i\rangle$, and it is, in fact, the nuclear model and structure information put into (5.1), which one wants to test in rearrangement reactions. The new element which is introduced by the exact description is that we also need this cluster decomposition for the calculation of the appropriate matrix elements of the optical potential operator U_i^F. It is really not a surprising result: In elastic scattering one needs the ground state matrix elements $\langle U_i^F\rangle$ of U_i^F, in inelastic scattering one needs the excited state matrix elements of U_i^F, and in rearrangement scattering one needs the "breakup" matrix elements of U_i^F. Equation (4.13) then ensures that these matrix elements are properly embedded in the dynamical description of the rearrangement process. Through further experience with Eq. (4.13) one should acquire a better intuitive feeling for this phase equivalent transformation. In particular, it is of interest to see how it affects the behavior of the distorted wave in the interior, and how it affects their momentum distributions. These aspects and a number of approximations, which circumvent the necessity of knowing the detailed structure of Ψ_i, will be discussed elsewhere.[34]

The equation for $\hat{U}_{ii}$ [Eq. (4.13)] can be written as a set of two coupled equations:

$$\hat{U}_{ii}=V_iG_0\tilde{U}_{ii} \tag{5.2}$$

and

$$\tilde{U}_{ii}=U_i^F+\tilde{U}_{ii}P_iG_0U_i^F\ . \tag{5.3}$$

In order to solve this equation for $\langle\hat{U}_{ii}\rangle$ we need to know $\langle G_0U_i^F\rangle$. Using (5.1) one finds

$$\langle\Psi_i\eta_i\vec{k}_i\,|\,G_0U_i^F\,|\,\Psi_i\eta_i\vec{k}'_i\rangle=\sum_{\mu,\nu}\!\!\int d\vec{p}\,f^{i*}_{\mu\nu}(\vec{p})\times\frac{1}{E+\epsilon^\mu_j+\epsilon^\nu_k+\epsilon_i-\dfrac{p^2}{2\mu_i}-\dfrac{k^2}{2M_i}+i0}\langle\eta^\mu_j\eta^\nu_k\eta_i\vec{p}\,\vec{k}_i\,|\,U_i^F\,|\,\Psi_i\eta_i\vec{k}'_i\rangle\ , \tag{5.4}$$

where ϵ^μ_j (ϵ^ν_k) is the binding energy of eigenstates μ (ν) cluster j (k). In configuration space this relation becomes

$$\langle \Psi_i \eta_i \vec{r}_i \mid G_0 U_i^F \mid \Psi_i \eta_i \vec{r}\,'_i \rangle = -\frac{M_i}{2\pi} \sum_{\mu,\nu} \int d\vec{p}\, f^{i*}_{\mu\nu}(\vec{p}) \times \int d\vec{r} \frac{\exp\{ i[2M_i(E+\epsilon_j^\mu+\epsilon_k^\nu+\epsilon_i-p^2/2\mu_i)]^{1/2} \mid \vec{r}_i-\vec{r} \mid \}}{\mid \vec{r}_i-\vec{r} \mid} \times \langle \eta_j^\mu \eta_k^\nu \eta_i \vec{p}\,\vec{r} \mid U_i^F \mid \Psi_i \eta_i \vec{r}\,'_i \rangle \ . \tag{5.5}$$

Of course, these expressions still look quite complicated; however, this is due to the fact that they are still completely general and exact. In actual practice, both the cluster wave functions and the expansion (5.1) are often trivial, e.g., in the case of nucleons and deuterons. For the nuclear wave functions the shell model usually gives a straightforward expansion.

One can now solve Eq. (5.3) by setting up integral equations for $\langle \eta_j^\mu \eta_k^\nu \eta_i \vec{p}\,\vec{k}_i \mid \tilde{U}_{ii} \mid \Psi_i \eta_i \vec{k}\,'_i \rangle$ in terms of the input $\langle \eta_j^\mu \eta_k^\nu \eta_i \vec{p}\,\vec{k}_i \mid U_i^F \mid \Psi_i \eta_i \vec{k}\,'_i \rangle$. These matrix elements of $\tilde{U}_{ii}$ are sufficient to determine $\langle \tilde{U}_{ii} \rangle$:

$$\langle \vec{k} \mid \langle \hat{U}_{ii} \rangle \mid \vec{k}\,' \rangle = \sum_{\mu,\nu} \int d\vec{p}\, f^i_{\mu\nu}(\vec{p})^* \ \frac{-(\epsilon_{jk}-\epsilon_j^\mu-\epsilon_k^\nu)-p^2/2\mu_i}{E+\epsilon_j^\mu+\epsilon_k^\nu+\epsilon_i-p^2/2\mu_i-k^2/2M_i+i0} \langle \eta_j^\mu \eta_k^\nu \eta_i \vec{p}\,\vec{k} \mid \tilde{U}_{ii} \mid \Psi_i \eta_i \vec{k}\,' \rangle \ . \tag{5.6}$$

Notice that for on-shell momenta ($k^2/2M_i = E+\epsilon_i+\epsilon_{jk}$) the p-dependent ratio is unity, so that the integral over cluster states collapses into Ψ_i. We can now use this potential to compute $\hat{\chi}_{i,\vec{k}_i}$ via a standard Schrödinger equation.

There is a more direct and simpler way to compute $\hat{\chi}_{i,\vec{k}_i}$, namely by expressing this distorted wave directly into the elastic wave function. Writing the distorted waves formally as

$$\hat{\chi}_{i,\vec{k}_i} = \mid \vec{k}_i \rangle + \langle G_i \rangle \langle \hat{U}_{ii} \rangle \hat{\chi}_{i,\vec{k}_i}$$

or, using Eq. (4.12),

$$\hat{\chi}_{i,\vec{k}_i} = (1+\langle G_i \rangle \langle \hat{A}_{ii} \rangle) \mid \vec{k}_i \rangle \ , \tag{5.7}$$

and using the formal expression for $\chi^F_{i,\vec{k}_i}$,

$$\chi^F_{i,\vec{k}_i} = (1-\langle G_i \rangle \langle U_i^F \rangle)^{-1} \mid \vec{k}_i \rangle \ , \tag{5.8}$$

one finds, with Eq. (3.11),

$$\hat{\chi}_{i,\vec{k}_i} = (1-\langle G_0 U_i^F \rangle) \chi^F_{i,\vec{k}_i} \ . \tag{5.9}$$

This is our basic result for the distorted waves. Considering the completely different structure of the operators we started with [U_{ii} and U_i^F in Eqs. (3.1) and (3.2)], it is quite amazing that these wave functions are so simply related. For a calculation of the distorted waves, Eq. (5.9) is clearly superior over the indirect procedure [Eqs. (5.2) and (5.3)]. Instead of solving integral equations in two continuous variables one only has to do a quadrature in one continuous variable. The potentials $\langle \hat{U}_{ii} \rangle$, however, remain useful for the calculation of recoupling terms in coupled channel equations, as these terms cannot easily be calculated in a different way.

In order to stress the simplicity of Eq. (5.9) we give it in more explicit notation:

$$\hat{\chi}_{i,\vec{k}_i}(\vec{r}_i) = \chi^F_{i,\vec{k}_i}(\vec{r}_i) - \int d\vec{r}\,' \langle \vec{r}_i \mid \langle G_0 U_i^F \rangle \mid \vec{r}\,' \rangle \chi^F_{i,\vec{k}_i}(\vec{r}\,') \ . \tag{5.9'}$$

This equation is very similar to the ordinary scattering equation, with the plane wave replaced by $\chi^F_{i,\vec{k}_i}(\vec{r}_i)$, and the Green's function G replaced by G_0. Notice, however, that there is a minus sign instead of the usual plus sign in front of the scattering term. This may imply that the usual elastic wave function $\chi^F_{i,\vec{k}_i}$ overestimates the distortion effects.[34] As stated before, one often does not require the information contained in Eq. (5.1) to get a reasonable determination of $\hat{\chi}_{i,\vec{k}_i}$. In particular, one can make approximations which limit the amount of required information to $\langle U_i^F \rangle$, or which are valid in the high energy limit.[34] Hence, an approximate calculation of $\hat{\chi}_{i,\vec{k}_i}$ is quite feasible.

VI. ANALYSIS OF THE TRANSITION POTENTIALS

In Sec. IV we derived a nonlinear integral equation [Eq. (4.23)] for the transition amplitude $\hat{T}_{ij}$ in

terms of the transition potentials $\langle V_j + U_{ji} \rangle$ and $\langle V_i + U_{ij} \rangle$. We explained how higher order terms in this equation (the so-called recoupling terms) could be evaluated using (modified) coupled channel methods. In this section we want to address the transition potentials themselves. These potentials contain familiar contributions, such as DWBA and coupled channel effects (contributions of intermediate excited states in either of the two channels); however, they also contain other contributions which have not been considered before in the context of coupled channel equations.

Provided our guiding principle applies, much of the strength of the original plane wave transition operator has been absorbed in the distorted waves, so that the remaining $\langle U_{ji} \rangle$ is indeed small. What is more, we will see that the same arguments which are used in favor of the smallness of $\langle U_{ji} \rangle$ (essentially the elimination of elastic degrees of freedom) can be used in comparing subsequent orders in the series for $\langle U_{ji} \rangle$, so that we expect to have a well-behaved, convergent series expansion for U_{ji}. We now want to derive this expansion.

The transition potential satisfies the following equation [cf. Eq. (3.4)]:

$$U_{ji} = V_j Q_i G_i U_{ii} + V_j G_k (V_k + U_{ki}), \quad k \neq i \neq j \neq k \tag{6.1}$$

where we did put $Q_k = 1$ as usual. Eliminating U_{ki} in favor of U_{ji} we obtain

$$U_{ji} = V_j G_0 T_k (1 + Q_j G_0 T_j) + V_j (1 + G_0 T_k) Q_i G_i U_{ii} + V_j G_0 T_k Q_j G_j U_{ji}, \quad k \neq i \neq j \neq k \ , \tag{6.2}$$

where we introduced the free two cluster T matrix

$$T_k = V_k + V_k G_0 (E + i0) T_k \ . \tag{6.3}$$

Equation (6.2) is our basic result for the expansion of the transition potential. We now want to present some arguments in favor of its convergence. In the Introduction we stated that many-body degrees of freedom (i.e., Q space) cannot be ignored, but possibly can be treated perturbatively. This statement was intently vague, as it is very important when such an expansion is introduced. For example, if the optical potential U_i^F is expanded in such a series, starting directly from Eq. (3.2), then one would obtain a first-order folding potential with free nucleon-nucleon potentials, which is generally considered inferior to the potentials obtained in multiple scattering theory.[5,15] In the latter case, one first introduces the two-nucleon T matrices which summarize more of the physics of the two nucleon collision. In addition, these T matrices implicitly contain some aspects of the many-body problem (since they are T matrices in the nuclear medium). Using these effective interactions one can then argue that the remaining series is basically a correlation expansion,[17] and can be expected to have good convergence properties (although a quantitative analysis has to confirm such an expectation ultimately). The question we are facing is, therefore: Is the equation for U_{ji} in Eq. (6.2) sufficiently developed that a perturbation expansion is appropriate? We believe that this is the case. The relevant operators in this equation, the optical potential U_{ii} and the intercluster T matrices T_k and T_j, summarize definite physical processes (more so than the individual potentials), and in our opinion provide an optimal starting point for the expansion in Q space. Notice that the Faddeev method has automatically ensured that all intercluster potentials can be replaced by intercluster T matrices. Not only do these T matrices probably ensure a better convergence of the series, they also are usually better behaved than the potentials and are closer to experiment, which is an advantage if we want to exploit experimental information on cluster-cluster scattering.

There is one aspect which seems to weaken the Q-space argument: Whereas in the elastic case one has to return to the original channel, thereby emphasizing the role of P space, in rearrangement scattering the initial and final channels are different, so that one can imagine that there are processes which owe their importance to their large overlap with both the initial and the final channel states. In fact, the term $V_j G_0 T_k$ in Eq. (6.2) is exactly such a process: it proceeds via the third (implicit) channel k, which is closely related to the initial and the final channel, but not identical to either of them. As we see from Eq. (6.2) this is the only contribution to U_{ji} which is not constrained to excited state propagation, and therefore we should put special emphasis on this term. It is another plus of the Faddeev description that it made this particular process so readily identifiable.

We are now ready to discuss the contributions to the transition potential more explicitly. The first term of Eq. (4.23) when used in Eq. (4.19b) produces a result which can easily be recognized as a microscopic form of the usual DWBA amplitude:

$$T_{ji}^{\mathrm{DWBA}}(\vec{k}'_j, \vec{k}_i, E) = \langle \Psi_j \eta_j \hat{\chi}^{(-)}_{j,\vec{k}'_j} \mid V_j \mid \Psi_i \eta_i \hat{\chi}^{(+)}_{i,\vec{k}_i} \rangle \ .$$

In the (p,d) reaction, V_j is either the short-range neutron-proton or the longer ranged neutron-core

interaction. Since it is more convenient to work with the former, one should choose j and i to correspond to the deuteron and proton channel, respectively, i.e., we would in this particular case consider $T_{31}^{\mathrm{DWBA}}(\vec{k}'_3,\vec{k}_1,E)$. This amplitude can be calculated with standard DWBA codes if one replaces $\chi^F_{i,\vec{k}_i}(\vec{r})$ with $\hat{\chi}_{i,\vec{k}_i}(\vec{r})$. Notice that by fixing j and i, we have also fixed the off-shell transformation (V_3G_0) and the nature of the inelastic excitations (Q_1U_{11}, i.e., in the proton channel) in Eq. (6.2). Since we know the deuteron wave function and the nucleon-nucleon potential quite well the off-shell transformation only presents a minor complication in the (p,d) case.

We now continue with the first term in U_{ji}:

$$T_{ji}^{\mathrm{resc}}(\vec{k}_j,\vec{k}_i,E)=\langle\Psi_j\eta_j\hat{\chi}^{(-)}_{j,\vec{k}_j}\mid V_jG_0T_k\mid\Psi_i\eta_i\hat{\chi}^{(+)}_{i,\vec{k}_i}\rangle\ . \tag{6.5}$$

This process is illustrated in Fig. 4 for ^{4}He$(p,d)^3$He. As noted before, this term deserves special attention as the intermediate state is not constrained to Q space. This term is absent in the usual DWBA and coupled reaction channel (CRC) calculations,[32,33] and may indicate a severe shortcoming of these phenomenological theories. The presence of triangle mechanisms in rearrangement reactions was also discussed by Vanzani[35] in the context of a three-body model of rearrangement reactions. Since he does not use projection techniques his "distorted waves" are not simple two-body wave functions and he has to make a further approximation to obtain distorted waves of the normal form. For the case of elastic scattering this rescattering term represents the exchange contribution, which underlines its importance. Using the same expansion [Eq. (5.1)] as before we can write (6.5) as follows:

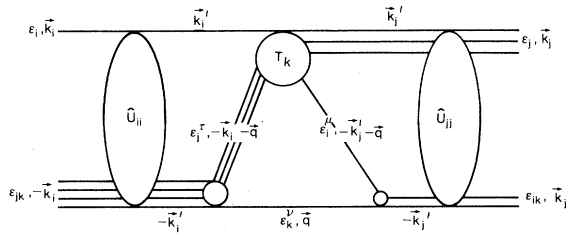

FIG. 4. Rescattering contribution to the (p,d) direct amplitude. The ellipses represent the effects of distortion on the initial (final) waves.

$$T_{ji}^{\mathrm{resc}}(\vec{k}_j,\vec{k}_i;E)=\sum_{\mu,\nu,\tau}\int d\vec{k}'_j\int d\vec{k}'_i\int d\vec{q}\,\hat{\chi}^{(-)}_{j,\vec{k}_j}(\vec{k}'_j)^*f^j_{\mu\nu}(\vec{p}'_j)^*\times\frac{-E_j+\epsilon_j+\epsilon_i^\mu+\epsilon_k^\nu-\vec{p}'^{\,2}_j/2\mu_j}{E+\epsilon_j+\epsilon_i^\mu+\epsilon_k^\nu-\vec{p}'^{\,2}_j/2\mu_j-\vec{k}'^{\,2}_j/2M_j}\times\langle\eta_j\eta_i^\mu\vec{p}_k\mid\hat{T}_k(\hat{E})\mid\eta_j^\tau\eta_i\vec{p}'_k\rangle f^i_{\nu\tau}(\vec{p}'_i)\hat{\chi}^{(+)}_{i,\vec{k}_i}(\vec{k}'_i)\ , \tag{6.6}$$

where

$$\begin{aligned}\vec{p}'_j&=\vec{q}+\frac{m_k}{m_k+m_i}\vec{k}'_j\ ,\\ \vec{p}'_i&=-\vec{q}-\frac{m_k}{m_k+m_j}\vec{k}'_i\ ,\\ \vec{p}_k&=-\vec{k}'_j-\frac{m_j}{m_j+m_i}\vec{q}\ ,\\ \vec{p}'_k&=\vec{k}'_i+\frac{m_i}{m_j+m_i}\vec{q}\ ,\\ \hat{E}&=E+\epsilon_k^\nu-\frac{q^2}{2M_k}\ .\end{aligned} \tag{6.7}$$

As is common in three-body theory we have indicated three-body operators which are reduced to two-body space with a hat ($T_k\to\hat{T}_k$). The physical region for the scattering process described by $\hat{T}_k$ is $\hat{E}>\max(-\epsilon_j-\epsilon_i^\mu,-\epsilon_j^\tau-\epsilon_i)$. For $\mu=\tau=0$, $\hat{T}_k$ describes elastic scattering. Notice that V_jG_0 only replaces the momentum space singularity of $f^j_{\mu\nu}$ (which is canceled by the numerator of the off-shell factor) by the three-cluster propagator, which is energy dependent. The wave function with its bound-state singularity removed is usually called the form factor or vertex function. It has been studied extensively in the case of the deuteron. Again, we should stress that the complexity of (6.6) is due to its complete generality; in actual cases the cluster wave functions and expansions are often quite trivial. Various approximations can be used

to make Eq. (6.6) more tractable. In Ref. 28 we have published results for ^{4}He$(p,d)^3$He, which indicate a fairly large contribution at intermediate energies and forward angles. These calculations were done in the plane wave approximation using only the elastic contribution $\mu=\tau=0$, so that we only had to perform one integral (which could be done analytically).

There are three other Born terms in Eq. (6.2), all of which proceed via excited channel states. The most familiar one is $V_j Q_i G_i U_{ii}$, which represents a typical coupled channel effect (see Fig. 5). Since it involves the coherent potential U_{ii} and no further rescattering, it may well be the most important one of the three remaining terms. Coupled channel effects have been calculated extensively, and they even seem to give important contributions at intermediate energies.[36] According to our theory this term is given by

$$T_{31}^{\rm exc}(\vec{k}'_3,\vec{k}_1;E) = \langle \Psi_3\eta_3\hat{\chi}^{(-)}_{3,\vec{k}'_3} \mid V_3 Q_1 G_1 U_{11} \mid \Psi_1\eta_1\hat{\chi}^{(+)}_{1,\vec{k}_1}\rangle . \tag{6.8}$$

Since excited states in channel 1 are naturally eigenstates of H_1, the Green's function $G_1=(E-H_1)^{-1}$ has a very simple form in this case. As expected we need inelastic matrix elements of the distorting potential to compute this coupled channel effect. Provided we know the excited states in terms of a cluster decomposition, such as (5.1), we can calculate $Q_i\hat{U}_{ii}P_i$ from the same matrix elements $\langle \eta_j^{\mu}\eta_k^{\nu}\eta_i\vec{p}\vec{k}_i \mid \underline{U}_{ii} \mid \Psi_i\eta_i\vec{k}_i\rangle$, which were needed for the computation of $P_i\hat{U}_{ii}P_i$. We then can evaluate (6.8) in the approximation $Q_i U_{ii} P_i \simeq Q_i\hat{U}_{ii}P_i$.

In principle, one can also calculate this coupled channel effect via coupled channel calculations. In Sec. IV we discussed a one-to-one relationship between the coupled channel equation (2.20) and the T-matrix equation (4.9). Such a relationship is not possible for the transition potential itself, among others, because of the "free" Green's function G_i which does not contain the optical potential. We therefore have to go back to Eq. (2.23) and carry the excited states throughout. The resulting equations are probably a straightforward matrix generalization of the present ones, which in the case of Eq. (5.9) will mean that we also have to know the inelastic wave function. Although this would be an interesting alternative to the present equations we feel that a direct evaluation of $T_{31}^{\rm exc}$, which exploits the simplicity of G_1, is preferable.

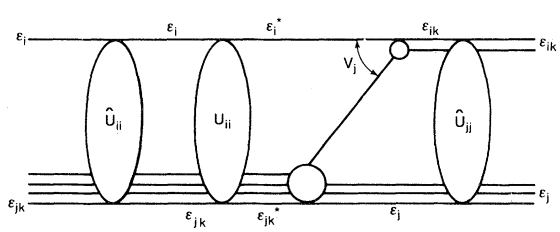

FIG. 5. Multistep effect. The excitation of the intermediate state can also involve the projectile if it is composite. This diagram represents initial excitation of the nucleus in the proton channel, followed by a transfer to the deuteron channel.

Let me point out one interesting feature of (6.8). It seems as if this expression only describes excitations in channel 1. This is quite different from Eq. (2.23), where both excitations in channel 1 and channel 3 occur. One can, however, bring out the excitations in channel 3 by using the left-hand version of Eq. (3.4):

$$U_{31}=V_3G_0T_2+U_{33}Q_3G_3V_{33}+U_{32}G_2V_2 . \tag{6.9}$$

However, now the excitations in channel 1 vanished. This duality reflects the fact that it is inconsistent to include excitations in both channels unless one renormalizes the potential U_{ii} at the same time [which is done implicitly in Eq. (2.23)]. Formally, this renormalization can be accomplished as follows. Assume, to be specific, that we want to include an excitation α in the deuteron channel. The excitation mode will be represented by P_3^{α}, and we also define $Q_3^{\alpha}=Q_3-P_3^{\alpha}$. We can now define a new potential operator $\underline{U}^{\alpha}$ through

$$\underline{\underline{U}}^{\alpha} = \underline{\underline{V}}\,\underline{\underline{G}}\,\underline{\underline{Q}}^{\alpha}\underline{\underline{V}} + \underline{\underline{V}}\,\underline{\underline{G}}\,\underline{\underline{Q}}^{\alpha}\underline{\underline{U}}^{\alpha} = \underline{\underline{V}}\,\underline{\underline{G}}\,\underline{\underline{Q}}^{\alpha}\underline{\underline{V}} + \underline{\underline{U}}^{\alpha}\underline{\underline{G}}\,\underline{\underline{Q}}^{\alpha}\underline{\underline{V}} , \tag{6.10}$$

so that

$$\underline{\underline{U}}=\underline{\underline{U}}^{\alpha}+(\underline{\underline{V}}+\underline{\underline{U}}^{\alpha})\underline{\underline{G}}(\underline{\underline{Q}}-\underline{\underline{Q}}^{\alpha})(\underline{\underline{V}}+\underline{\underline{U}}) . \tag{6.11}$$

Thus, we obtain

$$U_{31}=U_{31}^{\alpha}+U_{33}^{\alpha}G_3P_3^{\alpha}(V_3+U_{31}) . \tag{6.12}$$

Using Eq. (6.11) we find

$$U_{31}= V_3G_0T_2+V_3G_2U_{21}^{\alpha}+V_3G_1Q_1U_{11}^{\alpha} + U_{33}^{\alpha}G_3P_3^{\alpha}(V_3+U_{31}) . \tag{6.13}$$

The structure of (6.13) is similar to that of Eq. (6.1) for $j=3$ and $i=1$, except that we have an additional coupled channel term in the deuteron channel and a different proton optical U_{11}^{α} and transition potential U_{21}^{α}. The relation of U_{11} and U_{11}^{α} is given by (6.11):

$$U_{11}^{\alpha}=U_{11}-(V_1+U_{13}^{\alpha})G_3P_3^{\alpha}(V_3+U_{31}) . \quad (6.14)$$

If the difference between U_{11}^{α} and U_{11} is small (it is a typical recoupling term) then one can replace U_{11}^{α} by U_{11} to good accuracy. In that case, we could add the deuteron channel excitation term to the original amplitude (6.1) without a serious overcounting problem. However, the overcounting problem becomes more serious if the excitation is stronger, and if more proton channel excitations are included. In Fig. 6 we give a schematic picture of the coupling scheme if the deuteron excitation is included in a rigorous way.

Two other Born terms appear in Eq. (6.2): $V_jG_0T_kQ_jG_0T_j$ and $V_jG_0T_kQ_iG_iU_{ii}$. These are shown in Fig. 7 for the case of ^{4}He$(p,d)^3$He. In the first diagram the proton first knocks out the neutron, but instead of forming a deuteron immediately, it then rescatters with the remaining core nucleons, and finally forms the deuteron. It is this type of diagram which has recently attracted much attention at intermediate energies,[37] since it spreads the momentum transfer over two steps, and therefore does not rely on the tiny large momentum components of the nuclear wave functions. Again, this term is neglected in standard DWBA and coupled channel equations. The present theory shows that it can be added to a stripping diagram without risk for overcounting, provided we use the projection operator Q_3 in the intermediate state, and provided we are using the distorted waves $\hat{\chi}_{i,\vec{k}_i}$ instead of $\chi^F_{i,\vec{k}_i}$. Future calculations will have to show whether this diagram is still so important if it is calculated in a consistent way.

The higher-order terms in Eq. (6.2) are essentially repetitions of the previous ones. With each higher order we introduce another operator $V_jG_0T_kQ_jG_j$, i.e., another rescattering process and an additional propagation via an excited state. The calculation of higher order diagrams may thus be quite irrelevant for comparisons with experiment; however, they are very valuable for testing the convergence rate of the series quantitatively.

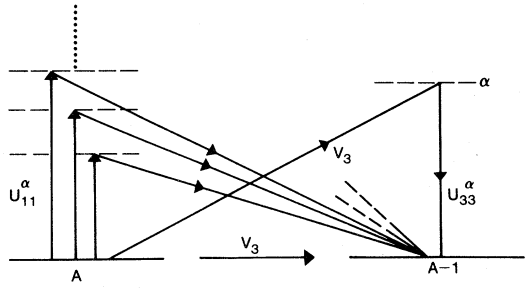

FIG. 6. Coupling scheme when one excitation in the final (deuteron) channel is included as well. Both the inelastic potential in the proton and deuteron channel have to be renormalized.

VII. SUMMARY AND DISCUSSION

We have developed a microscopic framework for calculating rearrangement amplitudes. The basic guiding principle in its development was that the distortion in the initial and final channel plays a central role in rearrangement scattering, and therefore should be treated in a consistent and constructive way. By constructive we mean that the distorting potential should be derived in the context of the rearrangement problem itself, rather than by considerations of other processes, such as elastic scattering, although we stress the importance of connecting the distorting potentials with the elastic optical potentials. Such an approach guarantees a constructive transition amplitude, i.e., an amplitude without counterterms [see Eqs. (4.7) and (6.2)]. The amplitude can then be expanded in a series with a well-defined hierarchic structure (not obscured by counterterms with a totally different structure), and therefore enables a straightforward analysis of its convergence properties. Also, each term in the expansion represents a clear dynamical process which can be represented diagrammatically.

In Fig. 8 we summarize the present theory. This

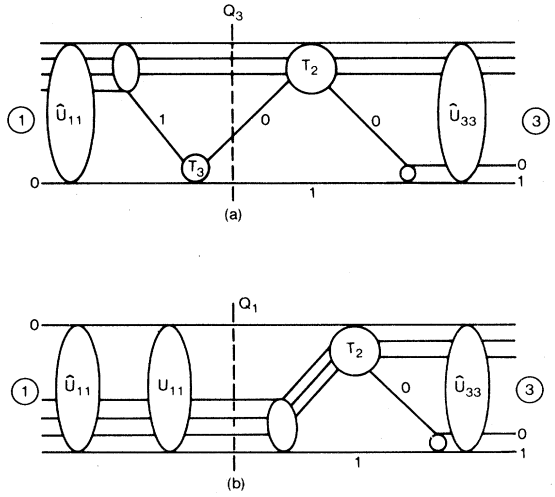

FIG. 7. (a) Two-nucleon mechanism. The proton rescatters before forming the deuteron. (b) Rescattering graph preceded by channel excitation.

diagram illustrates how the problem of constructing the physical amplitudes is reduced to a number of separate subproblems, which are much easier to deal with individually. The central input for both direct and exchange amplitude is the optical potential operator U_i^F. The specific cluster structure of the rearrangement process (i.e., direct or exchange) determines which matrix elements of U_i^F are needed. These matrix elements, together with the cluster structure of the asymptotic bound states, enable a microscopic calculation of the distorted waves and distorting potentials. The cluster structure of the asymptotic states is also required for the calculation of the DWBA and the driving terms in the transition potential. We see that the Born terms further to the right require an increasing amount of input, e.g., nuclear structure information about excited states and T matrices for intercluster scattering. The simplest calculation is the one using the two external lines. This is the standard DWBA. We see that this approximation requires replacing the distorted wave by χ_i^F. Although it is natural to approximate the transition potential in first order by the DWBA, there is no obvious reason why $\chi^F_{i,\vec{k}_i}$ should be a good approximation to $\hat{\chi}_{i,\vec{k}_i}$. A further analysis of Eq. (5.9) may reveal when this is the case, and may provide a better understanding of the successes and failures of the DWBA.

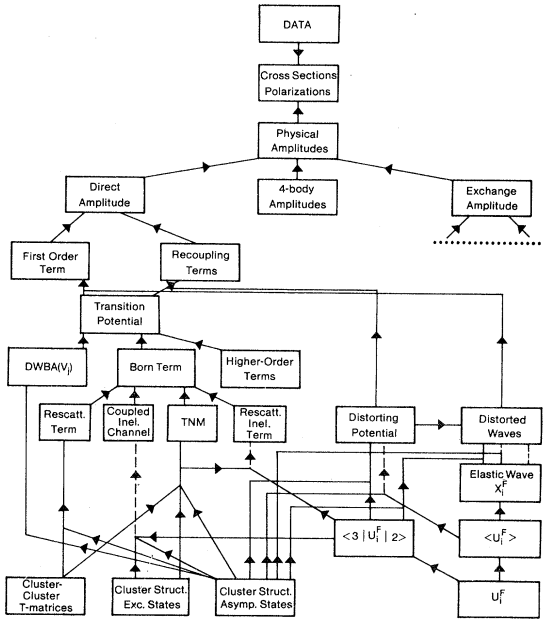

FIG. 8. Schematic summary of the present theory. This scheme clearly shows the rich structure of the physical amplitude. Broken lines represent possible approximations. The decomposition of the exchange amplitude is completely analogous, although the cluster structure will be different. The decomposition of four-body amplitudes, which are not present for reactions involving an asymptotic nucleon-nucleus channel, are not discussed in this paper. Since *all* of the terms in the transition potential require knowledge of the potential V_j [the neutron-proton interaction in the (p,d) reaction] we did not specify this explicitly. Notice the special role of the DWBA. This term does not occur in the Born term of the expansion for the transition potential.

One of the practical advantages of this theory is that it enables one to add various terms [recoupling effects, inelastic excitations, two nucleon mechanisms (TNM), etc.] to the DWBA amplitude without the risk of double counting. Such consistency is especially necessary at intermediate energies, where one is considering higher-order diagrams (in particular the TNM) which distribute the momentum transfer over various steps. The danger for inconsistencies in phenomenological methods is demonstrated at various occasions in this investigation. Firstly, we show that standard coupled channel calculations, which employ unsubtracted potentials ($\langle \hat{U}_{ii} \rangle$) are only valid for evaluating the first-order amplitudes, and give the wrong sign for the second-order recoupling term. Fortunately, we are able to suggest a slight modification of these calculations which can easily be incorporated in existing codes. Secondly, we show that coupled channel calculations which include excitations in both the initial and final channel suffer from double counting. Again, we are able to suggest a modification (in this case of the optical potentials) which resolves this problem, in principle. Since we do not expect that the double counting in this case is too serious, the main virtue of this portion of the theory may be that it provides a tool to check the importance of this inconsistency. Thirdly, we find a new rescattering graph (Fig. 4) which has been omitted from coupled channel calculations so far, probably out of fear for double counting. There is still a remote possibility that all these objections do not apply to the standard calculations because these employ the distorting potential $\langle U_i^F \rangle$, and not the (phase equivalent) $\langle \hat{U}_{ii} \rangle$. However, we find it very hard to accept that this phase equivalent transformation would resolve any of these three inconsistencies. At the very least we have demonstrated that there may exist severe inconsistencies in the standard coupled channel calculations, and we have presented a theory which is free of such problems.

Since the off-shell transformations are responsi-

ble for many of the nonstandard features in the theory, we may ask whether one could have avoided these by using a different starting point. For example, we could have started with the so-called distorted Faddeev equations and introduced the microscopic optical potential $\langle U_i^F \rangle$ in the left-hand side of Eq. (2.9), so that this would constitute the distorting potential in lowest order. However, this approach has the common disadvantages of a nonconstructive approach, and leads to counterterms in the right-hand side of Eq. (2.9). Also, after projecting out the Q states we still end up with additional (very complicated) contributions to the distorting potential and introduce a host of nonorthogonality terms (characterized by products of P_i and P_j without interactions in between). Another approach—advanced for inelastic proton scattering and $(p,2p)$ reactions by Picklesimer, Thaler, and Tandy[38]—is to introduce the elastic wave functions directly into the exact matrix elements. Apparently, the resulting transition operator in the new basis still has a constructive nature in the cases they consider,[38] however, its application to rearrangement reactions would most certainly lead to a nonconstructive transition operator. It therefore seems that some kind of off-shell or phase equivalent transformation is necessarily part of any microscopic theory of rearrangement reactions,[25] unless one is willing to accept nonorthogonality terms, uncertain counterterms, and a large amount of arbitrariness in the theory.

Let us now comment on possible variations of the present approach and its relation to some other methods. Since we are dealing with a three-body problem embedded in a many-body problem, one might ask whether it would not have been more appropriate to develop true three-body equations instead of the effective two-body equations. The problem is, however, that the corresponding projector P_0, which projects out product wave functions of the three clusters, defines a small space, which, in general, does not contain the asymptotic bound cluster states. Only if these asymptotic two-cluster states can be written as a product wave function would this be the case. If, however, we would allow all possible excitations of three clusters to P_0, then P_0 would just be the unit operator. Hence, such a theory only becomes practical if we employ a three-body *model* of the reaction which ignores important many-body aspects of the reaction. Naturally, the three-body scattering problem is also an order of magnitude harder to solve than the two-body problem. The embedding of the three-body problem in the many-body context is also a central theme in the theory of Tandy, Redish, and Bolle.[39] However, their theory aims at a simultaneous description of elastic scattering, knockout, and pickup, which in our opinion is too ambitious and too inflexible as far as the pickup problem is concerned. One price they have to pay for this ambitious scheme is that their optical potential is limited to the single-scattering approximation. Also, the three-body nature of their theory limits its practicality.

Finally, we want to discuss the further development and application of this theory. First, one will have to develop reliable approximations[34] to calculate the distorted waves from χ_i^F. If we succeed in this, then we can use rearrangement scattering to test the (microscopic) optical potential $\langle U_i^F \rangle$ which is put in. In a sense this is already done at present when one modifies the optical potentials in DWBA calculations to fit the data. However, now that we understand the role of the optical potential microscopically, we can perform such a test with more confidence. Other aspects of the theory which could be tested are the convergence of the series expansion for the transition amplitude and the transition potential. Especially, the latter test will not be easy; however, the very existence of this series expansion may give enough encouragement to such an undertaking. At intermediate energies one would put more emphasis on certain rescattering terms, such as the two-nucleon mechanism. In this respect we notice that relativistic kinematics can easily be introduced in the present theory, as most of the processes can be represented by diagrams, which can be formulated with relativistic kinematics. This feature was already employed in the application in Ref. 28.

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APPENDIX A

In this Appendix we prove the identity

$$(1+P_iG_i\langle \hat{A}_{ii} \rangle)(1+P_iG_i\langle A_{ii} \rangle)^{-1}=1+P_iG_i\langle \Delta_i \rangle \ . \tag{A1}$$

Both $\langle A_{ii} \rangle$ and $\langle \hat{A}_{ii} \rangle$ can be expressed in terms of $\langle U_{ii} \rangle$, which enables us to relate them. From (4.6) we have

$$\langle U_{ii}\rangle=\langle A_{ii}\rangle(1+P_iG_i\langle A_{ii}\rangle)^{-1}\,, \tag{A2}$$

and from Eqs. (3.7) and (3.12) we infer that

$$\langle U_{ii}\rangle=(1+\langle\hat{A}_{ii}\rangle P_iG_i)^{-1}(\langle\hat{A}_{ii}\rangle-\langle\Delta_i\rangle)\,. \tag{A3}$$

Eliminating $\langle U_{ii}\rangle$ we obtain:

$$\langle A_{ii}\rangle=\langle\hat{A}_{ii}\rangle-\Delta_i(1+P_iG_i\langle A_{ii}\rangle)\,. \tag{A4}$$

One easily verifies that this is equivalent to (A1).

APPENDIX B: PROOF OF EQUATION (4.17)

We have to prove that

$$(1+P_iG_i\langle\Delta_i\rangle)[1-P_i\tilde{G}_i(1+\langle\hat{A}_{ii}\rangle P_iG_i)^{-1}\langle\Delta_i\rangle]-1=0\,. \tag{B1}$$

The left-hand can be written easily as follows:

$$\text{lhs}=P_iG_i[1-(G_i^{-1}+\langle\Delta_i\rangle)\tilde{G}_i(1+\langle\hat{A}_{ii}\rangle P_iG_i)^{-1}]\langle\Delta_i\rangle\,. \tag{B2}$$

We now consider the expression in brackets and write

$$[\,.\,.\,.\,.\,]=\{1+\langle\hat{A}_{ii}\rangle P_iG_i-P_iG_i{}^{-1}\tilde{G}_i-\langle\Delta_i\tilde{G}_i\rangle\}\times(1+\langle\hat{A}_{ii}\rangle P_iG_i)^{-1}\,. \tag{B3}$$

Using the equation

$$1=P_iG_i{}^{-1}\tilde{G}_i-\langle U_{ii}\rangle P_i\tilde{G}_i\,, \tag{B4}$$

one can write

$$\{\,.\,.\,.\,.\,\}=\langle\hat{A}_{ii}\rangle P_iG_i-(\langle U_{ii}\rangle+\langle\Delta_i\rangle)P_i\tilde{G}_i\,. \tag{B5}$$

Using

$$G_i\tilde{G}_i{}^{-1}=1-G_i\langle U_{ii}\rangle \tag{B6}$$

one easily finds that

$$\{\,.\,.\,.\,.\,\}=(\langle\hat{A}_{ii}\rangle-\langle\hat{A}_{ii}\rangle P_iG_i\langle U_{ii}\rangle-\langle U_{ii}\rangle-\langle\Delta_i\rangle)P_i\tilde{G}_i\,, \tag{B7}$$

which according to Eq. (3.7) is zero, Q.E.D.

APPENDIX C: IDENTITY OF DISTORTED WAVE AND PLANE WAVE TRANSITION MATRIX ELEMENTS

The transition matrix element in the distorted wave basis is given by Eqs. (4.20) and (4.15):

$$T_{fi}=\langle\hat{\chi}_{j,\vec{k}_j}|\langle V_j+U_{ji}\rangle P_i[1-\tilde{G}_i\langle V_i+U_{ij}\rangle P_j\tilde{G}_j\langle V_j+U_{ji}\rangle]\,|\,\hat{\chi}_{i,\vec{k}_i}\rangle\,. \tag{C1}$$

Using the identity (4.10) one rewrites this as

$$\begin{aligned}T_{fi}&=\langle\hat{\chi}_{j,\vec{k}_j}|\langle V_j+U_{ji}\rangle\tilde{G}_iP_i[G_i{}^{-1}-U_{ii}-(1+\hat{A}_{ii}P_iG_i)^{-1}\langle\Delta_i\rangle]\,|\,\hat{\chi}_{i,\vec{k}_i}\rangle\\&=\langle\hat{\chi}_{j,\vec{k}_j}|\langle V_j+U_{ji}\rangle\tilde{G}_iP_i[G_i{}^{-1}-(1+\hat{A}_{ii}P_iG_i)^{-1}\hat{A}_{ii}]\,|\,\hat{\chi}_{i,\vec{k}_i}\rangle\\&=\langle\hat{\chi}_{j,\vec{k}_j}|\langle V_j+U_{ji}\rangle\tilde{G}_iP_iG_i{}^{-1}(1+P_iG_i\hat{A}_{ii})^{-1}\rangle\,|\,\hat{\chi}_{i,\vec{k}_i}\rangle\,,\end{aligned} \tag{C2}$$

where Eq. (3.7) has been used to obtain the second line. From (4.10) and the explicit form of $\langle\Delta_i\rangle$ one easily derives the identity

$$P_i(V_i+U_{ij})P_j\tilde{G}_j=P_i(1+\hat{A}_{ii}P_iG_i)^{-1}(\hat{A}_{ij}+V_i)G_jP_j\,, \tag{C3}$$

so that (C2) can be written as follows [interchange i and j in (C3)]:

$$T_{fi}=\langle\hat{\chi}_{j,\vec{k}_j}|(1+\hat{A}_{jj}P_jG_j)^{-1}(\hat{A}_{ji}+V_j)(1+P_iG_i\hat{A}_{ii})^{-1}\,|\,\hat{\chi}_{i,\vec{k}_i}\rangle=\langle\vec{k}_j\,|\,\hat{A}_{ji}+V_j\,|\,\vec{k}_i\rangle\,. \tag{C4}$$

Using Eqs. (3.6) and (3.8) one can show that A_{ij}^F satisfies the Faddeev-type equations

$$A_{ij}^F=\bar{\delta}_{im}T_m\bar{\delta}_{mj}+\bar{\delta}_{im}T_mG_0A_{mj}^F\,, \tag{C5}$$

so that $V_j+A_{ji}^F$ is the exact transition amplitude in the plane wave basis. Therefore, $\hat{A}_{ji}+V_j$ is on-shell equivalent to the exact T matrix.

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[1]L. D. Faddeev, *Mathematical Aspects of the Three-Body Problem in the Quantum Scattering Theory* (Israeli Program for Scientific Translations, Jerusalem, 1965).

[2]H. Feshbach, Ann. Phys. (N.Y.) 5, 357 (1958); 19, 287 (1962).

[3]For a concise discussion of N-body methods see V. Vanzani, in *Few Body Nuclear Physics* (IAEA, Vienna, 1975), pp. 57–112.

[4]K. M. Watson, Phys. Rev. 89, 575 (1953); L. L. Foldy, *ibid.* 67, 107 (1945).

[5]See the discussion in J. M. Eisenberg and D. S. Koltun, *Theory of Meson Interactions with Nuclei* (Wiley, New York, 1980), Chap. 4.

[6]P. E. Shanley and A. Aaron, Ann. Phys. (N.Y.) 44, 363 (1967); A. S. Reiner and A. I. Jaffe, Phys. Rev. 161, 935 (1967); A. N. Mitra, Nucl. Phys. 32, 529 (1962); Phys. Rev. 139B, 1472 (1965); D. Bouldin and F. S. Levin, Phys. Lett. 37B, 155 (1971); 42B, 167 (1972).

[7]N. Austern, J. P. Farrell, K. Kabir, and C. M. Vincent, Phys. Rev. C 18, 1577 (1978); N. Austern, C. M. Vincent, and J. P. Farrell, Ann. Phys. (N.Y.) 114, 93 (1978).

[8]J. M. Greben and F. S. Levin, Phys. Rev. C 20, 437 (1979).

[9]J. M. Greben and F. S. Levin, Nucl. Phys. A235, 145 (1979).

[10]S. R. Cotanch and C. M. Vincent, Phys. Rev. C 14, 1739 (1976).

[11]N. Austern, *Direct Nuclear Reaction Theories* (Wiley, New York, 1970).

[12]W. Tobocman, Phys. Rev. 115, 98 (1959); 94, 1655 (1964); *The Theory of Direct Nuclear Reactions* (Oxford University Press, London, 1961).

[13]G. Takeda and K. M. Watson, Phys. Rev. 97, 1336 (1955).

[14]A. K. Kerman, H. McManus, and R. M. Thaler, Ann. Phys. (N.Y.) 8, 551 (1959).

[15]H. Feshbach and J. Hüfner, Ann. Phys. (N.Y.) 56, 268 (1970); H. Feshbach, A. Gal, and J. Hüfner, *ibid.* 66, 20 (1971); E. Lambert and H. Feshbach, *ibid.* 76, 80 (1973); J. J. Ullo and H. Feshbach, *ibid.* 82, 156 (1974).

[16]E. Kujawski, Phys. Rev. C 1, 1651 (1970).

[17]D. J. Ernst, J. T. Londergan, G. A. Miller, and R. M. Thaler, Phys. Rev. C 16, 537 (1977).

[18]L. Ray, Nucl. Phys. A335, 443 (1980) and references therein.

[19]See, for example, R. Goldflam and K. L. Kowalski, Phys. Rev. C 22, 949 (1980).

[20]H. Feshbach, in *Reaction Dynamics,* edited by F. S. Levin and H. Feshbach (Gordon and Breach, New York, 1973).

[21]This property is well known by now and is discussed in various papers, e.g., Ref. 9.

[22]J. R. Taylor, in *Scattering Theory* (Wiley, New York, 1972), p. 462.

[23]D. J. Kouri and F. S. Levin, Nucl. Phys. A253, 395 (1975).

[24]E. F. Redish, P. C. Tandy, and M. L'Huillier, Phys. Lett. 61B, 413 (1976).

[25]We thank P. C. Tandy for discussing this point.

[26]The application of Feshbach projection techniques to (Schrödinger) wave function components was first proposed by Hahn (Ref. 27). Its application to Faddeev components has been discussed by the author (Ref. 28), and more recently by Gy. Bencze (Ref. 29).

[27]Y. Hahn, Phys. Rev. 142, 603 (1966).

[28]J. M. Greben, Proceedings of the Workshop on Nuclear Structure with Intermediate-Energy Probes, Los Alamos, LASL Report LA-8303-C, 1980, p. 402.

[29]Gy. Bencze, Phys. Lett. 98B, 331 (1981).

[30]T. Tamura, Rev. Mod. Phys. 37, 679 (1965).

[31]E. F. Redish, Phys. Lett. 90B, 188 (1980); Gy. Bencze and E. F. Redish, Central Research Institute for Physics, Budapest, KFKI Report 1979-64.

[32]T. Ohmura, B. Imanishi, M. Ichimura, and M. Kawai, Prog. Theor. Phys. 41, 391 (1969); 43, 347 (1970); 44, 1242 (1970); T. Udagawa, H. H. Wolter, and W. R. Coker, Phys. Rev. Lett. 31, 1507 (1973).

[33]B. Imanishi, M. Ichimura, and M. Kawai, Phys. Lett. 52B, 267 (1974).

[34]J. M. Greben (unpublished).

[35]V. Vanzani, Lett. Nuovo Cimento 2, 706 (1969).

[36]J. Källne and A. W. Obst, Phys. Rev. C 15, 477 (1977); S. D. Baker *et al.*, Phys. Lett. 52B, 57 (1974).

[37]J. Källne, A. N. Anderson, J. L. Beveridge, J. Rogers, D. A. Hutcheon, and W. J. McDonald, Phys. Rev. C 21, 675 (1980); B. K. Jain, Bbabha Atomic Research Centre report, 1981.

[38]P. C. Tandy and R. M. Thaler, Phys. Rev. C 22, 2321 (1980); A. Picklesimer, P. C. Tandy, and R. M. Thaler, Phys. Rev. C (to be published).

[39]P. C. Tandy, E. F. Redish, and D. Bolle, Phys. Rev. C 16, 1924 (1977).