PHYSICAL REVIEW C VOLUME 25, NUMBER 6 JUNE 1982

α-particle D-state effects in (d, α) reactions

F. D. Santos*
University of Wisconsin, Madison, Wisconsin 53706

S. A. Tonsfelt, T. B. Clegg, E. J. Ludwig, Y. Tagishi,† and J. F. Wilkerson
Department of Physics and Astronomy, University of North Carolina, Chapel Hill, North Carolina 27514 and Triangle Universities Nuclear Laboratory, Duke Station, Durham, North Carolina 27706
(Received 9 July 1981)

It is shown that the tensor analyzing powers for (d, α) reactions are senstive to the D state component in the α-particle wave function. The D to S state asymptotic ratio extracted from T_{20} and T_{22} data in $J = L \pm 1$ transitions is discussed using an α-particle D state generated with the Jackson and Riska model.

[NUCLEAR REACTIONS Polarization ^{32}S, ^{36,38}Ar$(\vec{d}, \alpha)$, $E = 16$ MeV; measured A_{xx}, A_{yy}; deduced effect of α-particle D state.]

Recent measurements of the tensor analyzing powers T_{20} and T_{22} in (d, α) reactions performed at the Triangle Universities Nuclear Laboratory (TUNL)[1] show systematic features in the angular distributions that distinguish between different values of the total angular momentum transfer J. These J-dependent effects could not be reproduced by calculations using the standard distorted wave Born approximation (DWBA). In particular, the effect of the deuteron spin dependent distortion does not explain[1] the observed J dependence. It is well known[2] that the deuteron D state is primarily responsible for the large tensor analyzing powers T_{2q}, $q = 0, 1, 2$, observed in (d,p) reactions. Also, the T_{2q} for (d,t) and $(d, {}^3\text{He})$ reactions[3,4] are, to a large extent, determined by the D states of ^{3}H and ^{3}He. In fact, the DWBA without spin dependent distortion (SDD) and D-state effects (DSE) predicts T_{2q} that are identically zero in (d,p), (d,t), and $(d, {}^3\text{He})$ reactions. The situation is different in (d, α) reactions since, with no SDD and no DSE, the T_{2q} are nonvanishing[5] except in transitions where the orbital angular momentum of the transferred deuteron L is zero.

In this Communication we show that the J-dependent effects in the T_{2q} for (d, α) reactions are a manifestation of the D state in the relative motion between the deuteron-deuteron clusters in the α particle. The asymptotic D- to S-state ratio ρ is discussed using the model of Jackson and Riska.[6]

In the one-step DWBA the transition amplitude for a (d, α) reaction depends on the internal structure of the α particle through

$$R(\vec{r}) = \langle \phi_1^{\sigma_1}(\vec{r}_{13})\phi_2^{\sigma_2}(\vec{r}_{24}) | \phi_\alpha \rangle \quad . \tag{1}$$

Here $\phi_i^{\sigma_i}$ $(i = 1, 2)$ and ϕ_α are the normalized internal wave functions of the two deuterons and α particle. The quantities $\vec{r}_{13}$, $\vec{r}_{24}$ $(\vec{r}_{ij} = \vec{r}_i - \vec{r}_j)$ are the internal coordinates of the deuterons, and $\vec{r} = (\vec{r}_{21} - \vec{r}_{43})/2$ is the separation between the centers of mass of deuterons 1 and 2. Because of angular momentum and parity selection rules, $R(\vec{r})$ contains only S- and D-state terms and can be written as

$$R(\vec{r}) = \frac{1}{2} \sum_{L' = 0,2} (-1)^{\sigma_2} (L'M1\sigma_1 | 1 - \sigma_2) \times u_{L'}(r) Y_{L'}^M(\hat{r}) \quad . \tag{2}$$

The quantities u_0 and u_2 are the radial wave functions that describe the relative motion between the deuteron clusters in the α particle. In the asymptotic region of large r they behave as

$$u_{L'}(r) = N_{L'} u_{L'}^N(r)/(\alpha r) \rightarrow -i^{L'} N_{L'} h_{L'}(i\alpha r), \quad r \rightarrow \infty \quad . \tag{3}$$

Here $h_{L'}$ is a spherical Hankel function, $N_{L'}$ are asymptotic normalization constants, and $\alpha = [2M(B_\alpha - 2B_d)/\hbar^2]^{1/2}$ is the wave number of the relative motion of the deuteron clusters, where B_α and B_d are the α-particle and deuteron binding energies. The superscript N indicates that $u_{L'}^N$ is independent of $N_{L'}$ for large r. The asymptotic D- to S-state ratio in the α particle is defined as $\rho = N_2/N_0$. The usual D-state parameter[2–4] D_2 is given by $D_2 = \rho\alpha^{-2}$ when the functions $u_{L'}$ are represented by their asymptotic forms.

In the DWBA the angular momentum coupling in a $A(d, \alpha)B$ transition amplitude is determined by the coefficient[7]

$$A_{l1J}^{L'L}=[3(2l+1)]^{1/2}W(LL'J1;l1) \quad . \tag{4}$$

Here L and L' are the orbital angular momenta of the transferred deuteron in the target nucleus and in the α particle. l is the total orbital angular momentum transfer in the reaction. For a zero spin target the total angular momentum transfer J is equal to the residual nucleus spin. Equation (4) shows that $L'=0$ implies $l=L$. For $L'=2$, considering only the dominant normal parity transitions,[8] in which $l+L$ = even, we obtain $l=L$, $L\pm 2$. Thus for $L=J$, there is only one allowed value of l, namely, $l=L$. However, for $J=L\pm 1$, there is a mixing of l values. In fact, $l=L,L-2$ are allowed for $J=L-1$, and $l=L,L+2$ are allowed for $J=L+1$. This selection rule on l and J is the origin of the J-dependent effects in the T_{2q}. In order to gauge the α particle D-state effects in (d,α) reactions, we make a plane-wave peripheral model for the transfer process where the scattering wave functions are represented by plane waves and the bound state wave functions by the asymptotic Hankel functions.[7] Using the methods developed in Refs. 7 and 8 we obtain[9]

$$T_{2q}=-(2\pi/5)^{1/2}F(L,J)Y_2^q(\hat{Q}) \quad . \tag{5}$$

Here $\vec{Q}=(m_A/m_B)\vec{k}_d-\vec{k}_\alpha$ ($\vec{k}_d$, $\vec{k}_\alpha$ are the asymptotic momenta) and

$$F(L,J=L)=-1 \quad , \tag{6a}$$

$$F(L,J=L\pm 1)=\left[L+(1\mp 1)/2+9\sqrt{2}\frac{J(J+1)}{2J+1}\Delta_L^{\pm}\left(1+\frac{[J+(1\pm 3)/2][2(L\pm 2)+1]^{1/2}}{2\sqrt{10}(2J+1)}\Delta_L^{\pm}\right)\right] \times\left(2J+1+\frac{9J(J+1)}{2(2J+1)}(\Delta_L^{\pm})^2\right)^{-1} \quad . \tag{6b}$$

The quantities $\Delta_L^{\pm}$ are given by

$$\Delta_L^{\pm}=-\rho\frac{\int j_{L\pm 2}(Qr)h_{L\pm 2}(i\beta r)r^2dr}{\int j_L(Qr)h_L(i\beta r)r^2dr}\left[\left(\frac{\alpha}{\beta}\right)^2-\rho\frac{L+(1\mp 1)/2}{\sqrt{2}[2(L\pm 1)+1]}\right]^{-1} \quad , \tag{7}$$

where β is the wave number of the transferred deuteron in the target.

In the absence of DSE, $\rho=\Delta_L^{\pm}=0$ and Eq. (6) give the well-known[1,5] L and J dependence of the T_{2q} as predicted by the DWBA with no SDD.

Instead of T_{20} and T_{22}, it is more convenient to consider the Cartesian polarization observables A_{xx} and A_{yy}, referred to the same Madison Convention coordinate system.[1] Equation (5) gives

$$A_{xx}=\tfrac{1}{4}(3\cos 2\gamma-1)F(L,J) \quad , \tag{8a}$$

$$A_{yy}=\tfrac{1}{2}F(L,J) \quad , \tag{8b}$$

where γ is the angle that $\vec{Q}$ makes with the z axis. Calculations usings Eqs. (6)–(8), and shown in Fig. 1, provide a qualitative description of the A_{xx}, A_{yy} angular distributions in transitions with $J=L\pm 1$. In all transfers only the predominant L value[1] was taken into account and the weak dependence of $\Delta_L^{\pm}$ on θ was neglected. The oscillatory pattern in A_{yy}, particularly noticeable in the $L=0$ transition, is a diffraction effect with a period of approximately $\Delta Q=\pi/R$, where R is the nuclear radius. Transitions with $J=L$ were not considered since they are expected to have considerably smaller DSE. The best fit to the $L=0$, $J=1$, data of Fig. 1 was obtained with $\rho=-0.19$: the $L=2$, $J=1$ and $L=2$, $J=3$ transitions give $\rho=-0.21$ and -0.22, respectively. The analysis of the present (d,α) data obtained at TUNL[1] gives a value of $\rho=-0.21\pm 0.06$, where the error includes an estimate of the effect of L mixing. This value of ρ should be considered as preliminary, since it results from a simplified reaction model. Full finite range DWBA calculations of (d,α) reactions, including the S and D states of the α particle, are clearly needed.

The experimentally estimated value of ρ is now compared with the result of a calculation using a particular α-particle wave function. Gerjuoy and Schwinger[10] were the first authors to give a classification of states in the α-particle ground state and to consider the effect of the tensor interaction in the 4 nucleon bound system. There are six independent ${}^5D_0$ states in the α particle.[11] To estimate ρ, we shall consider only the principal D state,[12] namely, ${}^5D_0{}^{(1)}$, which is believed to have the largest probability because it has maximum symmetry in the tensor interaction operator. Further, the perturbative approach of Jackson and Riska,[6] used here, generates only a ${}^5D_0{}^{(1)}$ state in the α particle. We write

$$|\psi_D\rangle=G_0\sum_{i<j}V_T^{ij}|\psi_s\rangle \quad , \tag{9}$$

where $|\psi_s\rangle$ is the normalized ${}^1S_0$ state, G_0 is the Green's operator of the free nucleon system, and

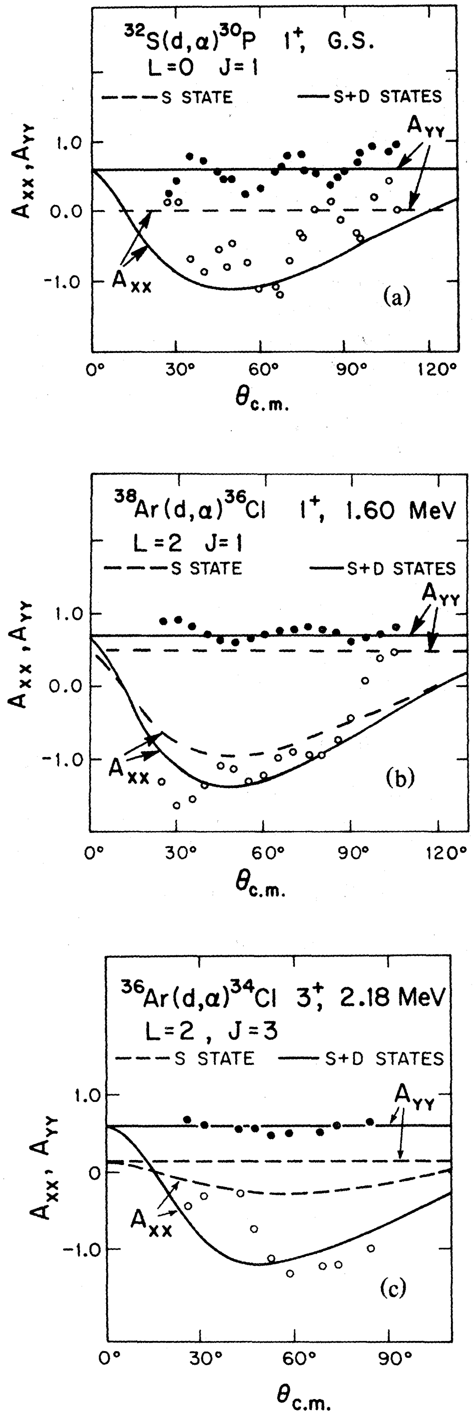

FIG. 1. Angular distributions of A_{xx} and A_{yy} for (a) ^{32}S$(d,\alpha)^{30}$P, ground-state transition, (b) ^{38}Ar$(d,\alpha)^{36}$Cl, 0.79 MeV transition; and (c) ^{36}Ar$(d,\alpha)^{34}$Cl, 2.18 MeV transition, all at $E_d = 16$ MeV. The solid (open) data points are for $A_{yy}(A_{xx})$ and the error bars are about the size of the circles. The solid curves are the result of calculations including the α-particle D state and the broken curves were obtained with a pure S-state α particle.

$V_T^{ij} = -V_T(r_{ij})S_{12}$ is the Reid[13] form of the one-pion-exchange tensor potential (OPEP), where S_{12} is the usual tensor operator. We assume that the $L' = 2$ part of the overlap function $R(\vec{r})$ is generated only by the α-particle D state given in Eq. (9). However, it is noted that some components in the α-particle S states give a contribution to the $L' = 2$ part of $R(\vec{r})$ through their overlap with the D states of the deuteron clusters. This type of contribution is relatively small in (d,t) reactions.[4]

The asymptotic ratio ρ is determined by the low momentum components of $R(\vec{r})$. For small k, the $L' = 2$ radial wave function is

$$\bar{u}_2(k) = -\left(\frac{2}{\pi}\right)^{1/2} \frac{1}{15} \int_0^\infty u_2(r) r^4 dr k^2 \times [1 + 0(k^2)] \ . \qquad (10)$$

The calculation of the k^2 coefficient in Eq. (10), using Eq. (9), gives the approximate formula[9]

$$\rho = \frac{\sqrt{8}S}{B} \int_0^\infty V_T(r) u_0^N(r) r^3 dr \Big/ \int_0^\infty u_2^N(r) r^3 dr \ , \qquad (11)$$

which can be applied to the $A = 2$, 3, and 4 nucleon bound systems. In Eq. (11), B is the binding energy of the A system and S is a spin-isospin factor: $S(d) = 1$, $S(^3\mathrm{H}) = S(^3\mathrm{He}) = -1$, and $S(\alpha) = -2$. The sign of S results from the attractive or repulsive character of the tensor interaction between the transferred one or two nucleons and the remaining nucleons of the A system. It determines the relative sign of the T_{2q} for (d,p) and (d,t), and (d,α) reactions, at low deuteron incident energies. As A goes from 2 to 4, the increase in binding energy pushes the wave function $u_0^N(r)$ to smaller radii. Hence, the form of the numerator in Eq. (11) implies that $\rho(\alpha)$ is very sensitive to the tensor interaction for $r\mu < 1$, where $\mu^{-1} = 1.43$ fm is the range of the OPEP. The reliability of Eq. (11) can be established by using it to estimate ρ for $A = 2$ and 3, using realistic deuteron and triton wave functions. With the Reid soft core deuteron wave function,[13] Eq. (11) gives $\rho(d) = 0.032$, which is in reasonable agreement with the exact Reid value $\rho(d) = 0.026$. For the triton using the wave function of Sasakawa *et al.*,[14] which has $\rho = -0.048$, Eq. (11) predicts $\rho = -0.060$, which is about 30% smaller.

Turning now to the α particle, we notice from Eq. (11) that the behavior of u_0 for small momentum is important in determining ρ. In fact, ρ is particularly sensitive to the finite range parameter β [defined in Eq. (8) of Ref. 15] that determines the coefficient of the k^2 term in the expansion of $\bar{u}_0(k)$. Furthermore, β^{-1} characterizes the range of the interaction that generates u_0. It is reasonable to assume that the effective interactions that generate u_0 in the triton and α particle have appoximately the same range β^{-1}. The triton wave function of Ref. 14 has $\beta = 1.5$ fm^{-1}. In the α particle we choose for u_0^N a parametrization of the form $u_0^N(r) = \exp(-\alpha r) - \exp[b(R-r) - \alpha R]$, where $R = 0.4$ fm is a hard core radius. The parameter b is determined through the condition that u_0^N has $\beta = 1.5$ fm^{-1}. With this wave function, and representing u_2^N by its asymptotic form, Eq. (11) gives $\rho(\alpha) = -0.31$. The inclusion of a cutoff factor,[16] $1-\exp(-Ar^2)$, with $A = 0.735$ fm^{-2}, in the OPEP tensor interaction, increases ρ to $\rho(\alpha) = -0.23$. Although rather crude, the model used to predict $\rho(\alpha)$ gives values which are in qualitative agreement with those extracted from the (d, α) reaction data. The marked increase of $\rho(A)$ with A results from its approximate proportionality to $S\alpha^4 B^{-1}$ in Eq. (11) and is a consequence of the increase in binding energy with A from $A = 2$ to 4.

The results reported in this Communication show that the tensor analyzing powers of (d, α) reactions are sensitive to D states in the α particle. Further experiments and refined theoretical calculations are required to improve the determination of $\rho(\alpha)$.

The authors are grateful to W. J. Thompson for many useful discussions and to T. Sasakawa and T. Igarashi for providing the triton wave function. One of us (F.D.S.) would like to thank the University of Wisconsin for its hospitality and the NATO Science Program (Grant No. 5-2-03B154) for its financial assistance. This work was supported in part by the U.S. Department of Energy.

*On leave from Laboratório de Física, Universidade de Lisboa, 1294 Lisboa Codex, Portugal.

†Present address: University of Tsukuba, Institute of Physics, Ibaraki, Japan 305.

[1]S. A. Tonsfeldt, T. B. Clegg, E. J. Ludwig, Y. Tagishi, and J. F. Wilkerson, Phys. Rev. Lett. 45, 2008 (1980).

[2]L. D. Knutson, in *Proceedings of the Fourth International Symposium on Polarization Phenomena in Nuclear Reactions, Zurich, Switzerland, 1975,* edited by W. Gruebler and V. König (Birkhauser-Verlag, Basel, 1976), p. 205.

[3]L. D. Knutson, B. P. Hichwa, A. Barroso, A. M. Eiró, F. D. Santos, and R. C. Johnson, Phys. Rev. Lett. 35, 1570 (1975).

[4]F. D. Santos, A. M. Eiró, and A. Barroso, Phys. Rev. C 19, 238 (1979).

[5]L. J. B. Goldfarb and R. C. Johnson, Nucl. Phys. 18, 353 (1960); 21, 462 (1960).

[6]A. D. Jackson and D. O. Riska, Phys. Lett. 50B, 207 (1974).

[7]F. D. Santos, Nucl. Phys. A212, 341 (1973).

[8]F. D. Santos and A. M. Goncalves, Phys. Lett. 101B, 219 (1981).

[9]F. D. Santos (unpublished).

[10]E. Gerjuoy and J. Schwinger, Phys. Rev. 61, 138 (1942).

[11]J. E. Beam, Phys. Rev. 158, 907 (1967).

[12]J. Irving, Proc. Phys. Soc. London Sect. A 66, 17 (1953).

[13]R. V. Reid, Ann. Phys. (N.Y.) 50, 411 (1968).

[14]T. Sasakawa and T. Sawada, Phys. Rev. C 19, 2035 (1979).

[15]A. M. Eiro and F. D. Santos, Nucl. Phys. A234, 301 (1974).

[16]D. O. Riska and G. E. Brown, Phys. Lett. 32B, 662 (1970).