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Nuclear medium effects in pion absorption potentials

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Nuclear medium effects in pion absorption potentials, that are calculated by using a two-nucleon mechanism, are estimated in this work by including the modification of the pion propagator and πNN absorption vertex. The results show that the imaginary part of *S*-wave pion-absorption potential changes significantly when these effects are incorporated.

[NUCLEAR REACTIONS Pion-absorption potential; nuclear-medium effects.]

Pion absorption on a pair of nucleons is the dominant mechanism for generating absorptive pion-nucleus potential at low pion energies ($T_\pi \lesssim 100$ MeV). This was first recognized by Ericson and Ericson,[1] who included complex terms involving the square of nuclear density ρ in zero (kinetic) energy pion-nucleus optical potential that was used for the analysis of pionic-atoms data. These terms, which are proportional to $\rho^2(r)$ and $\nabla \cdot [\rho^2(r)\nabla]$, describe *S*- and *P*-wave pion absorption and the coefficients of these two terms are generally written as B_0 and C_0, respectively. Widths of pionic atoms determined imaginary parts of B_0 and C_0 rather well[2] [$\mathrm{Im}B_0 = (0.04-0.06)m_\pi^{-4}$, $\mathrm{Im}C_0 = (0.036-0.07)m_\pi^{-6}$]. The real parts, on the other hand, cannot be determined realiably from pionic-atoms data.[3]

A large number of microscopic calculations have been done[4–6] to determine B_0 and C_0 as a function of incident pion energy by using a specific two-nucleon model for the pion-absorption process. For example, B_0 is calculated in Ref. 4 by considering *S*-wave scattering of incident pion on the first nucleon followed by absorption on the second nucleon (see Fig. 1). Imaginary parts of B_0 and C_0 have also been calculated by using "quasideuteron" models where pion-absorption data on deuteron serves as an input.[7] One of the common features of most of these calculations is that the computed value of $\mathrm{Im}B_0$ at threshold is somewhat smaller than the experimental value[8] [$(0.02-0.03)m_\pi^{-4}$ vs $(0.04-0.06)m_\pi^{-4}$]. This probably indicates that the two-nucleon absorption operator is modified in nuclear medium and these modifications should be included in pion-absorption potential calculations.

In the present work I have estimated the effect of nuclear-medium corrections on *S*-wave pion-absorption potential. This has been done by calculating two-nucleon absorption operator in spin-isospin symmetric nuclear matter having uniform density ρ_0 ($=0.17$ fm^{-3}). Thus, the present calculation includes "kinematical" corrections such as Pauli blocking, nuclear binding, etc., which have been included in earlier calculations.[4,6] In addition, the present calculation also includes "dynamical" effects such as the modification of the propagator of exchanged pion and the renormalization of pion-nucleon vertices. I have included the renormalization of the πNN vertex only; the change in pion-nucleon *S*-wave scattering vertex is expected to be small,[9] since the range of the *S*-wave pion-nucleon interaction is small (approximately the inverse of nucleon mass) in comparison with other characteristics lengths (m_π^{-1} or k_F^{-1}). The present calculation shows that introduction of a nuclear-medium pion propagator increases $\mathrm{Im}B_0$ whereas the πNN vertex renormalization has an opposite effect. The net result of nuclear-medium corrections is an increase of $\mathrm{Im}B_0$ by roughly a factor of 2.5 at threshold.

In the absence of "dynamical" nuclear-medium effects, the *S*-wave pion-absorption operator can be determined by employing some model for *S*-wave pion-nucleon scattering. I have used the (phenomenological) Woodruff Hamiltonian[10] for this

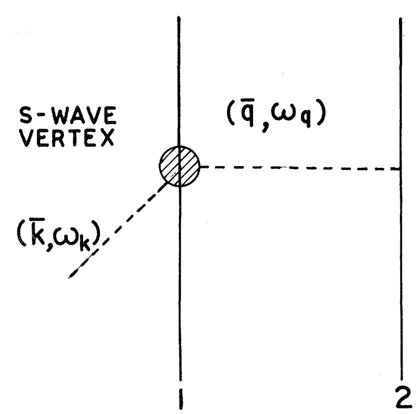

FIG. 1. Model for *S*-wave pion absorption.

purpose. This Hamiltonian has been used earlier by Brack *et al.*[11] in their $\pi^+d\rightarrow pp$ reaction cross section calculation and by a number of authors[4–6] to calculate $\mathrm{Im}B_0$. With this Hamiltonian the S-wave absorption operator for (say) a π^+ meson is

$$R_{01}^{S}(\pi^+)=\frac{i4\pi f}{m_\pi^2(\omega_k)^{1/2}}g_s(q)g(q)D_\pi^0(q,\omega_q)\left\{\vec\sigma^2\cdot\vec q\left[2\lambda_1\tau_+^2-i\frac{\lambda_2}{m_\pi}(\omega_k+\omega_q)(\vec\tau^{\,1}\times\vec\tau^{\,2})_+\right]+\vec\sigma^1\cdot\vec q\left[2\lambda_1\tau_+^1-i\frac{\lambda_2}{m_\pi}(\omega_k+\omega_q)(\vec\tau^{\,2}\times\vec\tau^{\,1})_+\right]\right\}\ , \qquad (1)$$

where $\vec\sigma^{\,i}$ and $\vec\tau^{\,i}$ are spin and isospin operators for ith nucleon, f is the pseudoscalar πNN coupling constant ($f^2/4\pi=0.081$), λ_1 and λ_2 are the coupling constants of the Woodruff Hamiltonian, and $g(q)$ and $g_s(q)$ are the form factors associated with πNN and S-wave vertices, respectively. Coupling constants λ_1 and λ_2 are fixed by fitting S-wave scattering lengths ($\lambda_1=0.003$ and $\lambda_2=0.05$). Form factors $g(q)$ and $g_s(q)$ are chosen to be

$$g(q)=g_s(q)=(\Lambda^2-m_\pi^2)/(\Lambda^2+q^2-\omega_1^2)$$

with $\Lambda=1$ GeV. $D_\pi^0(q,\omega_q)$ in Eq. (1) above is the free pion propagator $(q^2+m_\pi^2-\omega_q^2)^{-1}$ and ω_q, the energy of exchanged pion is taken to be zero.[11] The expression for $\mathrm{Im}B_0$ in terms of the absorption operator defined earlier is

$$\mathrm{Im}B_0=\frac{2f^2m}{\pi^2}\int d^3r\,d^3r'\,f(r)f(r')\frac{\sin\hat Q|\vec r-\vec r'|}{|\vec r-\vec r'|}\hat r\cdot\hat r' X^0(r)X^0(r')\times\left\{(\hat\lambda_2^2+2\lambda_1^2)\hat j_1^2\left(k_F\left|\frac{\vec r-\vec r'}{2}\right|\right)\cos\left(\vec k_0\cdot\frac{\vec r-\vec r'}{2}\right)+\lambda_1\left[(\hat\lambda_2+\tfrac{1}{2}\lambda)\hat j_1^2\left(k_F\left|\frac{\vec r-\vec r'}{2}\right|\right)+(\hat\lambda_2-\tfrac{1}{2}\lambda)\hat j_1^2\left(k_F\left|\frac{\vec r+\vec r'}{2}\right|\right)\right]\cos\left(\vec k_0\cdot\frac{\vec r+\vec r'}{2}\right)\right\}\ , \qquad (2)$$

where $f(r)$ is the Jastrow correlation function,[12] $\hat Q=(mE_{\mathrm{av}})^{1/2}$ is the (average) relative momentum of pair of nucleons after pion absorption,

$$\hat j_1(x)=3(\sin x/x^2-\cos x/x)/x\ ,$$

$$\hat\lambda_2=\omega_k\lambda_2/m_\pi\ ,$$

and k_F is the nucleon Fermi momentum ($k_F=1.36\ \mathrm{fm}^{-1}$). Equation (2) above has been obtained by considering pion absorption on correlated nucleon pair in spin-isospin symmetric nuclear matter. The correlation function has been chosen to be

$$f(r)=0,\quad r<0.4\ \mathrm{fm}\ ,$$
$$f(r)=1-\exp[-(r-0.4)/0.2],\quad r>0.4\ \mathrm{fm}\ . \qquad (3)$$

One could use the realistic correlation function computed in variational calculations (e.g., Ref. 13). However, the results are not expected to change much since $\mathrm{Im}B_0$ is not very much sensitive to the detailed behavior of $f(r)$. (In fact an earlier calculation[4] shows that $\mathrm{Im}B_0$ decreases by only a few percent when the correlation function is included.) The function $X^0(r)$ is given by

$$X^0(r)=\frac{2}{\pi m_\pi^2}\int q^3\,dq\,j_1(qr)\times D_\pi^0(q,0)g(q)g_s(q)\ . \qquad (4)$$

The "dynamical" nuclear-medium effects essentially account for the interaction of exchanged pion with nuclear medium. One of the effects of such an interaction is the modification of pion propagator in nuclear medium. The modified pion propagator can be expressed in the form $D_\pi(q,\omega_q=0)=[q^2+m_\pi^2+\pi(q,\omega_q=0)]^{-1}$, where $\pi(q,\omega_q)$ is the pion self-energy (or equivalently, the pion optical potential in nuclear matter). In the present work the pion self-energy has been calculated by keeping nucleon pole term. Thus

$$\pi(q,\omega_q=0)=\frac{-q^2AF(q)g^2(q)}{1+\xi AF(q)g^2(q)}\ , \qquad (5)$$

where $A=f^2mk_F/m_\pi^2\pi^2$. $F(q)$ is the nuclear Lindhard function,

$$F(q)=\left\{1-\frac{k_F}{q}\left[1-\left(\frac{q}{2k_F}\right)^2\right]\ln\left|\frac{1-q/2k_F}{1+q/2k_F}\right|\right\} \qquad (6)$$

and ξ is the Lorenz-Lorentz parameter[1] which essentially takes care of short-range nucleon-nucleon correlations. The value of ξ depends on the detailed short-distance behavior of nucleon-nucleon interaction. Here ξ has been chosen to be 0.5. The contribution of the Δ_{33} resonance and the S-wave πN interaction to pion self-energy has not been included here. These contributions are small when $\omega_q=0$ and

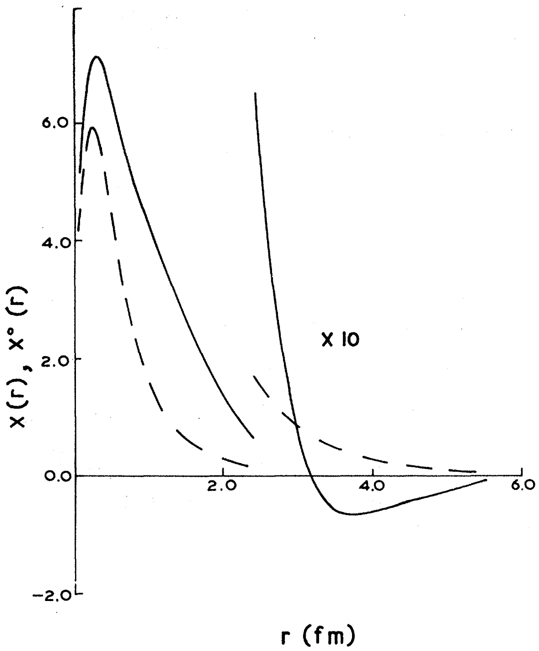

FIG. 2. Plot of $X(r)$ (continuous curve) and $X^0(r)$ (broken curve) vs r.

are not expected to produce qualitative changes in final results.

The effect of the nuclear-medium pion propagator on $\mathrm{Im}B_0$ can be seen from the behavior of $X(r)$, which is obtained by replacing $D_\pi^0(q,0)$ by $D_\pi(q,0)$ in Eq. (4). Functions $X(r)$ and $X^0(r)$ are plotted in Fig. 2 and this figure shows that $X(r)$ is larger in magnitude then $X^0(r)$. Also, $X(r)$ has oscillations that are damped. This behavior of $X(r)$ is due to the enhancement of the (momentum-space) pion propagator with the inclusion of pion self-energy. (Ericson[14] has shown that a coordinate-space nuclear-medium pion propagator also has damped oscillatory behavior below pion condensation threshold.) Since the major contribution to the integral in Eq. (2) comes from the region $r,r' \lesssim 3$ fm, where $X(r)$ is larger than $X^0(r)$, it is evident that $\mathrm{Im}B_0$ would increase when nuclear-medium correction to pion propagator is included.

In addition to the modification of pion propagator, the nuclear-medium effects renormalize the πNN absorption vertex.[15] The vertex renormalization is due to short-range nucleon-nucleon interaction (excluding pion-exchange contribution) and its effect can be approximately included by replacing the free πNN coupling constant, f, by an effective coupling constant:

TABLE I. Calculated values of $\mathrm{Im}B_0(m_\pi^{-4})$ for different pion kinetic energies (MeV).

T_π (MeV)	No medium correction (m_π^{-4})	With medium correction (m_π^{-4})
0	+0.0280	+0.0714
20	+0.0332	+0.0805
40	+0.0383	+0.0895
60	+0.0433	+0.0961
80	+0.0480	+0.1008
100	+0.0527	+0.1042

$$f_{\mathrm{eff}} = f\,\{1-\xi AF(q=0)/[1+\xi AF(q=0)]\} \quad (7)$$

in Eq. (2). Clearly, $f_{\mathrm{eff}} < f$ and, as a result, the effect of vertex renormalization is to decrease $\mathrm{Im}B_0$.

The imaginary part of B_0 is obtained by evaluating the integrals in Eq. (2) numerically. The results, which are displayed in Table I, show that with "dynamical" nuclear-medium effects, the threshold value of $\mathrm{Im}B_0$ is increased by roughly a factor of 2.5, thus becoming larger than phenomenological $\mathrm{Im}B_0$. The increase in $\mathrm{Im}B_0$ is somewhat smaller at larger pion energies.

The present work clearly shows the importance of nuclear-medium corrections in pion-absorption potential calculations. However, at this stage I would like to point out that the present calculation, to some extent, overestimates nuclear-medium effects because of the following reason. Here the nuclear-medium effects are introduced by using a nuclear-matter pion propagator. Clearly this procedure is not valid in the nuclear surface, where nuclear density is smaller than the central density ($\sim 0.17\ \mathrm{fm}^{-3}$). Since pion self-energy decreases with the decrease in nuclear density, one would expect that the change in $\mathrm{Im}B_0$ would be smaller when the variation of nuclear density is properly included. In this context, one should note that phenomenological $\mathrm{Im}B_0$ is determined from pionic-atoms data for very light nuclei, which are essentially surface nuclei.

To sum up, nuclear-medium effects on S-wave pion-absorption potential are estimated in this work. The calculation shows that nuclear-medium effects increase the imaginary part of S-wave pion-absorption potential by roughly a factor of 2.5.

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