PHYSICAL REVIEW C VOLUME 25, NUMBER 5 JUNE 1982

Possible energy parameters for continuum angular distributions.

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(Received 28 September 1981)

The shapes of continuum angular distributions have previously been described using the energy of the emitted particle as the principal parameter, but phenomenological methods are here found to be unable to distinguish conclusively between this and other related energy parameters. This points to the need for more theoretical work. Slight evidence for a shift by the ejectile binding energy is noted.

[NUCLEAR REACTIONS Investigated alternate energy parameters for describing shapes of continuum angular distributions.]

In a recent paper,[1] hereafter referred to as I, a study was made of the systematics of continuum angular distributions for particles emitted in light ion induced nuclear reactions. In that work it was observed that to first order the shapes of the angular distributions are determined by the center of mass energy, ϵ, of the outgoing particle. The other pertinent parameter appears to be the fraction of the cross section which is due to statistical multistep direct (MSD) as opposed to statistical multistep compound (MSC) processes. MSD processes are ones in which the system passes through a series of configurations each of which has at least one unbound particle degree of freedom. The resulting angular distributions are expected to be forward peaked. Whenever the system passes through a configuration where all of the particles are bound, the resulting process is said to be MSC and should exhibit an angular distribution which is symmetric about 90° in the center of mass.

The systematics observed in I are very general. The dependence on the energy of the emitted particle could be clearly differentiated from something as different as a momentum dependence, but no effort was made to investigate other variables more closely related to the emission energy. Results[2] on charged particle emission in proton induced reactions, however, suggest that it might be useful to shift the emission energy by the ground state Q value of the reaction or by the binding energy of the emitted particle. These new shifted parameters may be designated as $e_1=\epsilon-Q$ and $e_2=\epsilon+B_b$. It was decided, therefore, to see if either of these options would give improved agreement with the more varied data set of I.

In order to specify either the Q value of the reaction or the binding energy of the emitted particle, it is necessary to know the identity of the emitting nucleus. In an energy domain in which only one particle can be emitted, all emission is from the initial composite nucleus, but when sequential emission is possible, the Q value and binding energy are often ambiguous in an inclusive experiment of the sort considered here. For the sake of simplicity, the first-particle-out values have been used across the entire spectrum.

It is clear that the $e_2=\epsilon+B_b$ is to be preferred over $e_1=\epsilon-Q$ on purely physical grounds. The very simple systematics observed in I indicate that the detailed reaction mechanism is not important in determining the shape of the angular distribution so that an overall Q-value dependence, particularly when sequential emission is possible, does not seem to be indicated. On the other hand, the emission angle and energy of the ejectile are largely determined before it leaves the intermediate nucleus so that it would be reasonable to expect the shape of the angular distribution to depend on the particle's energy *inside* rather that outside the nucleus. Nevertheless, the effects of replacing ϵ with both e_1 and e_2 have been investigated.

In I the continuum angular distributions for a reaction $A(a,b)$ were found to be adequately described in terms of the regular Legendre polynomials, $P_l(\cos\theta)$, by the relation

$$\frac{d^2\sigma}{d\Omega\,d\epsilon}(a,b)=a_0(\mathrm{MSD})\sum_{l=0}^{6} b_l(\epsilon)P_l(\cos\theta)+a_0(\mathrm{MSC})\sum_{\substack{l=0\\ \Delta l=2}}^{6} b_l(\epsilon)P_l(\cos\theta) \qquad (1)$$

TABLE I. Data used in parameter search.

Reaction	Projectile energy (MeV)	Ejectile energy (MeV)	No. of energies	No. of angles	Ref.	Q	B_b
^{120}Sn(p,p')	62	8–55	11	8	4	0.00	5.78
^{59}Co(α,p)	42	10–32	5	8	5	−0.34	6.12
^{103}Rh(α,p)	42	10–32	5	8	5	−2.98	5.80
^{54}Fe(α,α')	59	7–45	9	6	6	0.00	6.40

with the zero order coefficients a_0 determined from the preequilibrium reaction model code PRECO–D[3]. The reduced polynomial coefficients of order l were found to be given by

$$b_l(\epsilon)=(2l+1)/[1+\exp A_l(B_l-\epsilon)] , \tag{2}$$

$$A_l=k_1+k_2 l(l+1) , \tag{3a}$$

$$B_l=k_3+k_4[l(l+1)]^{-1/2} , \tag{3b}$$

with the four k parameters determined empirically using a nonlinear least squares fitting routine on a subset of the data considered. The data[4–6] used in the fitting are given in Table I. The values of k_1 to k_4 and the reduced chi square found in I are reproduced in Table II.

In the present work the fitting procedure was repeated with the parameter ϵ replaced by $e_1=\epsilon-Q$ and by $e_2=\epsilon+B_b$. The results of the new searches are also given in Table II. Except for a change of 6 MeV in k_3 to compensate for the average binding energy B_b when e_2 is used there is no significant change in either the coefficients or the chi-square value. This insensitivity is related to the uniformity of Q and B_b for the systems considered.

When, however, the full data set of I is considered, the situation changes. For reactions involving deuterons or tritons, the Q values and binding energies can differ significantly from those shown in Table I, thus causing the angular distributions calculated using e_1 or e_2 to differ from those obtained in I.

Angular distributions have been calculated for all

TABLE II. Results of least squares fitting.

Energy variable	k_1 (MeV^{-1})	k_2 (MeV^{-1})	k_3 (MeV)	k_4 (MeV)	reduced χ^2
ϵ	0.036	0.0039	92	−90	5.32
$e_1=\epsilon-Q$	0.036	0.0039	92	−90	5.16
$e_2=\epsilon+B_b$	0.036	0.0039	98	−90	5.26

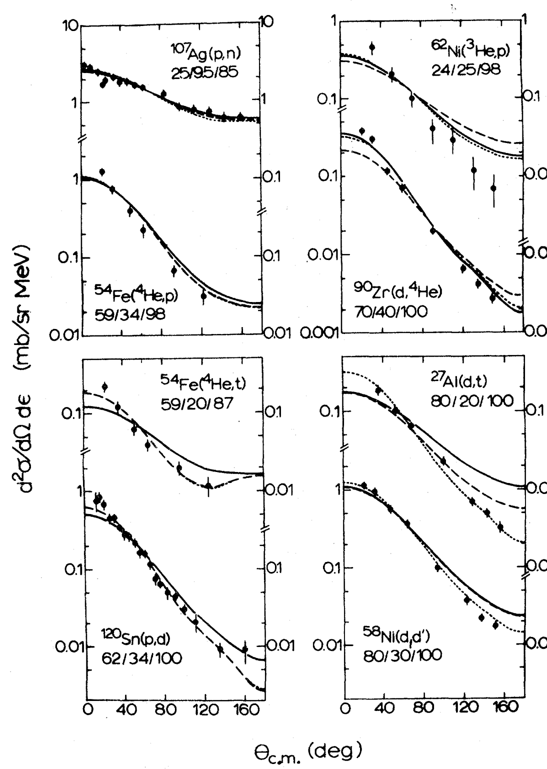

FIG. 1. Comparison of experimental and calculated angular distributions. The solid points indicate the data which were taken from Refs. 4 and 6–9. The solid curves were calculated in I using the energy parameter ϵ. The long and short dashed curves were obtained by replacing ϵ with $e_1=\epsilon-Q$ and $e_2=\epsilon+B_b$, respectively. The k values of Table II were employed.

TABLE III. Comparison of energy parameters.

Projectile	Ejectile	Similar[a] angular distributions	No. of angular distributions	Acceptable fits[b] ϵ	e_1	e_2	Better fits[c] ϵ	e_1	e_2
p,α	n,p,α	ϵ,e_1,e_2	122	108	110	108	3	6	3
p,α	$d,t,{}^3$He	e_1,e_2	58	43	51	51	12	30	30
$d,{}^3$He	p,α	ϵ,e_2	14	3	0	3	10	1	5
			(10	3	0	3	6	1	5)[d]
$d,{}^3$He	$d,t,{}^3$He	ϵ,e_1	8	0	1	3	1	1	5

[a]For systems with average binding energies, these parameters tend to yield similar angular distributions.
[b]Curves pass through at least half of all data points.
[c]Fits are clearly better on visual examination than those obtained with other parameters. Where two parameters yield nearly identical curves, both are listed.
[d]Results for systems with $A > 200$ and with α particle ejectiles eliminated. Removal of the one ^{209}Bi(p,α) system would not significantly alter the results of the first line of the table. Alpha projectile systems in this mass range have not been considered.

of the data[4-10] considered in I using (1)–(3) but with ϵ replaced first by $e_1=\epsilon-Q$ and then by $e_2=\epsilon+B_b$. As in I the ratio a_0(MSD)/a_0(MSC) was taken from the results of the reaction code PRECO–D while the sum a_0(MSD)+a_0(MSC) was adjusted to facilitate comparisons of the shapes of the calculated and experimental results. The values of k_1, k_2, k_3, and k_4 shown in Table II were employed. The systems include targets ranging from ^{12}C to ^{232}Th, light ion ($A \leq 4$) projectiles with energies from 18 to 80 MeV, and neutron and light ion ejectiles in the energy range of 4 to 60 MeV. Some additional angular distributions for incident deuterons from Ref. 8 were also included in this study.

The results for some sample systems are shown in Fig. 1. Because of binding energy trends, they may be divided first by the nature of the projectile and then of the ejectile. Since protons and alpha particles tend to have similar binding energies they are considered together. Similarly, d, t, and ^{3}He particles are considered together. A summary of the number of systems in the different classes for which acceptable fits were obtained with the various energy parameters is given in Table III. "Acceptable" here means passing through at least half of the data points. Also shown are the number of angular distributions which show significantly better fits with one parameter (or a pair of parameters yielding nearly identical results) than the others.

The first group of reactions is not at all sensitive to the choice of an energy parameter. For the remaining three groups, only $e_2=\epsilon+B_b$ is consistently one of the parameters yielding the largest number of acceptable fits, indicating a slight preference for this variable. This is not the case when the number of significantly better fits is considered until the (d,α) data on ^{208}Pb and ^{232}Th are removed from consideration. These systems have unusually low alpha particle binding energies. This suggests that if more such systems involving unusually high or low binding energies had been considered, the preference for e_2 would be even less clear.

In conclusion, then, it appears that the systematics of continuum angular distributions observed in I is quite general but also somewhat approximate. The data are not particularly sensitive to replacing the emission energy by other closely related energy parameters. At present there is no evidence for preferring a Q-value shifted energy, $\epsilon-Q$, while evidence for a binding energy shift exists but is far from overwhelming. Similarly there are plausible but not compelling physical arguments for preferring $\epsilon+B_b$ to ϵ itself. A better resolution of this question must await a theoretical explanation of the simple systematics observed in I.

This work was supported in part by Hanford Engineering Development Laboratory (HEDL). HEDL is operated by Westinghouse Hanford Company, a subsidiary of Westinghouse Electric Corporation, under DOE Contract No. DE-AC-14-76FF02170. The author would like to acknowledge the hospitality of the Triangle Universities Nuclear Laboratory.

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