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Particles and holes equivalence for generalized seniority and the interacting boson model

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An apparent ambiguity was recently reported in coupling either pairs of identical fermions or hole pairs. This is explained here as being due to a Hamiltonian whose lowest eigenstates do not have the structure prescribed by generalized seniority. It is shown that generalized seniority eigenstates can be equivalently constructed from correlated $J=0$ and $J=2$ pair states of either particles or holes. The interacting boson model parameters recently calculated can be unambiguously interpreted and then are of real interest to the shell model basis of the interacting boson model.

NUCLEAR STRUCTURE Shell model description of interacting boson model; particle-hole equivalence in generalized seniority.

In a recent paper[1] the derivation of parameters of the interacting boson model (IBM) from the shell model is considered. It is claimed there that they depend sensitively on whether shell model states are constructed from correlated pairs of nucleons or pairs of holes. In Ref. 1, this is presented as an ambiguity and is attributed to "the shell-model truncation inherent in the IBM." It is concluded that "it is crucial to include renormalization effects from outside the *S-D* subspace."

The aim of the present report is to clarify the origin of the effects observed in Ref. 1. We show that states with generalized seniority[2] $J=0$, $v=0$ and $J=2$, $v=2$ can be equivalently obtained by coupling either identical particle pairs or hole pairs. It then follows that the lowest $J=0$ and $J=2$ eigenstates of the Hamiltonian of Ref. 1 do not have the structure prescribed by generalized seniority as assumed there. There is no ambiguity in the mapping of shell model states on boson states, which is the microscopic foundation of IBM.[3] It is based on the lowest $J=0$ and $J=2$ states of valence protons and of valence neutrons having well defined generalized seniorities.

The lowest $J=0$ states of identical valence nucleons, like ground states of semimagic nuclei, were constructed by[2]

$$(S^\dagger)^N|0\rangle\,,$$
$$S^\dagger=\Sigma\alpha_j S_j^\dagger=\Sigma\alpha_j\left(\tfrac{1}{2}\Sigma(-1)^{j-m}a_{jm}^\dagger a_{j-m}^\dagger\right)\,. \tag{1}$$

In (1), $|0\rangle$ has only closed shells, $a_{jm}^\dagger$ are nucleon creation operators, and the summation extends over j orbits in a major shell. Such states as (1) are eigenstates of the (one body and two-body) shell model Hamiltonian H (normalized by $H|0\rangle=0$) provided

$$HS^\dagger|0\rangle=[H,S^\dagger]|0\rangle=V_0S^\dagger|0\rangle\,, \tag{2}$$
$$[[H,S^\dagger],S^\dagger]=\Delta(S^\dagger)^2\,, \tag{3}$$

where V_0 and Δ are numbers. The parameters α_j can be calculated from the eigenvector of the $J=0$ submatrix of H, with lowest eigenvalue V_0, in the two nucleon configuration. Such states as (1), assigned generalized seniority $v=0$, can be mapped onto boson states $(s_\pi^\dagger)^N|0\rangle$ of valence protons or $(s_\nu^\dagger)^N|0\rangle$ of valence neutrons.

Similarly, lowest states with $J=2$ of semimagic nuclei were constructed by[2]

$$(S^\dagger)^{N-1}D_M^\dagger|0\rangle\,,$$
$$D_M^\dagger=\sum_{j<j'}\beta_{jj'}(1+\delta_{jj'})^{-1/2}\times\Sigma(jmj'm'|jj'2M)a_{jm}^\dagger w_{j'm'}^\dagger\,. \tag{4}$$

Also such states, assigned generalized seniority $v=2$, are eigenstates of H if, in addition, the following conditions are satisfied

$$HD_M^\dagger|0\rangle=V_2D_M^\dagger|0\rangle\,, \tag{5}$$
$$[[H,S^\dagger],D_M^\dagger]=\Delta S^+D_M^\dagger\,. \tag{6}$$

The parameters $\beta_{jj'}$ in (4) can be calculated from (5). This amounts to calculating the eigenvector with lowest eigenvalue V_2 of the $J=2$ submatrix of H in the two nucleon configuration. The states (4) can be mapped onto boson states $(s_\pi^\dagger)^{N-1}d_{M_\pi}^\dagger|0\rangle$ for valence protons or $(s_\nu^\dagger)^{N-1}d_{M_\nu}^\dagger|0\rangle$ for valence neutrons. It was shown in Ref. 2 that a consequence of conditions (2), (3) and (5), (6) is that the separation between the $J=0$ states (1) and the corresponding $J=2$ states (4) is independent of N and equal to V_2-V_0. In the boson model this separation is identified with the energy difference $\epsilon=\epsilon_d-\epsilon_s$ between single d boson and single s-boson energies.

In Ref. 1 matrix elements of a certain quadrupole operator were calculated between states constructed like (1) and (4), properly normalized. When such states were constructed by coupling hole pairs the results turned out to be different. Obviously, the states constructed in Ref. 1 with hole pairs were not identical to those in which particle pairs were used. Let us now consider the case that (1) and (4) are eigenstates of the shell model Hamiltonian and construct hole states corresponding to them.

The (only) state, with $J=0$, of the completely closed major shell is obtained by putting $N=\Omega$ in (1), where $2\Omega=\Sigma(2j+1)$. The hole pair creation operator S' operating on that state should give a state proportional to $(S^\dagger)^{\Omega-1}|0\rangle$. By induction we obtain[2]

$$S'(S^\dagger)^\Omega|0\rangle=\Omega(S^\dagger)^{\Omega-1}[S',S^\dagger]|0\rangle+\tfrac{1}{2}\Omega(\Omega-1)(S^\dagger)^{\Omega-2}[[S',S^\dagger],S^\dagger]|0\rangle\ . \tag{7}$$

We can express S' as a linear combination of the Hermitian conjugates of $S_j^\dagger$ as $\Sigma\alpha_j'S_j^-$. The well known commutation relations of the quasispin operators $S_j^\dagger$, S_j^-, and $S_j^0=\frac{1}{2}[S_j^\dagger,S_j^-]$ yield for the commutators in (7)

$$\begin{aligned}[S',S^\dagger]&=\Sigma\alpha_j'\alpha_j[S_j^-,S_j^\dagger]\\&=-2\Sigma\alpha_j'\alpha_jS_j^0\ ,\\ [[S',S^\dagger],S^\dagger]&=-2\Sigma\alpha_j'\alpha_j{}^2[S_j^0,S_j^\dagger]\\&=-2\Sigma\alpha_j'\alpha_j{}^2S_j^\dagger\ .\end{aligned} \tag{8}$$

The rhs of (7) will be proportional to $(S^\dagger)^{\Omega-1}|0\rangle$ if, for every nonvanishing value of α_j we set $\alpha_j'=1/\alpha_j$. Apart from normalization, the hole pair creation operator is

$$S'=\Sigma\frac{1}{\alpha_j}S_j^-\ . \tag{9}$$

When the operator S', given by (9), operates on the state $(S^\dagger)^N|0\rangle$ with any value of $1\le N\le\Omega$, it gives a state proportional to $(S^\dagger)^{N-1}|0\rangle$ as seen by replacing Ω by N in (7). Hence, eigenstates of H with $\bar{N}=\Omega-N$ hole pairs can be obtained by $\bar{N}$ successive applications of S'. In particular, the coefficients $\alpha_j'=1/\alpha_j$ will be obtained by diagonalizing the $J=0$ submatrix of H in the two hole configuration.

The $J=2$ states with generalized seniority $v=2$ can be obtained by operating on $J=0$, $v=0$ states with a single nucleon quadrupole operator Q_M given by[4,5]

$$D_M^\dagger=\tfrac{1}{2}[Q_M,S^\dagger]\ . \tag{10}$$

Using this Q_M and S' we similarly construct the $J=2$ hole-pair creation operator

$$D_M'=\tfrac{1}{2}[Q_M,S']\ . \tag{11}$$

To see it we first calculate with the help of (8) and the Jacobi identity

$$[D_M',S^\dagger]=[D_M^\dagger,S']\ ,\quad [[D_M',S^\dagger],S^\dagger]=2D_M^\dagger\ . \tag{12}$$

We obtain by induction, making use of (12),

$$\begin{aligned}D_M'(S^\dagger)^N|0\rangle&=N(S^\dagger)^{N-1}[D_M',S^\dagger]0|\,\rangle+\tfrac{1}{2}N(N-1)(S^\dagger)^{N-2}[[D_M',S^\dagger],S^\dagger]|0\rangle\\&=N(N-1)(S^\dagger)^{N-2}D_M^\dagger|0\rangle\ .\end{aligned} \tag{13}$$

The term linear in N in (13) does not contribute as can be readily verified [cf. Eq. (11) in Ref. 2]. Hence, the operation of $(S')^{\bar{N}-1}D_\mu'$ on the state with closed shells $(S^\dagger)^\Omega|0\rangle$ yields a state proportional to $(S^\dagger)^{N-1}D_\mu^\dagger|0\rangle$, where $\bar{N}=\Omega-N$. All these states are eigenstates of H, and in particular, the state $D_\mu'(S^\dagger)^\Omega|0\rangle$. Hence, the exact form of D_μ' [the coefficients similar to those in (4)] can be obtained by diagonalizing the $J=2$ submatrix of the Hamiltonian in the configuration of two nucleon holes.

The shell model basis for IBM is the mapping of states constructed from $J=0$ and $J=2$ pairs onto states with s and d bosons. Since states with different numbers n_d of d bosons are orthogonal, the corresponding shell model states should also have this property. This is achieved by removing out of any state constructed by n_d operators $D^\dagger$ components with generalized seniorities lower than $2n_d$.[3,6] The latter have the general form $S^\dagger B^\dagger|0\rangle$. When n_d is larger than Ω all states have this form and will not survive the procedure. Therefore, beyond the middle of the shell the mapping onto states with N bosons is of shell model states constructed by N pairs of nucleon *holes*.[3,6]

The parameters of the quadrupole operators of protons and neutrons which appear in the boson Hamiltonian are determined from states with generalized seniorities $v=0$ and $v=2$. They can thus be obtained from those states constructed from either particle pair or hole pair states. No ambiguity will arise from using the boson description throughout the shell. With the convention made above, for every N, boson states correspond to definite nucleon states. Also in the exact middle of the shell, $N=\Omega/2$, states with $J=0$, $v=0$ and $J=2$, $v=2$ are the same, whether obtained from particle pairs or hole pairs. Thus, the boson model and its parameters are well defined also in that case.

Let us now return to the Hamiltonian of Ref. 1 and see why its lowest eigenstates do not have the form of (1) and (4). The two-body interaction used there is the surface delta interaction (SDI).[7] Arvieu and Moszkowski have shown[8] that the lowest $J=0$ states calculated with SDI have indeed the form of (1) with all coefficients α_j equal [more precisely, $\alpha_{lj}=(-1)^l$ which causes a trivial change in the definition of the quasispin operators]. This is true, however, only if all single nucleon energies are equal, which is far from the situation given in Ref. 1. As a result, eigenstates have no more definite seniorities. This fact can be checked by comparing the separation between the $J=0$ ground state and the lowest $J=2$ state, in the two particle and the two hole configurations. Using the Hamiltonian of Ref. 1, they are not equal, whereas they should be exactly equal for Hamiltonians whose eigenstates have definite generalized seniorities.[2] SDI is very ingenious and easy to work with. A realistic effective interaction between identical nucleons should, however, reproduce the rather constant 0-2 separation which is a characteristic feature of semimagic nuclei also where single nucleon energies are far from degenerate.

The results of Ref. 1 are still of great interest. Once the pair creation operators $S^\dagger$ and $D^\dagger$ were obtained from SDI in the two nucleon configuration, we may well disregard that interaction for other nucleon numbers. Instead, we can construct another shell model Hamiltonian (Hamiltonian I) which will have the same $S^\dagger|0\rangle$ and $D^\dagger|0\rangle$ states as eigenstates and also satisfy conditions (3) and (6) (cf. Ref. 4). The states $(S^\dagger)^N|0\rangle$ and $(S^\dagger)^{N-1}D_M^\dagger|0\rangle$ will then be eigenstates of this new Hamiltonian, with generalized seniorities $v=0$ and $v=2$. These are the states actually used in Ref. 1 and, as shown above, may be used throughout the shell. The other set of states defined in Ref. 1 as arising from hole pairs may be similarly interpreted. They can also be eigenstates of a certain shell model Hamiltonian (Hamiltonian II) which is different from the one introduced above (and is not the one with SDI).

Matrix elements calculated in Ref. 1 and resulting IBM parameters can be interpreted as being due to two different Hamiltonians I and II. Both have eigenstates with well defined generalized seniorities but differ in their two-body interactions *and* single nucleon energies. With this interpretation, we see for the first time, the IBM parameters calculated from a semirealistic case of several nondegenerate j orbits. It is very interesting to compare the dependence of the parameters κ_ν and χ_ν of the quadrupole operator with those obtained for a single j orbit (or equivalently, equal α_j parameters).[3,10]

For equal α_j the dependence of κ_ν on neutron number n is given by $\sqrt{2\Omega-n}$ for $n\leq\Omega$. The mapping of bosons on hole pair states beyond the middle of the shell leads to $\sqrt{n}$ dependence for $n>\Omega$. The behavior of κ_ν given in Fig. 1(a) of Ref. 1 is not symmetric but does not differ very much from the case of equal α_j. This dependence on n is seen in both cases I and II and is similar to the behavior of κ_ν determined by fitting experimental data.[9,10]

Much more interesting are the results of Ref. 1 for the parameter χ_ν. In the case of equal α_j its dependence on neutron number n is given by[3,10] $(\Omega-n)/\sqrt{2\Omega-n}$. It should decrease, in absolute value, for increasing n, go through zero, and change sign in the middle of the shell. The values of χ_ν determined empirically[9,10] show a similar behavior. On the other hand, values of χ_π so determined[11] are appreciably different. It was conjectured that if the

α_j are far from equal the dependence of χ could be very different. The results of Ref. 1 give a beautiful demonstration that this is indeed the case. Both sets of values in Fig. 1(b) of Ref. 1 show a dependence very different from the one with equal α_j. The set corresponding to Hamiltonian I changes sign twice, whereas set II changes sign only at the very end of the shell. These results show that generalized seniority is a powerful scheme on which the interacting boson model can be based. This can be done provided the α_j are properly chosen in agreement with the experimental data.

[1]S. Pittel, P. D. Duval, and B. R. Barrett, Phys. Rev. C 25, 2834 (1982).

[2]I. Talmi, Nucl. Phys. A172, 1 (1971).

[3]A. Arima, T. Otsuka, F. Iachello, and I. Talmi, Phys. Lett. B66, 205 (1977); B76, 141 (1978).

[4]S. Shlomo and I. Talmi, Nucl. Phys. A198, 81 (1972); I. Talmi, Riv. Nuovo Cimento 3, 85 (1973).

[5]J. N. Ginocchio and I. Talmi, Nucl. Phys. A337, 431 (1980).

[6]T. Otsuka, A. Arima, and F. Iachello, Nucl. Phys. A309, 1 (1978).

[7]I. M. Green and S. A. Moszkowski, Phys. Rev. 139, B790 (1965).

[8]R. Arvieu and S. A. Moszkowski, Phys. Rev. 145, 830 (1966).

[9]F. Iachello, G. Puddu, O. Scholten, A. Arima, and T. Otsuka, Phys. Lett. 89B, 1 (1979).

[10]G. Puddu, O. Scholten, and T. Otsuka, Nucl. Phys. A348, 109 (1980).

[11]R. Bijker, A. E. L. Dieperink, O. Scholten, and R. Spanhoff, Nucl. Phys. A344, 207 (1980).