Equilibrium deformations of heavy nuclei

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The equilibrium deformations of nuclei in the rare earth and transition region ($146 \leq A \leq 192$) are studied with the self-consistent quadrupole plus pairing interaction model, by considering all the nucleons in the nucleus explicitly in the configuration space of the first seven major oscillator shells for both protons and neutrons. The present results are in agreement with those obtained earlier by assuming an inert core and employing the renormalized strengths for the quadrupole and pairing interactions. The experimental $B(E2)$ values are correctly reproduced by employing bare nucleon charges in large configuration space.

[NUCLEAR STRUCTURE Equilibrium deformations, rare earth and transition nuclei, self-consistent $Q+P$ model, $B(E2)$ values.]

The equilibrium deformations of nuclei has been a topic of sustained interest in nuclear physics for many years. The equilibrium deformations of heavy nuclei were first studied[1-4] by employing the Nilsson model. The generalized Nilsson model calculations[3,4] led to the conclusion that all the rare-earth nuclei favor an axially symmetry prolate shape ($\gamma_0=0$) with deformation $\beta_0 \simeq 0.3$, whereas in the still heavier nuclei in the transition region ($A \sim 190$), the deformation is small ($\beta_0 \simeq 0.15$) and could very well be axially asymmetric ($\gamma_0 \neq 0$). The equilibrium deformations of heavy nuclei were also studied extensively with the self-consistent quadrupole plus pairing ($Q+P$) interaction model[5] by considering only the valence nucleons outside an "inert core" of 40 protons and 70 neutrons. The equilibrium deformation parameters β_0 and γ_0 obtained from the $Q+P$ model[5] and the generalized Nilsson model[3,4] are found to be very similar over the entire mass region $150 \leq A \leq 200$. There is, however, a clear distinction between the calculations based on the Nilsson and the $Q+P$ models to obtain equilibrium deformations. In the Nilsson model, all the nucleons must be taken into account explicitly,[3] while in the $Q+P$ model, one can assume an inert core and obtain equilibrium deformations by a proper choice of the strengths of the quadrupole and pairing interactions.

The $Q+P$ model has recently been applied[6] to study the properties of the high spin states of the nuclei in the rare earth and the transition region. It was found[6] from these detailed investigations that the assumption of an inert core necessitates the modification of the nucleon charges and excitation energies by a suitable renormalization to reproduce the experimental $B(E2)$ values and energy spectra. In order to justify this renormalization prescription in the $Q+P$ model, it is necessary to investigate the effects of the neglected core on the properties of the nuclear states projected from the intrinsic states of nuclei. Before considering the effect of the neglected core on the properties of the nuclear states, it is thus worthwhile to find its effects on the equilibrium deformations of the nuclei. It is first necessary to know whether proper equilibrium deformations can be obtained in the $Q+P$ model by considering all the nucleons in the nucleus explicitly, without the assumption of an inert core. The present calculations in a large configuration space of the first seven major oscillator shells indicate that it is possible to obtain proper equilibrium deformations by a suitable choice of the strengths of the quadrupole and pairing interactions. Furthermore, the experimental $B(E2)$ values are also correctly reproduced by our calculations with bare nucleon charges for all the nucleons.

The many-body Hamiltonian H for the nuclear system can be written as

$$H=\sum T_{\alpha\beta}a_\alpha^\dagger a_\beta+\frac{1}{4}\sum V_{\alpha\beta\gamma\delta}a_\alpha^\dagger a_\beta^\dagger a_\delta a_\gamma \ , \qquad (1)$$

where T is the kinetic energy operator and $V_{\alpha\beta\gamma\delta}$ is the antisymmetrized matrix element of the nucleon-nucleon (NN) interaction. The spherical basis states are denoted by Greek letters and the deformed states will be denoted by Latin letters. The subscript α in Eq. (1) denotes all the quantum numbers $(n_\alpha, l_\alpha, j_\alpha, m_\alpha)$ necessary for the specification of a single particle state $|\alpha\rangle$. The general Hartree-Fock-Bogoliubov (HFB) equations can be obtained by introducing an auxiliary Hamiltonian $H-\lambda\hat{N}$, where $\hat{N}$ is the number operator and λ the Lagrangian multiplier to conserve the number of nucleons in the system. The HFB equations can be simplified by employing the quadrupole (V^Q) and pairing (V^P) interactions with their matrix elements given by

$$V^Q_{\alpha\beta\gamma\delta}=-\sum_{\tau\tau'\mu}\sqrt{\chi_\tau\chi_{\tau'}}\{(q^\tau_\mu)_{\alpha\gamma}(q^{\tau'}_\mu)_{\delta\beta}-(q^\tau_\mu)_{\alpha\delta}(q^{\tau'}_\mu)_{\alpha\beta}\}\ , \quad (2)$$

$$V^P_{\alpha\beta\gamma\delta}=-\sum_\tau G_\tau\delta_{\alpha\bar\beta}\delta_{\gamma\bar\delta}\ , \quad (3)$$

where $\tau(\tau')$ stands for neutron or proton, χ_τ and G_τ are the strengths of quadrupole and pairing interactions, respectively, and $q_\mu=r^2Y_\mu{}^2$ is the quadrupole operator. The barred state $\bar\beta$ in Eq. (3) is obtained from the state β by the time reversal operator. The HFB equations can be further simplified by neglecting[5] (i) the exchange term in $V^Q_{\alpha\beta\gamma\delta}$, (ii) the contribution of $V^Q_{\alpha\beta\gamma\delta}$ to the pairing potential, and (iii) the contribution of $V^P_{\alpha\beta\gamma\delta}$ to the HF potential. The validity of these approximations has been justified.[7] With these approximations, the HF potential $\Gamma^\tau_{\alpha\gamma}$ and the pairing potential $\Delta^\tau_{\alpha\gamma}$ can now be expressed as

$$\Gamma^\tau_{\alpha\gamma}=-\chi\sum_\mu Q_\mu(\tilde{q}^\tau_\mu)_{\alpha\gamma}\ , \quad (4)$$

where the intrinsic mass quadrupole moment

$$Q_\mu=\sum_{\tau\beta\delta}(\tilde{q}^\tau_\mu)_{\delta\beta}C^{i\tau}_\beta C^{i\tau}_\delta(v_{i\tau})^2 \quad (5)$$

and

$$\Delta^\tau_{\alpha\gamma}=\Delta_\tau\delta_{\gamma\bar\alpha}\ , \quad (6)$$

where the pairing gap

$$\Delta_\tau=G_\tau\sum_i u_{i\tau}v_{i\tau}\ . \quad (7)$$

The strength χ and the operator $\tilde{q}$ in Eq. (4) are related to the corresponding quantities in Eq. (2) by

$$\chi=\chi_\tau(b/\theta_\tau)^4;\ \ \tilde{q}^\tau=q^\tau(\theta\tau/b)^2\ , \quad (8)$$

where

$$b^2=\hbar/m\omega=\tfrac{4}{5}(\tfrac{2}{3})^{1/3}r_0{}^2A^{1/3}\ ,$$
$$\theta_p=(2Z/A)^{1/3};\ \ \theta_n=(2N/A)^{1/3}\ . \quad (9)$$

The coefficients $C^{i\tau}_\alpha$ and the occupation probabilities $v_{i\tau}{}^2$ $(=1-u_{i\tau}{}^2)$ in Eqs. (5) and (7) are to be determined self-consistently by solving the equations

$$\sum_\gamma(T^\tau_{\alpha\gamma}+\Gamma^\tau_{\alpha\gamma}-\lambda_\tau\delta_{\alpha\gamma})C^{i\tau}_\gamma=\eta^\tau_i C^{i\tau}_\alpha\ , \quad (10)$$

$$v_{i\tau}{}^2=\tfrac{1}{2}\{1-[1+(\Delta\tau/\eta^\tau_i)^2]^{-1/2}\}\ . \quad (11)$$

The chemical potentials λ_τ and the pairing gaps Δ_τ are determined from the number conservation condition

$$N_\tau=\tfrac{1}{2}\sum_i\{1-[1+(\Delta_\tau/\eta^\tau_i)^2]^{-1/2}\} \quad (12)$$

and

$$G_\tau{}^{-1}=\tfrac{1}{4}\sum_i[(\eta^\tau_i)^2+\Delta_\tau{}^2]^{-1/2}\ . \quad (13)$$

The HFB energy is then given by

$$E^{\rm HFB}=\sum_{i\tau}T^\tau_{ii}v_{i\tau}{}^2-\tfrac{1}{2}\chi\sum_\mu Q_\mu{}^2-\sum_\tau\Delta_\tau{}^2/G_\tau\ . \quad (14)$$

The HFB calculations in this $Q+P$ model are to be carried out self-consistently to determine the chemical potentials λ_τ, pairing gaps Δ_τ, and intrinsic quadrupole moment Q_μ at the minimum of energy $E^{\rm HFB}$. The equilibrium deformation parameters β_0 and γ_0 of the generalized Nilsson model are[3,4] related to the intrinsic quadrupole moment Q_μ by the relations

$$\chi Q_0=\hbar\omega\beta_0\cos\gamma_0\ ,$$
$$\chi Q_2=\hbar\omega\beta_0\sin\gamma_0\ , \quad (15)$$

where $\hbar\omega$ is given by Eq. (9). The $B(E2)$ values for the γ transitions are computed in a variational projection formalism[6] which determines the wave function ψ^J projected from the intrinsic HFB state corresponding to the minimum in energy $\langle\psi^J|H|\psi^J\rangle$.

The HFB calculations with the $Q+P$ model reported here are carried out in the configuration space of the first seven major shells for both kinds of nucleons to obtain deformation parameters for nuclei in the mass region $146\le A\le 192$. The minimum of energy $E^{\rm HFB}$ and the corresponding parameters β_0 and γ_0 are obtained by solving Eqs. (10)–(15) self-consistently by an iterative procedure. The single particle energies corresponding to the spherical basis states employed in the present calculations are the same as those employed in Refs. 3 and 4. The present realistic calculations in large configuration space show that the strength of the quadrupole interaction required to obtain appropriate equilibrium deformations of nuclei in this mass region depends sensitively on the mixing caused by the quadrupole operator across the major shells. It is necessary to include the mixing across the major shells in order to investigate the renormalization effects caused by the assumption of an inert core. The HFB calculations are, however, substantially involved if one includes the mixing across the major shells since the dimensions of the HFB matrices in Eq. (10) increase. For the configuration space of seven major shells employed in the present calculations, the dimensions of the matrices for both protons and neutrons are 50×50 for positive parity states and 34×34 for negative parity states.

Following Kumar and Baranger[5] we assume the same A dependence of the strengths of the quadrupole and pairing interactions as

$$\chi=\chi_0 A^{-1.4};\quad G_p=G_{p0}A^{-1};\quad G_n=G_{n0}A^{-1}\ .$$

By assuming an inert core of 40 protons and 70 neutrons and the configuration space of $N=4$ and 5 shells for protons and $N=5$ and 6 shells for neutrons, Kumar and Baranger[5] obtain reasonable values for the deformation parameters and the pairing gaps Δ_p and Δ_n by employing $\chi_0=70$ MeV, $G_{p0}=27$ MeV, and $G_{n0}=22$ MeV. The values of these three constants χ_0, G_{p0}, and G_{n0} have to be determined, so as to obtain similar results from the present calculations performed without the assumption of an inert core. We have performed a series of self-consistent calculations to obtain the equilibrium deformation parameters and pairing gaps Δ_p and Δ_n by varying the strengths χ_0, G_{p0}, and G_{n0}. We find that the equilibrium deformation β_0 and both the pairing gaps Δ_p and Δ_n are sensitively dependent on the strength χ_0 of the quadrupole interaction. The variation of G_{p0} and G_{n0} (χ_0 remaining fixed) has a very insignificant effect on the equilibrium deformation parameters. However, the pairing gaps Δ_p and Δ_n increase significantly with the increase in G_{p0} and G_{n0}, respectively. After studying the variations of the equilibrium deformation parameters and pairing gaps, we find that our results obtained with $\chi_0=36.5$ MeV, $G_{p0}=18.5$ MeV,

TABLE I. The equilibrium deformation parameters β_0 and γ_0 (in degrees), the pairing gaps Δ_p and Δ_n, the intrinsic quadrupole moment Q, and the deformation energy ΔE for a few nuclei are tabulated. These results are obtained with the values (in MeV) $\chi_0=36.5$, $G_{p0}=18.5$, and $G_{n0}=16.5$, in the configuration space of $N=0$ to 6 major shells for both protons and neutrons. The values in parentheses are those obtained with the assumption of core and employing the values (in MeV) $\chi_0=70$, $G_{p0}=27$, and $G_{n0}=22$.

Nucleus	β_0	γ_0	Δ_p (MeV)	Δ_n (MeV)	Q (e b)	ΔE (MeV)
^{146}Sm	0.00(0.00)	0.0(0.0)	1.77(1.78)	1.00(0.86)	0.25	0.00
^{154}Sm	0.39(0.31)	0.0(0.0)	1.01(1.01)	1.04(0.90)	8.05	15.03
^{148}Gd	0.00(0.00)	0.0(0.0)	1.77(1.79)	0.97(0.85)	0.26	0.00
^{156}Gd	0.38(0.31)	0.0(0.0)	0.99(1.04)	1.02(0.90)	7.87	14.48
^{160}Gd	0.35(0.33)	0.0(0.0)	0.95(0.94)	0.86(0.84)	7.44	14.02
^{160}Dy	0.36(0.31)	0.0(0.0)	0.89(0.98)	0.91(0.87)	7.70	13.65
^{164}Dy	0.33(0.33)	0.0(0.0)	0.86(0.90)	0.81(0.75)	7.33	13.03
^{164}Er	0.34(0.31)	0.0(0.0)	0.83(0.93)	0.83(0.82)	7.53	12.46
^{168}Er	0.31(0.33)	0.0(0.0)	0.82(0.85)	0.74(0.68)	7.10	11.69
^{166}Yb	0.33(0.30)	0.0(0.0)	0.83(0.94)	0.83(0.82)	7.41	10.98
^{170}Yb	0.30(0.32)	0.0(0.0)	0.82(0.85)	0.73(0.68)	6.94	10.45
^{166}Hf	0.31(0.25)	0.0(0.0)	0.99(1.06)	0.91(0.91)	6.95	8.91
^{170}Hf	0.29(0.29)	0.0(0.0)	0.93(0.96)	0.80(0.76)	6.64	9.01
^{172}W	0.26(0.26)	0.0(0.0)	0.96(0.96)	0.83(0.82)	5.79	7.11
^{176}W	0.25(0.28)	0.0(0.0)	0.89(0.92)	0.73(0.75)	5.74	6.76
^{186}Os	0.17(0.20)	0.0(0.0)	0.73(0.76)	0.75(0.97)	4.08	2.22
^{188}Os	0.15(0.18)	10.0(21.4)	0.75(0.79)	0.76(0.99)	4.10	1.50
^{192}Os	−0.11(−0.15)	0.0(0.0)	0.84(0.85)	0.74(0.93)	−2.65	0.40
^{184}Pt	0.18(0.20)	0.0(0.0)	0.79(0.75)	0.77(0.94)	3.85	2.28
^{188}Pt	0.14(0.17)	10.0(20.2)	0.71(0.67)	0.82(1.05)	3.69	1.15
^{192}Pt	−0.11(−0.14)	0.0(0.0)	0.69(0.68)	0.82(0.82)	−2.51	0.34

TABLE II. As in Table I, these results are obtained with the values $\chi_0=39$ MeV, $G_{p0}=18.5$ MeV, and $G_{n0}=16.5$ MeV in the configuration space of $N=0$ to 5 major shells for protons and $N=0$ to 6 major shells for neutrons with the reduction factor (see text) for the uppermost shell for protons and neutrons.

Nucleus	β_0	γ_0	Δ_p (MeV)	Δ_n (MeV)	Q (e b)	ΔE (MeV)
^{146}Sm	0.00	0.0	1.77	1.00	0.25	0.00
^{148}Gd	0.00	0.0	1.77	0.97	0.26	0.00
^{160}Gd	0.34	0.0	0.96	0.90	6.46	18.88
^{164}Dy	0.32	0.0	0.89	0.86	6.34	17.51
^{168}Er	0.30	0.0	0.83	0.78	6.22	15.89
^{170}Yb	0.29	0.0	0.82	0.78	6.03	14.33
^{170}Hf	0.28	0.0	0.87	0.85	5.68	12.44
^{176}W	0.24	0.0	0.82	0.78	5.01	9.59
^{186}Os	0.17	0.0	0.72	0.79	3.63	3.76
^{188}Os	0.15	10.0	0.75	0.80	3.90	2.50
^{192}Os	−0.11	0.0	0.82	0.73	−2.70	1.14
^{184}Pt	0.18	0.0	0.70	0.82	3.42	3.87
^{188}Pt	0.15	10.0	0.65	0.81	3.32	2.26
^{192}Pt	−0.12	0.0	0.66	0.81	−2.65	1.11

and $G_{n0}=16.5$ MeV are similar to those obtained[5] by assuming an inert core with $\chi_0=70$ MeV, $G_{p0}=27$ MeV, and $G_{n0}=22$ MeV. In Table I, we show our values for the deformation parameters β_0 and γ_0, the pairing gaps Δ_p and Δ_n, the intrinsic quadrupole moment Q obtained from Eq. (5) by summing only over proton states, and the deformation energy ΔE which is the difference between the energies corresponding to the spherical and equilibrium shapes for a number of nuclei in mass region $146 \le A \le 192$.

It should be pointed out here that the earlier results[5] with the assumption of an inert core are obtained by introducing an arbitrary reduction factor to decrease the influence of the uppermost shell ($N=5$ for protons and $N=6$ for neutrons). It may be worthwhile to examine the effect of such a reduction factor on the results of our calculations by introducing the same reduction factor $\tau=(2N+1)/(2N+3)$ for the uppermost shell for protons and neutrons. We find that the introduc-

TABLE III. The $B(E2)$ values (in e^2b^2) for the γ transition $2^+\rightarrow0^+$ and $4^+\rightarrow2^+$ states obtained by employing bare nucleon charges in a configuration space of seven major shells are listed for some typical nuclei.

Nucleus	$B(E2;2^+\rightarrow0^+)$ Calc.	Expt. (Ref. 9)	$B(E2;4^+\rightarrow2^+)$ Calc.	Expt.
^{154}Sm	0.86	0.92±0.04	1.22	
^{156}Gd	0.89	0.93±0.04	1.27	
^{164}Dy	1.07	1.11±0.01	1.53	1.39±0.04
^{166}Er	1.10	1.16±0.04	1.56	
^{168}Yb	1.07	1.09±0.05	1.52	
^{166}Hf	0.75	0.70±0.03	1.07	1.06±0.06
^{170}Hf	0.90	1.01±0.03	1.28	1.43±0.15
^{184}W	0.61	0.73±0.03	0.87	
^{186}Os	0.47	0.58±0.08	0.66	
^{192}Os	0.40	0.42±0.04	0.58	
^{192}Pt	0.34	0.34±0.02	0.48	0.58±0.03
^{196}Pt	0.28	0.30±0.02	0.41	

tion of this reduction factor results in the increase of the value of χ_0 from 36.5 to 39 MeV so as to obtain the same equilibrium deformations over the whole range of nuclei. The results obtained in the configuration space of $N=0$ to 5 shells for protons and $N=0$ to 6 shells for neutrons with the reduction factor τ are displayed in Table II.

The assumption of an inert core necessitates the introduction of effective nucleon charges,[5,8] in order to explain the observed $E2$ transition probabilities. It is, therefore, interesting to see whether $B(E2)$ values can be reproduced by ascribing only bare nucleon charges in large configuration space as employed in the present calculations. We have determined the $B(E2)$ values by employing the bare nucleon charges in large configuration space and the results are shown in Table III for some typical nuclei in the mass region of interest. The values are in good agreement with the experimental data wherever available.

The present results thus indicate that it is possible, within the framework of the $Q+P$ model, to arrive at similar results on equilibrium deformations of the nuclei in the rare earth and transition region by performing calculations with and without the assumption of an inert core. The only difference is in the strength of the quadrupole and pairing interactions to be employed in the two calculations. It is gratifying to note that the observed $E2$ transition probabilities are correctly reproduced by employing the bare charges for all the nucleons in the present calculations in the large configuration space of the first seven major shells.

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