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Excluded bound state in the $S_{1/2}$ α-N interaction and the three-body binding energies of ^{6}He and ^{6}Li

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The effect on the ground-state $A=6$ three-body binding energies of representing the $S_{1/2}$ α-N interaction by a repulsive potential compared to an attractive, excluded-bound-state potential is examined. The theory underlying construction of an attractive, excluded-bound-state potential is reviewed and then applied in the construction of an $S_{1/2}$ α-N interaction. The role of Levinson's theorem from potential theory as opposed to its modified form for composite-particle scattering is stressed in comparing the repulsive and attractive-excluded-bound-state interactions. The $A=6$ three-body equations are generalized to accommodate an attractive $S_{1/2}$ α-N interaction with a forbidden bound state and it is shown that the limit to exclude the forbidden state leads to a set of well-defined three-body equations containing no spurious, deeply-bound solutions. Results for the ^{6}He and ^{6}Li binding energies suggest that the attractive, excluded-bound-state interaction gives a better representation of Pauli-exclusion effects in the $S_{1/2}$ α-N interaction than the repulsive form, based on the marked improvement in the predicted ^{6}He binding energy compared to experiment with essentially no degradation in the ^{6}Li value. This conclusion can be further tested by using the new wave functions to calculate the $^6\text{Li} \to \alpha + d$ momentum distribution which is sensitive to the components in the ^{6}Li wave function that are present solely due to the $S_{1/2}$ α-N interactions.

[NUCLEAR STRUCTURE ^{6}He and ^{6}Li, Pauli-exclusion effects, three-body calculations, Levinson's theorem.]

I. INTRODUCTION

The $A=6$ nuclei, ^{6}He and ^{6}Li, are especially interesting, because their structure appears to be understandable from three-body dynamics; that is, from the dynamics of two nucleons and an alpha particle treated as an elementary particle.[1–4] Such a model also seems to work well for elastic and inelastic alpha-deuteron scattering below the threshold for formation of ^{3}He and ^{3}H in the final state.[5–7] Although the alpha particle is taken to be elementary, its compositeness is imbedded in the alpha-nucleon (α-N) phase shifts that are used to generate α-N potentials for the three-particle dynamics. In particular, the $S_{1/2}$ α-N phase shift has a form that emphasizes that the Pauli exclusion principle plays a role in this wave of the α-N interaction. The $S_{1/2}$ phase shift appears to originate from either a repulsive interaction or an attractive interaction that supports a single bound state. The absence of a five-nucleon bound state in nature has led most investigators to represent the $S_{1/2}$ α-N interaction by a repulsive potential, with the source of the repulsion being the Pauli principle: The fifth nucleon resides outside the filled 1s shells of the alpha particle. Phase shifts, $\delta(E)$, where E is the scattering energy, originating from such repulsive potentials, satisfy Levinson's theorem[8] in the form

$$\delta(0)-\delta(\infty)=0\ . \tag{1}$$

It can be argued that this form of Levinson's theorem is incorrect, since we are dealing with the scattering of composite systems. Specifically, when the Pauli exclusion principle comes into play, Levinson's theorem is modified to

$$\delta(0)-\delta(\infty)=n\pi=(n_e+n_a)\pi\ , \tag{2}$$

where the number of bound states, $n=n_e+n_a$, is the sum of the number of bound states excluded by Pauli's principle and the number of actual bound states.[9,10] The correctness of this form of Levinson's theorem for composite-particle scattering has been demonstrated by Sloan.[11]

Sloan compared the s-wave doublet and quartet phase shifts for neutron-deuteron scattering from an exact three-body calculation (Amado model). In the doublet channel, there is a bound state, ^{3}H, and we expect the phase shift to satisfy Eq. (2) with $n=n_a=1$. Qualitatively, the Pauli exclusion principle is satisfied in the doublet channel by having the two neutrons with *opposite* spins and s-wave relative orbital angular momentum, quite in contrast to the quartet channel where the two neutron spins are necessarily *aligned* forcing p-wave relative orbital angular momentum. Thus, the strong attractive interactions present in the doublet configuration due to primarily s waves for the relative motions are lost in the quartet and the possible bound state is excluded. Sloan's calculations support this picture: Both the quartet and doublet phase shifts satisfy Eq. (2) with $n=1$, where for the former $n=n_e=1$ and for the latter $n=n_a=1$. To back up his calculations and to see more clearly how the results come about, Sloan applies a simple analytically solvable model that has two spin-$\frac{1}{2}$ N particles and an infinitely heavy P particle. The two N particles do not interact when their spins are aligned, but the N-P interaction is strong enough to support a single N-P bound state. There is no bound state of the NNP system for aligned N spins, because the Pauli principle keeps the two N's from occupying the same two-body bound state with the P. Nevertheless, the presence or absence of an N-particle bound to the target P particle makes no observable difference on the scattering of an N particle from the target, thus the modified Levinson theorem is satisfied for N scattering from the bound N-P system. Clearly, Levinson's theorem is modified for composite systems by the Pauli principle and effort should be made to construct $S_{1/2}$ α-N interactions that are used in $A=6$ three-body models that satisfy it in modified form.

The modified form of Levinson's theorem cannot be satisfied by a repulsive potential for the $S_{1/2}$ α-N interaction, but simply resorting to an attractive potential that reproduces the $S_{1/2}$ α-N phase shift leads to a single spurious $A=5$ bound state. Clearly, a projection procedure must be applied that completely removes the spurious $A=5$ bound state without altering the phase shift so that the modified Levinson theorem is satisfied. Several projection methods exist in the current literature,[12,13] two of which are of interest to us here.[14,15] The key element of all projection methods is that the scattering wave function be orthogonal to the excluded state(s) as required by the Pauli principle. The well-known method of Saito[14] implements this requirement explicitly by adding such (a) boundary condition(s) to the Schrödinger equation, thus, in effect, making the Schrödinger equation inhomogeneous. Saito's procedure amounts to constructing a potential that produces excluded states and then eliminating from the potential all terms which project onto the excluded states. Similar in spirit, but different in application, is the pseudopotential method of Krasnopol'skii and Kukulin.[15] Their method adds to the original potential a projector onto the excluded states, each excluded-state term multiplied by a constant parameter. The original potential plus the projector make up the pseudopotential. In the limit that the multiplicative parameters go to infinity, the forbidden states are removed and the allowed states are unchanged. Unlike the Saito method, where the total Green's function has a pole at $E=0$ corresponding to the excluded states, the method of Krasnopol'skii and Kukulin leads to the forbidden states being completely removed from the spectral decomposition of the Green's function.[13] For this reason and the fact that the method of Krasnopol'skii and Kukulin is much easier to implement in practice than that of Saito, we adopt their method to construct an $S_{1/2}$ α-N interaction that satisfies the modified Levinson theorem without introducing a spurious $A=5$ bound state.

The purpose of this paper is to examine the effect on the ^{6}He and ^{6}Li three-body binding energies of using an attractive excluded-bound-state representation of the $S_{1/2}$ α-N interaction compared to the standard repulsive interaction.[16,17] After constructing such an interaction the practical question arises as to implementing the projection limit (parameter to infinity) in the three-body equations. We show that this limit is well-defined in the three-body equations and that all spurious states are removed.[13] The projected equations are solved with the result that an attractive excluded-bound-state representation of the $S_{1/2}$ α-N interaction, relative to the repulsive form, affects ^{6}He and ^{6}Li differently, primarily owing to the difference in size of their respective three-body binding energies.

The structure of the text is as follows: In Sec. II, we review the theory of Krasnopol'skii and Kukulin for projecting excluded states from a two-body problem and apply it to set up an $S_{1/2}$ α-N interaction. Section III contains an illustration of how a projected two-body interaction is implemented in three-body theory and how it modifies our previously derived ^{6}He and ^{6}Li three-body equations.[1,2] Following this development, the results obtained from numerically solving the new equations are compared to our earlier results[1,2] obtained with a repulsive $S_{1/2}$ interaction. These results along with discussion are in Sec. IV. Finally, a summary and conclusion are given in Sec. V.

II. EXCLUDED BOUND STATE IN TWO-BODY PROBLEM

A. Review of theory

In this subsection, we briefly review the pseudopotential projection method of Krasnopol'skii and Kukulin.[15] We limit our discussion to the case of a single forbidden state, $|\varphi_f\rangle$, for simplicity, but it is easy to generalize to n forbidden states. Three aspects are of interest to us: (1) Schrödinger's equation, (2) the full resolvent, and (3) the t matrix.

Let us begin from Schrödinger's equation

$$(H_0+V-E)|\varphi\rangle=0\ , \tag{3}$$

where H_0 is the free two-body Hamiltonian, E is the energy, and V is the two-body interaction that generates the forbidden state. More specifically, we shall assume that V generates one forbidden state, one allowed bound state, and the continuum. The pseudopotential equation is obtained by replacing

$$V\to V+\eta|\varphi_f\rangle\langle\varphi_f|\ , \tag{4}$$

where $|\varphi_f\rangle$ is the forbidden state and η is a parameter that controls the projection of $|\varphi_f\rangle$. The pseudo-Schrödinger's equation is

$$(H_0+V+\eta|\varphi_f\rangle\langle\varphi_f|-\tilde{E})|\tilde{\varphi}\rangle=0\ , \tag{5}$$

where

$$|\tilde{\varphi}_f\rangle=|\varphi_f\rangle \text{ when } \tilde{E}=E_f+\eta\ , \tag{6}$$

$$|\tilde{\varphi}_a\rangle=|\varphi_a\rangle \text{ when } \tilde{E}=E_a<0\ , \tag{7}$$

and

$$|\tilde{\varphi}_c\rangle=|\varphi_c\rangle \text{ when } \tilde{E}=E_c>0\ . \tag{8}$$

$|\varphi_a\rangle$ and $|\varphi_c\rangle$ represent the allow bound state and the continuum states, respectively. It is clear that the allowed bound state and the continuum states appear in the pseudoproblem unchanged from the way in which they appear in the original problem. This means that the scattering phase shifts remain unaltered. In addition, however, through the parameter η, $|\varphi_f\rangle$ can be moved about in the spectrum; for example, it can be made degenerate with $|\varphi_a\rangle$ by choosing $\eta=E_a-E_f$ or it can be imbedded in the continuum by choosing $\eta>-E_f>0$. We shall see that by letting $\eta\to\infty$, $|\varphi_f\rangle$ is completely projected from the problem; that is, the corresponding pole is removed from the t matrix and $|\varphi_f\rangle$ is not part of the spectral resolution of the resolvent.

With the pseudo-Hamiltonian defined as

$$\tilde{H}=H+\eta|\varphi_f\rangle\langle\varphi_f|\ , \tag{9}$$

$$=H_0+V+\eta|\varphi_f\rangle\langle\varphi_f|\ , \tag{10}$$

the pseudoresolvent is

$$\tilde{G}(E)=(E-\tilde{H})^{-1}\ , \tag{11}$$

$$=\frac{|\varphi_f\rangle\langle\varphi_f|}{E-E_f-\eta}+\frac{|\varphi_a\rangle\langle\varphi_a|}{E-E_a}+\int dE'\frac{|\varphi_c(E')\rangle\langle\varphi_c(E')|}{E-E'+i\epsilon}\ , \tag{12}$$

$$=G(E)+\frac{\eta|\varphi_f\rangle\langle\varphi_f|}{(E-E_f)^2\left[1-\dfrac{\eta}{E-E_f}\right]}\ , \tag{13}$$

where

$$G(E)=(E-H)^{-1}\ . \tag{14}$$

The pseudo-t matrix is defined from the pseudoresolvent:

$$\tilde{G}(E)\equiv G_0(E)+G_0(E)\tilde{t}(E)G_0(E)\ , \tag{15}$$

where

$$G_0(E)=(E-H_0)^{-1}\ . \tag{16}$$

Clearly, the pseudo-t matrix can be written as

$$\tilde{t}(E)=-(E-H_0)+(E-H_0)\tilde{G}(E)(E-H_0)\ , \tag{17}$$

$$\equiv t(E)+\Delta t(E)\ , \tag{18}$$

where

$$\Delta t(E)=(E-H_0)\frac{\eta|\varphi_f\rangle\langle\varphi_f|}{(E-E_f)^2\left[1-\dfrac{\eta}{E-E_f}\right]}(E-H_0)\ . \tag{19}$$

For the fully on-shell t matrix ($E\neq E_f$), $\Delta t(E)\equiv 0$ and

$$\tilde{t}(E)=t(E)\ ; \tag{20}$$

the phase shifts remain unaltered. Off shell, $\tilde{t}(E)$ and $t(E)$ differ markedly due to their different singularity structure. Moreover, in the limit $\eta\to\infty$, the pseudoresolvent and pseudo-t matrix become

$$\tilde{G}(E)=G(E)-\frac{|\varphi_f\rangle\langle\varphi_f|}{E-E_f} \tag{21}$$

and

$$\tilde{t}(E)=t(E)-(E-H_0)\frac{|\varphi_f\rangle\langle\varphi_f|}{E-E_f}(E-H_0)\ , \tag{22}$$

respectively. The forbidden state $|\varphi_f\rangle$ has been completely removed from the problem, but Levinson's theorem applies in its modified form, Eq. (2).

B. Application of theory to $S_{1/2}$ α-N interaction

For the $S_{1/2}$ α-N interaction, the theory of subsection A applies without the allowed bound state. We begin with an attractive s-wave separable potential

$$\langle \vec{k}\,|\,V\,|\,\vec{k}'\rangle=-\frac{\Lambda}{2\mu}h(k)h(k')\ , \tag{23}$$

where

$$h(k)=(k^2+\beta^2)^{-1}\ , \tag{24}$$

$\mu=4M/5$, M is the mass of a nucleon, Λ is the interaction strength, and β is the inverse range of the interaction. The parameters Λ and β are determined by fitting the phase-shift parametrization of Arndt and Roper.[18] Their parametrization covers the laboratory-energy range of 0 to 21 MeV. Our values for Λ and β are given in Table I under "attractive" and our fit is compared to the Arndt-Roper parametrization in Fig. 1. These parameters yield a single forbidden bound state at 13.46 MeV which must be excluded in any three-body applications of this interaction. The forbidden-bound-state wave function is

$$\varphi(k)=\frac{Nh(k)}{k^2+\gamma^2}\ , \tag{25}$$

where N is the normalization constant and γ is related to the binding energy by $E_f=5\gamma^2/8M=13.46$ MeV. Therefore, the pseudopotential to be used in

TABLE I. $S_{1/2}$ α-N interaction parameters.

Interaction	Λ (fm^{-3})	β (fm^{-1})	E_f (MeV)	γ (fm^{-1})	N ($\text{fm}^{-5/2}$)
Attractive	0.7277	1.482	13.46	0.7203	1.074
Repulsive	−0.6373	0.7496			

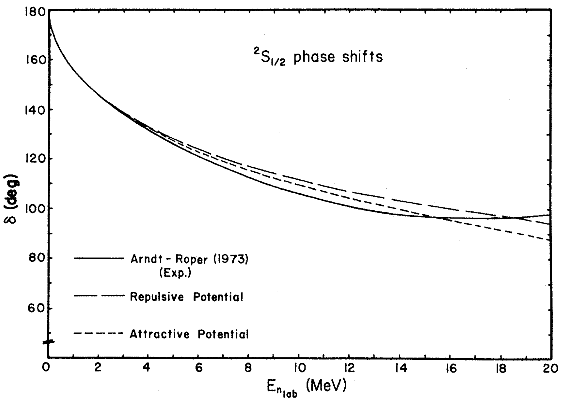

FIG. 1. Comparison of fitted phase shifts to Arndt-Roper phase-shift analysis (Ref. 18).

the three-body problem is

$$\langle \vec{k}\,|\,\tilde{V}\,|\,\vec{k}'\rangle=-\frac{\Lambda}{2\mu}h(k)h(k')+\eta\varphi(k)\varphi(k')\ , \tag{26}$$

where the limit $\eta\to\infty$ is applied to the resultant wave equation.

For comparison purposes, we also use a repulsive $S_{1/2}$ α-N interaction, the same one used in our earlier work.[1,2] The parameters are given in Table I under "repulsive." The low-energy phase shifts predicted by this repulsive interaction are essentially identical to those of the attractive interaction (see Fig. 1). At higher energies, they differ markedly as can be seen in Fig. 2. The repulsive interaction satisfies Levinson's theorem of Eq. (1), but the attractive interaction satisfies the modified form of Levinson's theorem, Eq. (2). However, this is not the only way the two interactions differ. For negative energies that arise in the bound-state solution of

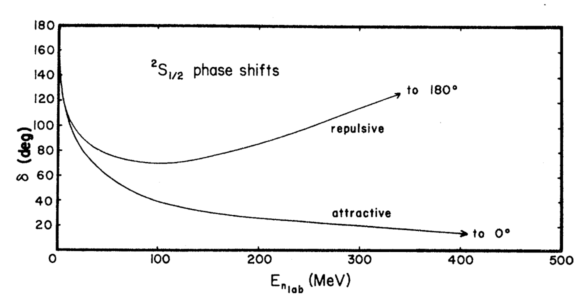

FIG. 2. Comparison of phase shifts from attractive and repulsive $^2S_{1/2}$ potentials beyond the threshold region.

the three-body problem, the off-shell behavior of their respective t matrices is quite different. The t matrix of the repulsive interaction is always positive for fixed k and k', and varies slowly with energy. The only role of the form factors $h(k)$ is to vary the amplitude. In contrast, the fully-projected, attractive-interaction t matrix has more structure. For k and k' small, it is negative, but as k and k' take on larger fixed values, as a function of the absolute energy, it starts out positive and then becomes negative. Thus, it imitates a repulsive interaction over a limited range of its variables, i.e., k and k' large enough and the absolute energy below a certain value.

III. BOUND-STATE THREE-BODY EQUATIONS WITH AN EXCLUDED-BOUND-STATE TWO-BODY INTERACTION

A. Spin independent case

In order to keep the details to a minimum, we consider the simple model problem of three spinless particles, two of them identical of mass M, and a third particle of mass $4M$. The two identical particles will be called the N particles and labeled 1 and 2, while the third particle will be the α particle. All interactions will be assumed representable by s-wave separable potentials and the α-N interaction will possess a single forbidden bound state. Specifically,

$$V_{12}=-\frac{\lambda_{NN}}{2\mu_{NN}}\,|g\rangle_3\,{}_3\langle g| \qquad (27)$$

and

$$V_{ij}=-\frac{\Lambda_{\alpha N}}{2\mu_{\alpha N}}\,|h\rangle_k\,{}_k\langle h|\ ,$$

$$(ij,k=31,2 \text{ or } 23,1)\ . \qquad (28)$$

We write for the pseudopotentials

$$\tilde{V}_{ij}=V_{ij}+\eta\,|\varphi_{ij}\rangle\langle\varphi_{ij}|\ , \qquad (29)$$

where $|\varphi_{ij}\rangle$ represents the forbidden bound state.

We begin from the three-body pseudo-Hamiltonian and write the Schrödinger equation as

$$(H_0+V_{12}+\tilde{V}_{31}+\tilde{V}_{23})\,|\tilde{\Psi}\rangle=-E\,|\tilde{\Psi}\rangle\ , \qquad (30)$$

where H_0 now represents the three-body free Hamiltonian and $E>0$. When Eqs. (27) and (29) are substituted into Eq. (30) spectator functions are defined as

$$G(p)={}_3\langle g\ \vec{p}\,|\tilde{\Psi}\rangle\ , \qquad (31)$$

$$F(p)={}_2\langle h\ \vec{p}\,|\tilde{\Psi}\rangle\ , \qquad (32)$$

$$\tilde{\Phi}(p)={}_2\langle \varphi_{31}\ \vec{p}\,|\tilde{\Psi}\rangle\ , \qquad (33)$$

we are led naturally to the wave function form

$$\tilde{\Psi}(\vec{k},\vec{p})=\Bigg\{\lambda g(k)G(p)+\frac{5\Lambda}{8}[h(k_{31})F(p_2)+h(k_{23})F(p_1)] + M\eta[\varphi(k_{31})\tilde{\Phi}(p_2)+\varphi(k_{23})\tilde{\Phi}(p_1)]\Bigg\}\Big/(k^2+\tfrac{3}{8}p^2+K^2)\ , \qquad (34)$$

symmetric under exchange of particles 1 and 2, and where $K^2=ME$, the subscripts on the lambdas have been suppressed, $\vec{k}$ is the relative momentum of particles 1 and 2, and $\vec{p}$ is the momentum of the third particle relative to the NN center of mass. The vectors $\vec{k}_{ij}$ and $\vec{p}_k$ have analogous meanings and can be expressed in terms of $\vec{k}$ and $\vec{p}$: $\vec{k}_{31}=(-4\vec{k}/5)-(3\vec{p}/5)$, etc. The coupled, homogeneous integral equations obtained from the Schrödinger equation for the spectator functions are

$$(1-\lambda I_{gg}(p))G(p)=\tfrac{5}{4}\Lambda\int d^3k\,A(k,p;\xi)\left[F(k)+\frac{8M\eta}{5\Lambda}\tilde{\Phi}(k)\right]\ , \qquad (35)$$

$$\begin{aligned}\tilde{\mathscr{D}}F(p)=&\lambda\int d^3k\{(1-M\eta I_{\varphi\varphi}(p))A(p,k;\xi)+M\eta I_{h\varphi}(p)\mathscr{A}(p,k;\xi)\}G(k)\\ &+\tfrac{5}{8}\Lambda\int d^3k\{(1-M\eta I_{\varphi\varphi}(p))B(k,p;\xi)+M\eta I_{h\varphi}(p)\mathscr{B}(p,k;\xi)\}F(k)\\ &+M\eta\int d^3k\{(1-M\eta I_{\varphi\varphi}(p))\mathscr{B}(k,p;\xi)+M\eta I_{h\varphi}(p)C(k,p;\xi)\}\tilde{\Phi}(k)\ ,\end{aligned} \qquad (36)$$

and

$$\begin{aligned}\tilde{\mathscr{D}}\tilde{\Phi}(p)=\lambda \int d^3k\{ \tfrac{5}{8}\Lambda I_{\varphi h}(p)A(p,k;\xi)+(1-\tfrac{5}{8}\Lambda I_{hh}(p))\mathscr{A}(p,k;\xi) \}G(k)\\ +\tfrac{5}{8}\Lambda \int d^3k\{ \tfrac{5}{8}\Lambda I_{\varphi h}(p)B(k,p;\xi)+(1-\tfrac{5}{8}\Lambda I_{hh}(p))\mathscr{B}(p,k;\xi) \}F(k)\\ +M\eta \int d^3k\{ \tfrac{5}{8}\Lambda I_{\varphi h}(p)\mathscr{B}(k,p;\xi)+(1-\tfrac{5}{8}\Lambda I_{hh}(p))C(k,p;\xi) \}\tilde{\Phi}(k) ,\end{aligned} \tag{37}$$

where

$$\tilde{\mathscr{D}}=(1-M\eta I_{\varphi\varphi}(p))(1-\tfrac{5}{8}\Lambda I_{hh}(p))-\tfrac{5}{8}\Lambda M\eta I_{h\varphi}(p)I_{\varphi h}(p) , \tag{38}$$

$$I_{gg}(p)=\int d^3k\frac{g^2(k)}{k^2+\frac{3}{8}p^2+K^2} , \tag{39}$$

$$I_{hh}(p)=\int d^3k\frac{h^2(k)}{\frac{5}{8}k^2+\frac{3}{5}p^2+K^2} , \tag{40}$$

$$I_{\varphi\varphi}(p)=\int d^3k\frac{\varphi^2(k)}{\frac{5}{8}k^2+\frac{3}{5}p^2+K^2} , \tag{41}$$

$$I_{h\varphi}(p)=I_{\varphi h}(p)=\int d^3k\frac{h(k)\varphi(k)}{\frac{5}{8}k^2+\frac{3}{5}p^2+K^2} , \tag{42}$$

$$A(k,p;\xi)=\frac{g(|\vec{k}+\frac{1}{2}\vec{p}|)h(|\frac{4}{5}\vec{k}+\vec{p}|)}{k^2+\frac{5}{8}p^2+K^2+\vec{k}\cdot\vec{p}} , \tag{43}$$

$$\mathscr{A}(k,p;\xi)=\frac{\varphi}{h}A(k,p;\xi) , \tag{44}$$

$$B(k,p;\xi)=\frac{h(|\vec{k}+\frac{1}{5}\vec{p}|)h(|\frac{1}{5}\vec{k}+\vec{p}|)}{\frac{5}{8}(k^2+p^2)+K^2+\frac{1}{4}\vec{k}\cdot\vec{p}} , \tag{45}$$

$$\mathscr{B}(k,p;\xi)=\frac{\varphi(|\frac{1}{5}\vec{k}+\vec{p}|)}{h(|\frac{1}{5}\vec{k}+\vec{p}|)}B(k,p;\xi) , \tag{46}$$

$$C(k,p;\xi)=\frac{\varphi}{h}\mathscr{B}(k,p;\xi) , \tag{47}$$

and $\xi=\hat{k}\cdot\hat{p}$. Now, we exclude the forbidden two-body state by taking the limit $\eta\rightarrow\infty$ in the above equations. Note that as $\eta\rightarrow\infty$, $\tilde{\mathscr{D}}$ is directly proportional to η. Thus, from the $\tilde{\Phi}(p)$ equation, we see that $\eta\tilde{\Phi}(p)$ remains finite as $\eta\rightarrow\infty$. Therefore, we define

$$\lim_{\eta\rightarrow\infty} M\eta\tilde{\Phi}(p)=\tfrac{5}{8}\Lambda\Phi(p) , \tag{48}$$

and derive in the $\eta\rightarrow\infty$ limit

$$\begin{aligned}\tilde{\Psi}(\vec{k},\vec{p})\rightarrow\Psi(\vec{k},\vec{p})=\{\lambda g(k)G(p)+\tfrac{5}{8}\Lambda[h(k_{31})F(p_2)+h(k_{23})F(p_1)]\\ +\tfrac{5}{8}\Lambda[\varphi(k_{31})\Phi(p_2)+\varphi(k_{23})\Phi(p_1)]\}/(k^2+\tfrac{3}{8}p^2+K^2) ,\end{aligned} \tag{49}$$

where

$$(1-\lambda I_{gg}(p))G(p)=\tfrac{5}{4}\Lambda\int d^3k\, A(k,p;\xi)(F(k)+\Phi(k)) , \tag{50}$$

$$\begin{aligned}\mathscr{D}F(p)=\lambda\int d^3k\{ I_{\varphi\varphi}(p)A(p,k;\xi)-I_{h\varphi}(p)\mathscr{A}(p,k;\xi) \}G(k)\\ +\tfrac{5}{8}\Lambda\int d^3k\{ I_{\varphi\varphi}(p)B(k,p;\xi)-I_{h\varphi}(p)\mathscr{B}(p,k;\xi) \}F(k)\\ +\tfrac{5}{8}\Lambda\int d^3k\{I_{\varphi\varphi}(p)\mathscr{B}(k,p;\xi)-I_{h\varphi}(p)C(k,p;\xi)\}\Phi(k) ,\end{aligned} \tag{51}$$

$$\mathscr{D}\Phi(p)=-\frac{8\lambda}{5\Lambda}\int d^3k\{\tfrac{5}{8}\Lambda I_{h\varphi}(p)A(p,k;\xi)+(1-\tfrac{5}{8}\Lambda I_{hh}(p))\mathscr{A}(p,k;\xi)\}G(k)$$

$$-\int d^3k\{\tfrac{5}{8}\Lambda I_{h\varphi}(p)B(k,p;\xi)+(1-\tfrac{5}{8}\Lambda I_{hh}(p))\mathscr{B}(p,k;\xi)\}F(k) \qquad (52)$$

$$-\int d^3k\{\tfrac{5}{8}\Lambda I_{h\varphi}(p)\mathscr{B}(k,p;\xi)+(1-\tfrac{5}{8}\Lambda I_{hh}(p))C(k,p;\xi)\}\Phi(k)\,,$$

$$\mathscr{D}=I_{\varphi\varphi}(1-\tfrac{5}{8}\Lambda I_{hh})+\tfrac{5}{8}\Lambda I_{h\varphi}{}^2\,. \qquad (53)$$

To summarize, we began from the three-body pseudo-Hamiltonian and derived the unprojected three-body bound-state equations. Then, we showed that the projection limit ($\eta\to\infty$) is well defined[13] and proceeded to derive the fully projected three-body equations, the latter being the basis for the work of this paper.

When one compares Eqs. (49)–(53) with those where the α-N interaction is represented by a one-term repulsive potential, the striking difference is the new spectator function $\Phi(p)$. The presence of this function adds extra terms to Ψ and leads to an extra integral equation.

Finally, it is not difficult to show that any contamination due to the forbidden state $|\varphi\rangle$ has been completely removed from Ψ. Specifically, we can prove that

$$\lim_{\eta\to\infty}{}_1\langle\varphi\,|\,\widetilde{\Psi}\rangle\to{}_1\langle\varphi\,|\,\Psi\rangle\equiv0\,. \qquad (54)$$

B. Application to ^{6}He and ^{6}Li

Once Eqs. (49)–(53) are at hand, it is straightforward to modify the equations of Ref. 1 (^{6}He) and Ref. 2 (^{6}Li) for the forbidden-state studies of this paper. Each wave function now has an extra term for *each* $S_{1/2}$ spectator function due to the presence of $\Phi(p)$ as in Eq. (49). Thus, the number of coupled equations to be solved increases from 4 to 5 for ^{6}He and from 9 to 11 for ^{6}Li. Owing to space limitations, we leave the details to the reader.

IV. BINDING ENERGY RESULTS

All the results discussed in this section are based on the N-N interactions and the P wave, form B α-N interactions described in Refs. 1 and 2. The reader will find all details about parameters and numerical methods in those papers. Only the $S_{1/2}$ α-N interaction is changed in the present work as described above. However, we do use the Shanley repulsive $S_{1/2}$ α-N interaction as a comparison.[5] Results quoted in this paper for that interaction are taken from Refs. 1 and 2.

Our results are displayed in Tables II and III. The most distinct feature is that the attractive (bound state excluded) interaction increases the binding energy of ^{6}He by $\lesssim 0.2$ MeV, but decreases the binding of ^{6}Li by ~ 0.16 MeV, compared to the repulsive interaction results. Qualitatively, this can be interpreted as a binding energy effect. The loosely bound ^{6}He nucleus is such that, on the average, the α-N pairs in the $S_{1/2}$ wave are far enough apart to experience more of the attractive part of the $S_{1/2}$ interaction (recall discussion at the end of Sec. II). On the other hand, the more tightly bound ^{6}Li nucleus is such that, on the average, the α-N pairs are closer together in the $S_{1/2}$ wave and experience more of the repulsion (due to the Pauli principle) that is present in the interaction. Percentage wise, the effect is much larger for ^{6}He (owing to the small-size of the binding energy) than for

TABLE II. ^{6}He binding energies.

$S_{1/2}$ interaction	N-N interaction	E (MeV)
Attractive (bound state excluded)	n-p best fit	0.735
Attractive (bound state excluded)	n-n best fit	0.556
Repulsive	n-p best fit	0.542
Repulsive	n-n best fit	0.359
Attractive (bound state *not* excluded, i.e., $\eta=0$)	n-p best fit	ground 42.3 excited 0.680
Attractive (bound state *not* excluded i.e., $\eta=0$)	n-n best fit	ground 41.5 excited 0.497
	Experiment	0.969

TABLE III. ^{6}Li binding energies.

$S_{1/2}$ interaction	N-N interaction	E (MeV)
Attractive (bound state excluded)	0%[a]	4.294
Attractive (bound state excluded	4%	3.903
Repulsive	0%	4.446
Repulsive	4%	4.070
Attractive (bound state *not* excluded, i.e., $\eta=0$)	0%	ground 55.6 excited 3.222
Attractive (bound state *not* excluded, i.e., $\eta=0$)	4%	ground 53.0 excited 3.098
	Experiment	4.53

[a]Percentage D-state component in the deuteron for the $^3S_1-{}^3D_1$ NN interaction.

^{6}Li; + 36% (np) or + 55% (nn) for ^{6}He and −3% (0%) or −4% (4%) for ^{6}Li.

An interesting question concerns what happens when the bound state is *not* excluded from the attractive interaction. For both ^{6}He and ^{6}Li, we find a deeply bound ground state of the order of 50 MeV and an excited state located close to the predicted ground states for the case where the forbidden state *is* excluded. It appears that the projection removes the spurious, deeply-bound ground state, while the excited state becomes the real ground state after a slight shift in value. A puzzling question is the reason for this slight shift in energy from the excited state to the ground state after projection. The answer probably resides in the different off-shell behaviors of the two-body interactions, but we have been unable to construct a convincing argument.

Does the use of an attractive excluded-bound-state $S_{1/2}$ α-N interaction alter the calculated spectator functions significantly compared to the repulsive form? Except for the $S_{1/2}$ spectator functions, the other spectator functions change only slightly. The $S_{1/2}$ spectator functions have changed dramatically compared to the repulsive case due to the presence of the $\Phi(p)$ term(s). The net effect of this restructuring of the $S_{1/2}$ component of the wave functions is not evident by simply examining the tabulated results, but must await testing of the wave functions by calculating ^{6}He β decay[3] and especially the $^6\text{Li}\rightarrow\alpha+d$ momentum distribution[4] which is sensitive to the $S_{1/2}$ component. Such work should also help decide whether a repulsive or attractive excluded-bound-state interaction gives the better representation of the Pauli principle in the α-N $S_{1/2}$ interaction.

V. SUMMARY AND CONCLUSION

The question addressed in this paper concerns the most appropriate representation of the Pauli-exclusion effects in the $S_{1/2}$ alpha-nucleon interaction: Are these effects best represented by a purely repulsive interaction or by an attractive, excluded-bound-state interaction? To shed light on this question, we compared the predicted three-body binding energies of ^{6}He and ^{6}Li from these two types of $S_{1/2}$ α-N interactions. Both interactions used give equally good fits to the low-energy (0 to 21 MeV laboratory energy) phase shifts, but they differ at higher energies. The repulsive-interaction's phase shifts satisfy Levinson's theorem in standard potential theory form, $\delta(0)-\delta(\infty)=0$, whereas the attractive, excluded-bound-state interaction yields phase shifts that satisfy the modified Levinson theorem, $\delta(0)-\delta(\infty)=n_e\pi=\pi$, where n_e represents the number of excluded bound states. We found that the attractive, excluded-bound-state interaction predicts $\sim$0.2 MeV *more* binding for ^{6}He and $\sim$0.16 MeV *less* binding for ^{6}Li compared to the repulsive interaction. These results are interpreted in terms of the absolute (theoretical) binding energies of the two nuclei: $\sim$0.5 MeV for ^{6}He and $\sim$4.0 MeV for ^{6}Li. The $S_{1/2}$ α-N pairs in ^{6}He are far enough apart on the average to experience mainly the attractive aspects of the attractive, excluded-bound-state interaction; whereas, just the opposite is true for ^{6}Li. In ^{6}Li, the $S_{1/2}$ α-N pairs experience primarily the Pauli repulsion present in the attractive interaction owing to the complete removal of the forbidden state. Besides the binding-energy results, we also noted that only the components of the ^{6}He and ^{6}Li wave functions which are directly associated with the $S_{1/2}$ α-N interaction changed appreciably. Thus, we conclude from the $A=6$ ground-state binding energies that the attractive, excluded-bound-state $S_{1/2}$ α-N interaction gives a better representation of Pauli effects than the repulsive interaction, since the binding of ^{6}He is increased by $\sim$40% to give better agreement with the experimental value while only shifting the binding of ^{6}Li away from experiment by $\lesssim 4\%$. Nevertheless, future work involving the wave functions e.g., the $^6\text{Li}\to\alpha+d$ momentum distribution which is particularly sensitive to the α-N $S_{1/2}$ interaction, should prove more decisive in distinguishing between the two forms of interaction.

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