PHYSICAL REVIEW C VOLUME 25, NUMBER 6 JUNE 1982

Deep-lying hole states in the optical model

S. P. Klevansky and R. H. Lemmer
Department of Physics, and Nuclear Physics Research Unit, University of the Witwatersrand, Johannesburg, 2001 South Africa
(Received 20 January 1982)

The strength function for deep-lying hole states in an optical potential is studied by the method of Green's functions. The role of isospin is emphasized. It is shown that, while the main trends of the experimental data on hole states in isotopes of Sn and Pd can be described by an energy independent optical potential, intermediate structures in these data indicate the specific nuclear polarization effects have to be included. This is done by introducing doorway states of good isospin into the optical model potential. Such states consist of neutron hole plus proton core vibrations as well as more complicated excitations that are analog states of proton hole plus neutron core vibrations of the parent nuclear system. Specific calculations for ^{115}Sn and ^{103}Pd give satisfactory fits to the strength function data using optical model and doorway state parameters that are reasonable on physical grounds.

[NUCLEAR STRUCTURE ^{115}Sn, ^{103}Pd; calculated single hole strength functions. Green's function method for optical potential, intermediate structure doorway states. Good isospin.]

I. INTRODUCTION

Deep-lying hole states in nuclei have been the subject of several recent experimental[1,2] and theoretical[3,4] studies. In the last few years, neutron pickup experiments for the tin and palladium isotopes have been extensively studied using (p,d) and $(^3\mathrm{He},\alpha)$ reactions, and the fragmentation of the $1g_{9/2}$ inner-hole strength is clearly observed in these data. To date, a full theoretical understanding of this strength function data is still lacking.

In this paper we present a method of calculating a strength function from an optical model point of view, as opposed to the previous nuclear structure calculations.[1,3] We regard the neutron hole as moving in an optical potential at negative energies. In its simplest form, this model depicts the observed gross structure in the data as a giant resonance in the optical potential.

In general, one knows that the optical potential seen by a nucleon in a nucleus, i.e., its self-energy $\Sigma(\omega)$, is energy dependent. $\Sigma(\omega)$ is obtained by energy averaging the nuclear polarization caused by the nucleon motion, and thus the energy dependence is associated with an underlying nuclear structure problem. If the nuclear states are essentially excited with random strength, the resulting energy variation of the nucleon self-energy is a smooth one; however, if doorway states play an important role in regulating these excitations, one expects energy variations in $\Sigma(\omega)$ over an energy range that is small compared with the giant resonance width. Such an energy dependence, which gives rise to intermediate structure in the strength function, represents a nuclear polarization involving select excitations that are strongly coupled to the nucleon state. It is clear from the structure inherent in the data that doorway mechanisms *are* present, and it has been suggested by Koeling and Iachello[3] that these mechanisms are provided by prominent vibrational states of the core. Therefore, by including an energy dependence characteristic of the doorways involved for the particular nuclear system, we obtain a natural description of the intermediate structure resonances observed.

The strength function to be compared with experiment is obtained directly from the method of Green's functions.[5] In this connection, the representation of the single particle Green's operator $G(\omega)$ has to be specified. The appropriate representation for our problem is given by eigenstates of the average nuclear potential piece of $\Sigma(\omega)$ that does not involve any polarization of the medium. Since we are dealing with a finite system, $G(\omega)$ is not di-

agonal in this representation. It thus proves convenient to obtain its matrix elements from equations that are reminiscent of the two-potential problem in scattering theory,[6] with the polarization part of $\Sigma(\omega)$ playing the role of the second potential. This method for constructing the Green's function is developed in Sec. II. While the procedure is quite general, it can only be implemented once assumptions are introduced about the behavior of $\Sigma(\omega)$ in coordinate space. Analytic results for a local, complex well form for $\Sigma(\omega)$ are given in Sec. III. This approximation, which is not too bad for deep-lying hole states, also indicates what form calculations using more realistic well shapes would take.

A problem of considerable importance is the isospin character of hole states formed during neutron pickup. Assuming that nuclei carry good isospin, one knows that pickup from states occupied by the $2T_0=(N-Z)$ excess neutrons produce states of unique isospin. However, this is not the case for deep lying neutrons, whose removal excites both analog and antianalog states of isospin $T=T_0\pm\frac{1}{2}$ in the residual nucleus. The role of isospin is discussed in Sec. IV in terms of neutron quasihole states of good isospin, in an effective single particle potential of the type proposed by Lane and Soper.[7]

Section V presents specific calculations for the ^{115}Sn and ^{103}Pd systems for which detailed strength function data relating to the $1g_{9/2}$ shell model state are available.[1,2] It is shown that the complex square well potential model of $\Sigma(\omega)$ is able to reproduce the gross features of these data satisfactorily. The question of intermediate structure is also discussed in terms of doorway states of good isospin that consist of coupling neutron and proton holes in ^{116}Sn to the collective vibrations of its proton and excess neutron shells, the latter coupling providing for *parent* doorways in ^{115}In, whose analogs give doorway states in ^{115}Sn. These calculations are schematic to the extent that the doorway widths and coupling strengths to the quasihole states have been regarded as adjustable parameters. However, the satisfactory description of the experimental data that results from using parameters whose values are also adjudged to be reasonable on physical grounds suggests that the main features of a more realistic calculation are already present in the model.

II. THE GREEN'S FUNCTION FOR HOLES

As outlined in the Introduction, we want to examine the spreading of hole states in nuclei, interpreted as giant resonances at negative energies in the nuclear optical model potential.

The optical model problem arises by considering the propagation of a particle or hole that has been added to an A-nucleon target system. By energy averaging over the many excited states of the $A\pm1$ compound systems that result, one retrieves an effective single-particle wave equation for the added particle or hole that has the general form[8]

$$[E-T-\Sigma(E)]\phi_E=0\ , \tag{2.1}$$

with T being the kinetic energy. The self-energy $\Sigma(E)$ describes the effects of the nuclear medium on the additional particle or hole. A formal theory of deriving $\Sigma(E)$ from first principles has been developed by many authors.[8,9] For our purposes, we merely note that $\Sigma(E)$ is, in general, nonlocal in the coordinates of the nucleon, in addition to depending on the energy E, and is complex:

$$\Sigma(\vec{r},\vec{r}',E)=V(\vec{r},\vec{r}',E)+iW(\vec{r},\vec{r}',E)\ . \tag{2.2}$$

While the result (2.2) for Σ is formally quite simple, actual calculations of the self-energy have until quite recently not received serious attention.[9] On the other hand, the many successes[10] of the optical model, using phenomenological *local* potentials to mimic Eq. (2.2), are by now well established for a variety of targets and projectiles. Moreover, recent approximate calculations[9] of the real and imaginary parts of Σ, using a parametrized form of the two-nucleon scattering matrices, have been surprisingly successful, both in fitting scattering data, as well as in providing a theoretical underpinning for the phenomenological approach. Thus, except for certain theoretical developments to follow, where the precise form of Σ will not matter, actual calculations in this paper will employ a local approximation for Σ. In fact, we take the approximation one step further for hole states, and employ square well potentials, because these can be handled analytically for arbitrary angular momenta. Later studies will employ more realistic potential shapes. The theoretical estimates of Σ referred to above, however, suggest that a constant potential in the nuclear interior (which determines the properties of deep hole states) is not unreasonable.

A direct access to the giant resonance character of the single particle or hole states in the complex potential Σ is provided by the Green's operator $G(\omega)$ associated with Eq. (2.1)

$$[\omega-T-\Sigma(\omega)]G(\omega)=1\ . \tag{2.3}$$

Here ω is an energy variable that ranges over the excited states of the $A-1$ system, with $\omega=0$ corre-

sponding to the ground state of the target system.

The distribution of strength of such giant resonances is given explicitly by the imaginary part of $G(\omega)$. Our interest is in the spreading of the hole state introduced by the imaginary part of Σ. We, therefore, consider $G(\omega)$ in the representation provided by the eigenstates of Eq. (2.1) with the imaginary part of Σ suppressed. Call these eigenstates ψ_n. Such eigenstates thus refer to single particle levels in a real potential well that are occupied in the target system. Removing a particle from the system creates an excited hole state that is then spread over the excited states of the $A-1$ system in a manner described by $\mathrm{Im}G_{nn}(\omega)$, where

$$G_{nn}(\omega)=\langle\psi_n\,|\,G(\omega)\,|\,\psi_n\rangle\ .$$

Since the Green's operator $G(\omega)$ is not diagonal in the representation ψ_n, Eq. (2.3) is not useful as it stands for calculating $G_{nn}(\omega)$. A convenient procedure is to split off the imaginary part of the self-energy operator explicitly, $\Sigma=V+iW$, and then express $G_{nn}(\omega)$ in a form reminiscent of the scattering by two potentials. Call $-\epsilon_n$, where $\epsilon_n>0$, the bound eigenenergies of the single particle Hamiltonian $H_1=T+V$, with eigenstates ψ_n. Then, using the standard operator identity

$$(\omega-H)^{-1}=(\omega-H_1)^{-1}+(\omega-H_1)^{-1}iW(\omega-H)^{-1}\ , \tag{2.4}$$

where

$$H=T+\Sigma=H_1+iW\ ,$$

one finds, by inverting Eq. (2.3), that the matrix elements $G_{n'n}(\omega)$ of $G(\omega)$ in the ψ_n basis satisfy

$$G_{n'n}(\omega)=(\omega+\epsilon_n)^{-1}\delta_{n'n}+(\omega+\epsilon_{n'})^{-1}\langle\psi_{n'}\,|\,iW(\omega-H)^{-1}\,|\,\psi_n\rangle\ . \tag{2.5}$$

Multiplying this equation by $(\omega+\epsilon_n)$, one obtains

$$(\omega+\epsilon_n)G_{n'n}(\omega)=\delta_{n'n}+(\omega+\epsilon_{n'})^{-1}\times\langle\psi_{n'}\,|\,iW\,|\,\Phi_n(\omega)\rangle\ , \tag{2.6}$$

where we have introduced the auxiliary vector

$$\Phi_n(\omega)=[1+(\omega-H)^{-1}iW]\psi_n=\psi_n+(\omega-H_1)^{-1}iW\Phi_n(\omega)\ . \tag{2.7}$$

Note that $\Phi_n(\omega)$ is offshell to the extent that $\omega\neq-\epsilon_n$, the energy of the "incident" state ψ_n, to use the analogy of scattering by two potentials, V and iW.

It is a simple matter to reexpress $\Phi_n(\omega)$ in terms of the operator

$$t(\omega)=iW+iW(\omega-H_1)^{-1}t(\omega)\ , \tag{2.8}$$

which might be called a t matrix due to the distorting field iW. The associated Möller operator $\Omega(\omega)$ connects $t(\omega)$ to iW, and $\Phi_n(\omega)$ to ψ_n

$$t(\omega)=iW\Omega(\omega) \tag{2.9a}$$

and

$$\Phi_n(\omega)=\Omega(\omega)\psi_n\ , \tag{2.9b}$$

which shows that the last term in Eq. (2.6) is just $t_{n'n}(\omega)$. Our final expression for $G_{n'n}(\omega)$ is thus given directly in terms of $t_{n'n}(\omega)$ as

$$(\omega+\epsilon_n)G_{n'n}(\omega)=\delta_{n'n}+(\omega+\epsilon_{n'})^{-1}t_{n'n}(\omega)\ . \tag{2.10}$$

This result will prove to be a very convenient way of actually constructing the $G_{n'n}(\omega)$ required for the strength function calculations in the next section.

We now specialize the indices in Eq. (2.10) to apply to a hole state $(n\ell j)^{-1}$ that is created in an even A target by removing a particle from an occupied state $(n\ell j)$ of V, where ℓ and j are orbital and total angular momenta, and n refers to the radial quantum number. Using this representation, $t_{n'n}(\omega)$ gets replaced by

$$t_{\ell j}(n',n,\omega)=\langle u_{n'\ell j}\,|\,iW\,|\,\phi_{n\ell j}(\omega)\rangle\ , \tag{2.11}$$

where $r^{-1}u_{n\ell j}(r)$ and $r^{-1}\phi_{n\ell j}(\omega,r)$ are the radial parts of ψ_n and $\Phi_n(\omega)$. The corresponding Green's function generated by $t_\ell(n',n,\omega)$ is $G_{\ell j}(n',n,\omega)$ and Eq. (2.10) still relates them, with ϵ_n becoming $\epsilon_{n\ell j}$. Equation (2.11) assumes that W cannot change the orbital or spin angular momentum of the particle.

III. THE STRENGTH FUNCTION FOR A COMPLEX SQUARE WELL

It is clear from Eqs. (2.10) and (2.11) that it is computationally advantageous to construct $G_{\ell j}(n',n,\omega)$ via $t_{\ell j}(n',n,\omega)$ since the wave functions entering the latter need only be known over the nuclear interior. This would be especially important where these wave functions can only be obtained numerically, as for example, in nuclear Hartree-Fock theory. In this section we construct $t_{nn}(\omega)=t_\ell(n,n,\omega)$ for a spinless hole state $\psi_{n\ell}=r^{-1}u_{n\ell}(r)Y_{\ell m}$ of angular momentum ℓ. For this we require the radial part of $\Phi_n(\omega,r)$

$=r^{-1}\phi_{n\ell}Y_{\ell m}$ in Eq. (2.7).

$$\phi_{n\ell}(\omega,r)=u_{n\ell}(r)+\int_0^\infty dr'dr''g_\ell(r,r',\omega)iW \times(r',r'',\omega)u_{n\ell}(r'') , \quad (3.1)$$

where $g_\ell(r,r',\omega)$ is the radial part of the full Green's operator, $G(\omega)=(\omega-T-\Sigma)^{-1}$. In Eq. (3.1), outgoing wave boundary conditions are required for ω positive and real, so $g_\ell(r,r',\omega)$ has the familiar structure (we have set $\hbar^2/2m=1$)

$$g_l(r_<,r_>,\omega)=(-)^{\ell+1}\phi_\ell(\sqrt{\omega},r_<)f_\ell \times(-\sqrt{\omega},r_>)/\sqrt{\omega} , \quad (3.2)$$

with

$$f_\ell(-\sqrt{\omega},r)\to\exp i\,(\sqrt{\omega}r-\tfrac{1}{2}\pi\ell)$$

for large r. This asymptotic form is also suitable for negative ω if the square root is taken as $i\sqrt{|\omega|}$, $\omega<0$, which ensures that g_ℓ remains bounded. Here ϕ_ℓ and f_ℓ are regular and irregular solutions of the homogeneous wave equation associated with g_ℓ. The former is given in terms of the latter and its linearly independent partner $f_\ell(\sqrt{\omega},r)$ by

$$\phi_\ell(\sqrt{\omega},r)=i\{f_\ell(-\sqrt{\omega})f_\ell(\sqrt{\omega},r)-f_\ell(\sqrt{\omega}) \times f_\ell(-\sqrt{\omega},r)\}/2f_\ell(-\sqrt{\omega}) , \quad (3.3)$$

where $f_\ell(\sqrt{\omega},0)=f_\ell(\sqrt{\omega})$ defines the Jost function.[11]

The solutions $f_\ell(\pm\sqrt{\omega},r)$ are easily obtainable analytically in terms of spherical Bessel and spherical Hankel functions of the second kind, $j_\ell(x)$ and $h_\ell^{(2)}(x)$; if Σ is approximated by a complex square well, $\Sigma=-V_0+iW_0$ for $r<R$, and $\Sigma=0$ otherwise. (Recall that $W_0>0$ for holes.) The resulting expression is

$$f_\ell(-\sqrt{\omega})\equiv f_\ell(-\sqrt{\omega},Q)=(-)^\ell\left[\frac{\sqrt{\omega}}{Q}\right]^\ell i\sqrt{\omega}Rj_\ell(QR)h_\ell^{(2)}(-\sqrt{\omega}R)\,[D_\ell(Q)-\Delta_\ell(-\sqrt{\omega})+is_\ell(-\sqrt{\omega})] \quad (3.4)$$

for the Jost function. Here $Q=(\omega+V_0-iW_0)^{1/2}$ is the complex wave number inside the well, while $D_\ell(Q)$ and $\Delta_\ell(-\sqrt{\omega})-is_\ell(-\sqrt{\omega})$ give the logarithmic derivatives of $xj_\ell(x)$ and $xh_\ell^{(2)}$ at $x=QR$ and $-\sqrt{\omega}R$, respectively. For ω on the energy shell, $\omega=k^2$, the Δ_ℓ and s_ℓ are the familiar shift and penetration factors for the scattering of a particle of wave number k. For negative ω the factor $s_\ell=0$.

It is now a simple matter to construct $\phi_{n\ell}(\omega,r)$ in terms of g_ℓ and the wave function $u_{n\ell}$, which is given by

$$u_{n\ell}=A_{n\ell}(K_nr)j_\ell(K_nr)$$

inside the real well. Then, for $r<R$

$$\phi_{n\ell}(\omega,r)=A_{n\ell}(K_nr)j_\ell(K_nr)-\frac{A_{n\ell}(K_nr)iW_0}{\omega+\epsilon_{n\ell}-iW_0}\left[\left[\frac{Q}{K_n}\right]^{l+1}\frac{f_\ell(-\sqrt{\omega},K_n)}{f_\ell(-\sqrt{\omega},Q)}\,Qrj_\ell(Qr)-K_nrj_\ell(K_nr)\right] , \quad (3.5)$$

where $-\epsilon_{n\ell}$ is the binding energy of the state $n\ell$ in the real square well, and $K_n=(-\epsilon_{n\ell}+V_0)^{1/2}$ is the associated wave number. As the notation suggests, $f_\ell(-\sqrt{\omega},K_n)$ is given by Eq. (3.4) with Q replaced by K_n. The $\epsilon_{n\ell}$ are given by the roots of the transcendental equation

$$f_\ell(-i\sqrt{\epsilon_{n\ell}})=0=D_\ell(K_n)-\Delta_\ell(-i\sqrt{\epsilon_{n\ell}}) .$$

$A_{n\ell}$ is a normalization constant.

The calculation of $t_\ell(n,n,\omega)\equiv t_\ell(\omega)$ now proceeds via Eqs. (3.5) and (2.11) to yield

$$t_\ell(\omega)=iW_0\frac{|A_{n\ell}|^2}{K_n}\left\{\tfrac{1}{2}(K_nR)^3\left[j_\ell^2(K_nR)-j_{\ell+1}(K_nR)j_{\ell-1}(K_nR)\right]\frac{\omega+\epsilon_{n\ell}}{\omega+\epsilon_{n\ell}-iW_0}\right.$$
$$\left.-\left[\frac{K_n}{Q}\right]^\ell(K_nR)j_\ell(K_nR)j_\ell(QR)[D_\ell(Q)-D_\ell(K_n)]\frac{f_\ell(-\sqrt{\omega},K_n)iW_0(V_0-\epsilon_{n\ell})}{f_\ell(-\sqrt{\omega},Q)(\omega+\epsilon_{n\ell}-iW_0)^2}\right\} , \quad (3.6)$$

where the normalization constant is given by the relation

$$-|A_{n\ell}|^2 V_0 R (K_n R)^2 j_{\ell+1}(K_n R) j_{\ell-1}(K_n R)/2\epsilon_{n\ell}=1 \ . \tag{3.7}$$

The Jost functions appearing in Eq. (3.6) are given explicitly by Eq. (3.4).

While somewhat complicated in appearance, Eq. (3.6) is an exact expression for $t_\ell(\omega)$ in closed form. The associated Green's function is, therefore, also known exactly. The singularities of $t_\ell(\omega)$, and, therefore, of $G_\ell(\omega)=G_\ell(n,n,\omega)$, can be read off directly. These are given by the zeroes of the Jost function $f_\ell(-\sqrt{\omega},Q)$ associated with the complex square well potential. Note that $t_\ell(\omega)$ is not singular at $\omega=-\epsilon_{n\ell}+iW_0$, and $G_\ell(\omega)$ is not singular at $\omega=-\epsilon_{n\ell}$ of Eqs. (3.6) and (2.10) might suggest. This can be seen by expanding the logarithmic derivatives in Eq. (3.6) about the above two values of ω where

$$[D_\ell(Q)-D_\ell(K_n)]$$

and

$$[D_\ell(K_n)-\Delta_\ell(-\sqrt{\omega})+i s_\ell(-\sqrt{\omega})]$$

vanish, respectively. Then, making use of the relation

$$f_\ell{}^2 \frac{d}{dx}\left[x\frac{d}{dx}(\ln f_\ell)\right]=-\frac{1}{x}[f_\ell{}^2-f_{\ell-1}f_{\ell+1}] \ , \tag{3.8}$$

where $f_\ell(x)$ is any spherical Bessel function, one obtains the approximate form

$$t_\ell(\omega)\cong i\gamma_{n\ell}(\omega+\epsilon_{n\ell})(\omega+\epsilon_{n\ell}-i\gamma_{n\ell})^{-1} \ , \tag{3.9}$$

for $t_l(\omega)$, leading to

$$G_\ell(\omega)\cong(\omega+\epsilon_{n\ell}-i\gamma_{n\ell})^{-1} \ , \tag{3.10}$$

for the Green's function. In these expressions $\gamma_{n\ell}=\alpha_{n\ell}W_0$, where $\alpha_{n\ell}$ is the probability of finding a particle inside the square well in state $n\ell$. The strength function, given by $1/\pi \mathrm{Im} G_\ell(\omega)$, then shows a Breit-Wigner resonance at $-\epsilon_{n\ell}$ of half width $\gamma_{n\ell}$, and total strength unity. The form of $G_\ell(\omega)$ given above, which is exact for infinite systems, is an approximation for the finite potential well case that involves replacing the self-energy contribution from the absorptive potential by its average value $i\langle\psi_{n\ell}|W|\psi_{n\ell}\rangle=i\alpha_{n\ell}W_0$.

IV. QUASIHOLE STATES OF GOOD ISOSPIN

We now improve on the assumption that a hole state carries neither spin nor isospin, by including these two degrees of freedom. Both effects can be incorporated in the square well potential model by making the well depth spin and isospin dependent, $V_0\equiv V_{\ell jT}$, where $j=\ell\pm\frac{1}{2}$, and $T=T_0\pm\frac{1}{2}$, where $2T_0=(N-Z)$ is the neutron excess of the target. However, the inclusion of isospin is more subtle than this simple prescription reveals. The importance of isospin derives from the fact that it is (nearly) a good quantum number for a heavy nucleus. However, only neutron holes in the valence shell of a target system $|C\rangle$ have good isospin. In contrast, neutron holes in any of the core states that are also occupied by protons do not carry good isospin. Instead, such neutron holes are only a component of the analog and antianalog states,

$$\psi_A=[|\bar{n}C\rangle-\sqrt{2T_0}\,|\bar{p}A\rangle](2T_0+1)^{-1/2} \tag{4.1a}$$

and

$$\tilde{\psi}_A=[\sqrt{2T_0}\,|\bar{n}C\rangle+|\bar{p}A\rangle](2T_0+1)^{-1/2} \ , \tag{4.1b}$$

which have good isospin $T=T_0\pm\frac{1}{2}$. Here, $|\bar{n}C\rangle$, $|\bar{p}A\rangle$, etc., refer to neutron or proton holes in the target system $|C\rangle$ or its analog $|A\rangle$. The additional component $|\bar{p}A\rangle$ in Eqs. (4.1a) and (4.1b) that is required to build a state of good T has a 2-hole-1-particle character. Thus, ψ_A and $\tilde{\psi}_A$ are no longer single-hole states. We, therefore, call such states quasihole states. They carry quantum numbers $(n\ell jT)$, and are approximate eigenstates with binding energy $-\epsilon_{n\ell jT}$ of the Lane potential[2]

$$V_{\ell jT}=V_{\ell j}+U_1(t\cdot T_0) \quad r<R \ , \tag{4.2}$$

where U_1 is negative for holes. With this interpretation ψ_A or $\tilde{\psi}_A$ provide the appropriate basis, previously called $\psi_{n\ell}$, in which to represent the Green's function. In this basis, $G_\ell(\omega)$ becomes $G_{\ell jT}(\omega)$, and satisfies Eq. (2.3) with the self-energy in that equation replaced by

$$\Sigma_{\ell jT}=[-V_{\ell jT}+iW_0\delta_{T,T_0-1/2}] \quad r<R \ . \tag{4.3}$$

The much higher level density of compound states with isospin $T_0-\frac{1}{2}$ over those carrying $T_0+\frac{1}{2}$ implies that absorption is essentially confined to the antianalog channel. This is the reason for the Kronecker delta in Eq. (4.3). The analytic results of Sec. III are retained with $V_{\ell jT}$ replacing V_0. The strength function is then given by

$$S_{\ell j}(\omega)=C^2S_{\ell jT}=\frac{1}{\pi}C^2\mathrm{Im}G_{\ell jT}(\omega) \tag{4.4}$$

with the appropriate Clebsch-Gordan coefficient C for $T=T_0\pm\frac{1}{2}$.

V. SPECIFIC CALCULATIONS

The strength function $S_{\ell j}(\omega)$ for the $1g_{9/2}$ neutron hole state has been measured in the $T_0-\frac{1}{2}$ isospin channel in isotopes[2] of Pd, and in both isospin channels in isotopes[1] of Sn. We have taken the neutron hole strengths as measured by the (^{3}He,α) reaction on ^{116}Sn and ^{104}Pd as typical of the data in this region. In all the figures to be discussed, the strength function $S_{\ell j}(\omega)$ is plotted as a function of the binding energy of the distributed quasihole state in the target nucleus. This means that $\omega=0$ corresponds to zero binding, and that larger negative binding corresponds to higher excitation energies of the residual system. It must also be noted that we have assumed in all the following calculations that W_0 is energy independent over the width of the resonance. Strictly speaking,[12] W_0 is a slowly varying function of energy, which in an infinite system would go to zero at the Fermi energy.[5] The low density of states in the finite system near its Fermi surface precludes a precise definition of W_0 in this region, so that the calculated curves only have a qualitative significance there.

A. Gross structure

Figure 1 presents a comparison between the optical model calculation for ^{115}Sn and the measured strength function of the $1g_{9/2}$ neutron hole in this nucleus. The calculated curve has been obtained from Eq. (4.4) using

$$\Sigma_{\ell jT}=[-49.1+0.96(t\cdot T_0)+i\,(1.0)]\ \mathrm{MeV}$$

for $r\leq R=1.2\,A^{1/3}=5.8$ fm for the self-energy that determines $t_{\ell jT}(\omega)$ in Eq. (3.6), and hence the associated Green's function using Eq. (2.10). The values for $\Sigma_{\ell jT}$, which have been chosen to give the correct binding energy of the $1g_{9/2}$ isospin analog-antianalog doublet, are typical for a neutron optical potential. The experimental data of Gerlic *et al.*[1] have been redrawn for Fig. 1 by averaging the measured strength in 0.5 MeV energy bins. These measurements locate about 45% of the total $1g_{9/2}$ strength allowed by the sum rule.[1] The calculated curve is given approximately by a Breit-Wigner shape, and thus encompasses 50% of the total strength within its own width. Judged on this criterion, the gross structure in the experimental data is well represented as a giant resonance in $S_{\ell jT}(\omega)$ due to a single quasihole in an optical potential. However, the substructure seen in the data cannot be so described. Such structure is made clearer by performing a running average of the data with a suitable weight function. We have used a Breit-Wigner weighting of width 0.3 MeV to generate the dotted curve in Fig. 1, which shows two prominent substructures. It has been suggested that this fragmentation comes about via some intermediate doorway[13] mechanism. Calculations[3] of Koeling and Iachello consider neutron holes coupled to collective vibrations of the neutron valence shell in Sn as possible doorways. These calculations neglect excitations of the proton core and hence, lead to states of mixed isospin. An alternative suggestion has been

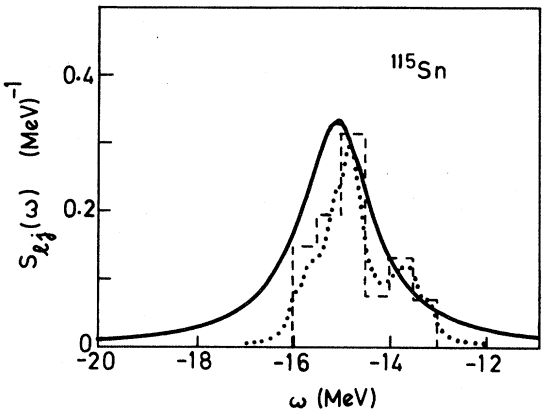

FIG. 1. Strength function for the $1g_{9/2}$ neutron hole component of the $T=\frac{15}{2}$ antianalog state in ^{115}Sn, plotted as a function of the binding energy. The solid curve represents the calculation with the hole self-energy given by $[-49.1+0.96t\cdot T_0+i\,(1.0)]$ MeV inside a nuclear radius $R=5.8$ fm. The experimental data (Ref. 1) are shown in two ways, as a dashed histogram and a dotted curve, which were obtained by averaging the individual measured strengths over discrete energy bins of 0.5 MeV, and with a running Breit-Wigner weight function of width 0.3 MeV.

made by Nomura,[4] who considers the doorways as consisting of collective vibrations of the *proton* core coupled to neutron holes in the valence shell in Sn. This scheme has the attractive feature of giving doorway states of good isospin $T_0-\frac{1}{2}$.

B. Intermediate structure

Nuclear structure calculations like those just mentioned provide an important link for establishing a possible structure for doorway states in ^{115}Sn. However, independent of the nature of their structure, one knows quite generally[13] that the presence of doorway states must cause $\Sigma_{\ell jT}$ to vary with energy over an energy range that is small compared with a giant resonance width $\sim 2W_0$. Such an energy variation, which leads naturally to intermediate structure in the strength function, may be incorporated by adding a "proper polarization part,"[5] due to the excitation of doorways, to $\Sigma_{\ell jT}$ in Eq. (4.3), in the nuclear interior

$$\Sigma_{\ell jT}(\omega)=-V_{\ell jT}+iW_0\delta_{T,T_0-1/2}+\sum_d \frac{|H_{nd}|^2}{\omega+\epsilon_n+E_d-i\Delta_d}\,. \tag{5.1}$$

The doorway states d in this expression lie at excitation energies E_d relative to the quasihole energy $\epsilon_n=\epsilon_{n\ell jT}$, to which they are coupled by H_{nd}. The doorway states are themselves not eigenstates of the nuclear Hamiltonian, and are consequently spread over the actual compound states in their vicinity by an amount symbolized by Δ_d. The poles of $\Sigma_{\ell jT}(\omega)$ are properly located for hole states by choosing Δ_d positive in the above expression.

While Eq. (5.1) follows from formal many-body theory[5] by considering the propagation of a quasihole in the nucleus, its imaginary part can be understood physically by recalling the expression[14] $(\pi/\Delta)\Sigma_c|H_{nc}|^2$ for the absorption of a shell model state n due to its coupling to compound states degenerate with it to within Δ. If, however, this state can only access the compound states via a doorway, then $|H_{nc}|^2$ and the coupling $|H_{nd}|^2$ are related by

$$|H_{nc}|^2=|a_{cd}|^2|H_{nd}|^2\,.$$

The distribution $|a_{cd}|^2$ of the doorway state over compound states in *its* vicinity has a Breit-Wigner shape on the average,[15] with a half width given by Δ_d. The contributions to $\mathrm{Im}\Sigma(\omega)$ from several such (nonoverlapping) doorways add incoherently, and one obtains the form (5.1). If we approximate the potentials H_{nd}, which are strictly nonlocal operators in coordinate space, by constants over the nuclear interior, we regain the complex square well model, but with real and imaginary potential depths that depend on the energy variable ω.

The well-studied vibrational levels[16] in the even isotopes of tin suggest that doorway states consist of hole excitations coupled to vibrations. One can construct two classes of such states for ^{115}Sn that carry good isospin. The first type of doorway state is formed by (quasi) neutron holes in the valence shell of ^{116}Sn coupled to its *proton* core vibrations that carry good isospin; the second type of doorway consists of the *analog* state to that formed by coupling (quasi) proton holes of the core of the parent ^{115}In to its *neutron* core vibrations. Note that this analog doorway state (ADS) carries good isospin to the extent that the charge exchange operator, and the operator creating the neutron core vibrations commute. Accompanying the ADS, which has isospin $T_0+\frac{1}{2}$, is an orthogonal antianalog doorway state (AADS) of isospin $T_0-\frac{1}{2}$.

The lowest proton core vibration is estimated[4] to lie at

$$S_p(^{115}\mathrm{Sn})-S_p(^{116}Sn)\cong 4.5\ \mathrm{MeV}$$

excitation. Consequently, if we couple the $2d_{5/2}$ neutron quasihole $S_n=9.56$ MeV to this, we build a core polarization vibration corresponding to a doorway of type 1 which lies below the "bare" $1g_{9/2}$ antianalog quasihole at 15.2 MeV binding, by 1.1 MeV. Since the $1g_{9/2}$ analog state[17] falls at $S_p+\Delta_c\cong 23$ MeV above the ground state of ^{116}Sn, including a 2^+ vibration at 1.27 MeV in the parent system will correspondingly place the ADS at 24.27 MeV. Likewise, the AADS will lie at 1.27 MeV above the quasihole energy. While a full nuclear structure calculation would involve several such states, we restrict ourselves to the above cases. The solid curve in Fig. 2 shows a calculation of the neutron hole strength function for ^{115}Sn in the analog channel with doorway states included. The values of the parameters for the interactions H_{nd} and the spreading widths for these doorway states have been inferred from the experimental data, and are listed in Table I. The other parameters of the optical potential model have remained the same as for Fig. 1. The influence of the AADS on the strength function is shown by the shoulder at around 17 MeV binding, and would probably not be visible as a resolved "state" in experimental data. On the other hand, the type 1 doorway state giving rise to the

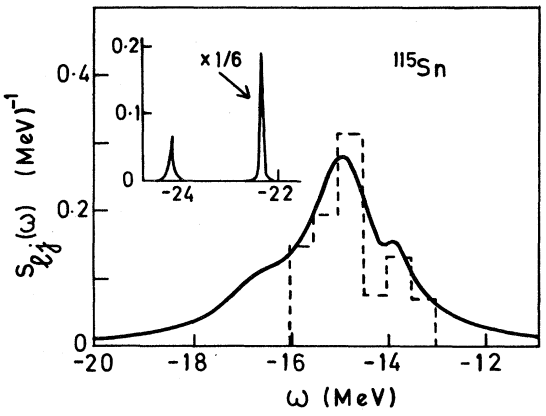

FIG. 2. The same calculation for ^{115}Sn as presented in Fig. 1, but with doorway states included in the self-energy of the hole. The relevant parameters are listed in Table I. The self-energy parameters determining the giant resonance are the same as for Fig. 1. The experimental data (broken histogram) have also been redrawn. The calculated strength function for the fragmented $1g_{9/2}$ neutron hole component in the $T=\frac{17}{2}$ analog channel is shown as an inset.

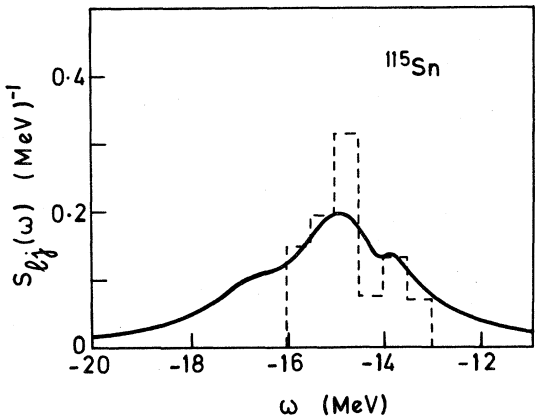

FIG. 3. Strength function calculation for ^{115}Sn as in Fig. 2, but with W_0 increased to 1.5 MeV. The analog states are not shown again, as they are unaffected by the choice of W_0.

lower bump at 14 MeV binding causes a definite structure, which can be clearly recognized in the data. The assumed difference in damping widths between these two states is 600 keV, which is in accordance with the expected increase in the optical potential over this energy range.[12] The inset in Fig. 2 shows the expected strength function for the analog channel, where an ADS is inserted into the self-energy.

It should be noted that the AADS and ADS necessarily have the same H_{nd} and E_d, but the ADS has a much smaller width. Hence, a rather unambiguous determination of H_{nd} and Δ_d can be made in the analog channel. The resulting width distribution of 31 to 89 keV between the lower and upper analog states compares well with the measured[1] values 31 ± 10 keV and 50 ± 15 keV. The fact that the higher state carries the larger width arises in the model from the assumption that absorption can only enter the analog channel via the damping of the doorway state component; see Eq. (5.1). It is also interesting to note that the value of H_{nd} required to reproduce the analog state splitting is close to the estimate (see Ref. 18, page 419).

$$H_{nd}=\langle j\,|\,r\frac{\partial V}{\partial r}\,|\,j\rangle[(2\lambda+1)\hbar\omega_\lambda/(8\pi C_\lambda)]^{1/2}\times(j\tfrac{1}{2}\lambda 0\,|\,j\tfrac{1}{2})\cong 0.8\text{ MeV}\,, \tag{5.2}$$

TABLE I. Doorway state parameters for ^{115}Sn.

Isospin	E_d (MeV)	Δ_d (MeV)	H_{nd} (MeV)
$T_0+\frac{1}{2}$	1.27	0.05	0.7
$T_0-\frac{1}{2}$	1.27	0.85	0.7
	−1.1	0.25	0.3

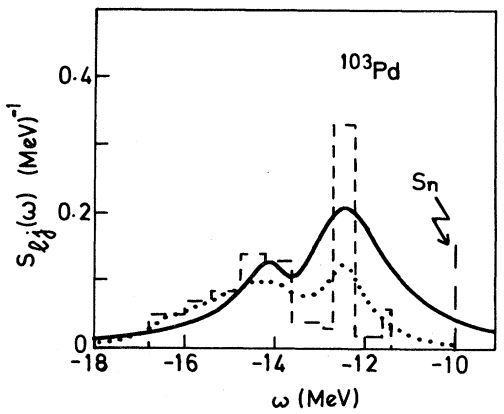

FIG. 4. The calculated strength function for ^{103}Pd using a single antianalog doorway state in the hole self-energy. Self-energy parameters determining the giant resonance parameters are the same as for Fig. 3, but with the smaller nuclear radius $R=5.6$ fm. Doorway parameters are described in the text. Experimental data (broken histogram) have been taken from Ref. 2, and also averaged with a Breit-Wigner weight function of width 0.8 MeV (dotted curve). The neutron binding energy $S_n=9.99$ MeV is also indicated.

for the coupling of the $\lambda=2^+$ surface phonon to a shell model state $j=\frac{9}{2}$ in ^{116}Sn (for which $\hbar\omega_2=1.27$, $C_2=2.53$, and $\langle j \mid r\,\partial V/\partial r \mid j\rangle \sim 50$, all in MeV). This value of the coupling also leads to the correct value of the polarization charge in ^{115}In (see Ref. 18, page 533). On the other hand, the value of the absorption strength is rather imprecisely determined by the antianalog channel data. Figure 3 shows the same calculation as for Fig. 2 with W_0 increased to 1.5 MeV. The fit does not differ essentially from that shown in Fig. 2, apart from the fact that the giant resonance is lower and more spread. the data is still well represented, and the intermediate structure remains visible.

A similar calculation for the $1g_{9/2}$ strength function in the antianalog channel has been carried out for ^{103}Pd; see Fig. 4. The same optical potential parameters as in Fig. 3 for ^{115}Sn have been used, except for $R=5.6$ fm. The type 1 doorway state in this case has the parameters $H_{nd}=0.63$, $\Delta_d=0.40$, and $E_d=1.0$, all in MeV. A satisfactory representation of the data is obtained.

VI. SUMMARY

One concludes from the results presented that the optical model furnishes a good description of deep-lying hole states. The use of a complex square well to model the optical potential is not a serious limitation for such states, and can be used to display the gross structure with reasonable physical values of the necessary parameters. Modifying the potential to something more physical, but less tractable mathematically, is not expected to change the present results qualitatively, since deep lying states are insensitive to surface effects.

Intermediate structure features are described by including doorway state excitations of good isospin in the self-energy. Our parametrization of such nuclear polarization effects on the motion of a hole in the nucleus yielded good representations of the intermediate structures, with doorway parameter values that are reasonable. Independent predictions of the coupling strength are in good agreement with the values used. The overall satisfactory results do suggest that the main features of the model will be retained in a more detailed treatment of the problem.

ACKNOWLEDGMENTS

We would like to thank Rory Leahy of the Nuclear Physics Research Unit for his assistance with computing. One of us (S.P.K.) thanks African Explosives and Chemical Industries, Propriety and the Council for Scientific and Industrial Research for financial assistance.

[1]E. Gerlic, G. Berrier-Ronsin, G. Duhamel, S. Galès, E. Hourani, H. Langevin-Joliot, M. Vergnes, and J. Van de Wiele, Phys. Rev. C 21, 124 (1980).
[2]O. Scholten, M. N. Harakeh, J. van der Plicht, L. W. Put, R. H. Siemssen, S. Y. van der Werf, and M. Sekiguchi, Nucl. Phys. A348, 301 (1980).
[3]T. Koeling and F. Iachello, Nucl. Phys. A295, 45 (1978).
[4]M. Nomura, Prog. Theor. Phys. 55, 622 (1976).
[5]See, for example, A. L. Fetter and J. D. Walecka, *Quantum Theory in Many-Particle Systems* (McGraw-Hill, New York, 1971).
[6]See, for example, M. L. Goldberger, and K. M. Watson, *Collision Theory* (Krieger, New York, 1975).
[7]A. M. Lane and J. M. Soper, Nucl. Phys. 37, 663 (1962).
[8]H. Feshbach, Ann. Phys. (N.Y.) 5, 357 (1958).
[9]Lecture Notes in Physics, *Microscopic Optical Potentials*, edited by H. V. von Geramb (Springer, Heidelberg, 1979), Vol. 89.
[10]See, for example, P. E. Hodgson, *The Optical Model of Elastic Scattering* (Clarendon, Oxford, 1963).
[11]R. G. Newton, J. Math. Phys. 1, 319 (1960).
[12]See, for example, P. E. Hodgson, *Nuclear Reactions and Nuclear Structure* (Clarendon, Oxford, 1971).
[13]H. Feshbach, A. K. Kerman, and R. H. Lemmer, Ann. Phys. (N.Y.) 41, 230 (1967).
[14]H. Feshbach, Annu. Rev. Nucl. Sci. 8, 49 (1958).
[15]B. Block and H. Feshbach, Ann. Phys. (N.Y.) 23, 47 (1963).
[16]L. S. Kisslinger and R. A. Sorenson, Rev. Mod. Phys. 35, 853 (1963).
[17]G. M. Temmer, in *Fundamentals in Nuclear Theory, B*, edited by A. deShalit and C. Villi (IAEA, Vienna, 1967).
[18]A. Bohr and B. R. Mottleson, Nuclear Structure (Benjamin, Reading, 1975), Vol. II.