Scattering of light ions by ^{6}Li

M. LeMere* and Y. C. Tang
School of Physics, University of Minnesota, Minneapolis, Minnesota 55455

H. Kanada†
School of Physics, University of Minnesota, Minneapolis, Minnesota 55455
and Department of Physics, Niigata University, Niigata, Japan
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The scattering of light ions (n, d, ^{3}He, and α) by ^{6}Li is considered with the single-channel resonating-group method. The cluster internal functions used are translationally invariant shell-model functions of the lowest configurations in harmonic-oscillator wells having different width parameters. The result shows that the essential features of the cross-section angular distributions in all these systems can be reasonably well explained. The use of different width parameters is found to be necessary in the low-energy region where sharp resonance levels exist, but is not essential at higher energies greater than about 10 MeV/nucleon. Effects of nucleon exchanges have also been carefully examined. Here one finds that, even in the present case where a non-closed-shell nucleus is involved, the importance of the core-exchange terms still depends predominantly on the nucleon-number difference of the interacting nuclei. In addition, this investigation shows that the phenomenon of blocking has also a profound influence; it gives rise to a channel-spin dependence for the odd-even phase-shift behavior and may cause a substantial reduction in the effects of core-exchange contributions.

[NUCLEAR REACTIONS ^{6}Li(n,n), ^{6}Li(d,d), ^{6}Li(^{3}He, ^{3}He), ^{6}Li(α,α); calculated phase shifts and $\sigma(\theta)$. Resonating-group method and effects of antisymmetrization.]

I. INTRODUCTION

One of the main purposes of resonating-group studies is to obtain a detailed understanding of the roles played by the Pauli exclusion principle in nuclear structure and nuclear reactions.[1,2] For a two-cluster $A+B$ system where the clusters A and B contain N_A and N_B ($N_A > N_B$) nucleons, respectively, this can be achieved by carefully examining the features of the exchange-kernel function $K(\vec{R}',\vec{R}'')$ which characterizes the nonlocal part of the inter-cluster interaction.[3–6] As has been explained in recent publications,[1,5] this kernel function can be expressed as a sum of nucleon-exchange terms, i.e.,

$$K(\vec{R}',\vec{R}'')=\sum_x \sum_q K_{xq}(\vec{R}',\vec{R}'') , \tag{1}$$

where the index x denotes the number of nucleons interchanged between A and B ($1 \le x \le N_B$) and the index q denotes the interaction type for each value of x (see Ref. 5 for a detailed explanation). For the facilitation of analytical derivation of the kernel function, it is convenient in actual calculation to choose, for the description of the internal structures of these two clusters, translationally-invariant shell-model functions in harmonic-oscillator wells having width parameters α_A and α_B. With such a choice, the function K_{xq} can then be further written in the following factorized form,

$$K_{xq}(\vec{R}',\vec{R}'')=P_{xq}\exp(-A_{xq}\vec{R}'^2-C_{xq}\vec{R}'\cdot\vec{R}''-B_{xq}\vec{R}''^2)+\text{h.c.} \tag{2}$$

with P_{xq} being a polynomial in $\vec{R}'^2$, $\vec{R}'\cdot\vec{R}''$, and $\vec{R}''^2$. The exponential factors in all the nucleon-exchange terms, depending on x and the nucleon numbers of the clusters involved, collectively determine the general features of antisymmetrization, while the polynomial factors contain information concerning more specific features, such as blocking and clustering effects,[7] which depend additionally

on the dynamical structures of the interacting clusters.

By utilizing the complex-generator-coordinate technique,[1,8] general expressions for the coefficients A_{xq}, B_{xq}, and C_{xq} can be derived.[5,9] Using these expressions, one can then examine the characteristic features of the exponential factors. The result shows that the various nucleon-exchange terms can be grouped together into two classes, distinguished by the sign of C_{xq}. For the term in class A with $C_{xq}<0$, the Born scattering amplitude is forward peaked and can be reproduced by an equivalent local potential having a Wigner character. On the other hand, for the term in class B with $C_{xq}>0$, the Born scattering amplitude is backward peaked and can be reproduced by an equivalent local potential having a Majorana character. In addition, the following important features have also been noted[1]: (i) the one-exchange ($x=1$) terms have generally the largest influence among all class-A terms, while the core-exchange ($x=N_B$) terms have generally the largest influence among all class-B terms, and (ii) the one-exchange terms are generally important in all scattering systems and over a wide range of energies, while the core-exchange terms are generally important only when the nucleon-number difference (N_A-N_B) of the interacting nuclei is rather small. All these features, obtained through a careful examination of the exponential factors in the kernel function $K(\vec{R}',\vec{R}'')$, have been explicitly verified by a number of resonating-group calculations (see Ref. 1 and references contained therein) and by the recent WKB investigations[10,11] of light-ion scatterings by the doubly-closed-shell nuclei ^{4}He, ^{16}O, and ^{40}Ca.

Although the properties of the exponential factors are now well established, there exists only scant information[12] concerning the polynomial factors. This is unfortunate, since the influence of blocking may appreciably affect antisymmetrization effects when non-closed-shell nuclei are involved. In a recent study[13] of $\alpha+^6$Li scattering, for example, it was found that the core-exchange terms become substantially less important because of the large probability for the presence of two nucleons in the nonclosed $1p$ shell of ^{6}Li. Thus, it would be very useful to gain further information about, in particular, the general properties of those terms in the polynomial factors which have high powers in $\vec{R}'^2$, $\vec{R}'\cdot\vec{R}''$, and $\vec{R}''^2$. However, in view of the fact that resonating-group kernels are generally not easy to derive, especially when the clusters involved have complicated internal structures, it is our opinion that this type of general information would be quite difficult to obtain. Therefore, it seems that, at this moment, the best way to learn the importance of blocking effects is to adopt the relatively tedious procedure of performing systematic resonating-group calculations in many systems. The present investigation is our first step in this direction, in which we shall study the scattering of various light ions (n, d, ^{3}He, and α) by the nucleus ^{6}Li. In later calculations, we shall also consider the nucleus ^{7}Li as a target and the scattering of ^{16}O ion by ^{17}O, ^{18}O, ^{19}F, and ^{20}Ne.[14]

In addition, we wish, of course, to examine if the present calculation can explain the essential behavior of these systems. For this purpose, we shall make our calculation as realistic as practicable by choosing unequal width parameters (i.e., $\alpha_A\neq\alpha_B$) for the interacting clusters. With this condition, the present investigation then represents a substantial improvement over the calculations reported previously[15] where equal cluster width parameters were used for simplicity. As is well known, the adoption of unequal width parameters does appreciably complicate the computation; however, the power of the complex-generator-coordinate technique is such that the resonating-group kernels can still be straightforwardly derived in an analytical way.

In the next section, we give a very brief description of the resonating-group formulation. Selected results for the phase shift and differential cross section are presented in Sec. III. In Sec. IV, we discuss the significance of adopting unequal width parameters in the calculation. Here we shall show that the results obtained with equal and unequal cluster width parameters are appreciably different in the low-energy region where sharp resonance levels exist, but become rather similar at higher energies greater than about 10 MeV/nucleon. Thus, the study of the main characteristics of exchange effects at these latter energies can, in fact, be carried out under the simplifying assumption of equal width parameters. This will be discussed in Sec. V. Finally, in Sec. VI, we summarize the findings of this investigation and make some concluding remarks.

II. FORMULATION

The formulation of a single-channel resonating-group calculation has already been thoroughly discussed elsewhere[1]; hence, only a very brief description will be given here. With the adoption of a

purely central nucleon-nucleon potential, the trial wave function may be written as

$$\psi_\lambda = \mathcal{A}[\phi_A \phi_B F_\lambda(\vec{R}) \xi_\lambda Z(\vec{R}_{\text{c.m.}})] , \tag{3}$$

where ϕ_A and ϕ_B represent the internal spatial structures of ^{6}Li (denoted as cluster A) and the light ion n, d, ^{3}He, or α (denoted as cluster B), respectively, and ξ_λ represents an appropriate spin-isospin function. The index λ denotes the channel-spin multiplicity $(2s+1)$, with s being the channel-spin angular-momentum quantum number of the system. Also, in the above equation, the function $F_\lambda(\vec{R})$ describes the relative motion between the two clusters and $Z(\vec{R}_{\text{c.m.}})$ is a normalizable function describing the total c.m. motion. As was discussed in Ref. 1, this latter function will be conveniently chosen to facilitate the computation with the complex-generator-coordinate technique.

The translationally-invariant internal functions ϕ_A and ϕ_B are chosen to have the lowest and most-space-symmetric configurations in suitable harmonic-oscillator wells. That is, we choose

$$\phi_A = (\vec{r}_5 - \vec{R}_A)\cdot(\vec{r}_6 - \vec{R}_A) \times \exp\left[-\tfrac{1}{2}\alpha_A \sum_{i=1}^{N_A} (\vec{r}_i - \vec{R}_A)^2\right] , \tag{4}$$

and

$$\phi_B = \exp\left[-\tfrac{1}{2}\alpha_B \sum_{i=N_A+1}^{N_A+N_B} (\vec{r}_i - \vec{R}_B)^2\right] , \tag{5}$$

with $\vec{R}_A$ and $\vec{R}_B$ being, respectively, the position vectors of the c.m. of clusters A and B. The width parameter α_A is taken as

$$\alpha_A = 0.305 \text{ fm}^{-2} , \tag{6}$$

which is obtained by considering the low-q^2 form-factor behavior of ^{6}Li.[16] For the width parameter α_B, we use

$$\alpha_B = 0.20,\ 0.37, \text{ and } 0.514 \text{ fm}^{-2} \tag{7}$$

for the d, ^{3}He, and α clusters, respectively. These values are chosen to yield the correct rms matter radii of the corresponding clusters and are the same as those used in our investigation of light-ion scatterings by ^{16}O.[3]

To examine if the simplifying assumption of equal cluster width parameters could yield reasonable results, we have also made computations adopting a common width parameter $\bar{\alpha}$ in each system. Using the criterion that the sum of the cluster mean-square radii should remain the same as that in the unequal-width-parameter case, we choose

$$\bar{\alpha} = 0.261,\ 0.327, \text{ and } 0.367 \text{ fm}^{-2} \tag{8}$$

in the d, ^{3}He, and $\alpha + {}^6$Li systems, respectively.

The linear variational or relative-motion wave function $F_\lambda(\vec{R})$ is determined from the projection equation

$$\langle \delta\psi_\lambda \,|\, H - E_T \,|\, \psi_\lambda \rangle = 0 , \tag{9}$$

where E_T is the total energy of the system composed of the relative energy E of the clusters in the c.m. frame and the internal energies of the clusters obtained by computing the expectation values of the cluster Hamiltonians with respect to the assumed internal functions. The Hamiltonian operator has the form

$$H = \sum_{i=1}^{N_A+N_B} T_i + \sum_{i<j=1}^{N_A+N_B} V_{ij} - T_{\text{c.m.}} , \tag{10}$$

with $T_{\text{c.m.}}$ being the total c.m. kinetic-energy operator and V_{ij} being a nucleon-nucleon potential. In this calculation, we choose V_{ij} to be that given by Eqs. (9)–(11) of a previous publication.[17] It contains an exchange-mixture parameter u which will be adjusted in each system by using carefully selected experimental information. This adjustment is necessary in order to make a crude compensation for the various simplifications made in our calculation, among which the most notable one being the lack of explicit consideration for the specific distortions of the clusters involved.

Using the procedure described in Ref. 1, one obtains an integrodifferential equation for $F_\lambda(\vec{R}')$. It has the form

$$\left[-\frac{\hbar^2}{2\mu}\nabla_{\vec{R}'}^2 + V_{D\lambda}(\vec{R}') - E\right] F_\lambda(\vec{R}') + \int K_\lambda(\vec{R}', \vec{R}'') F_\lambda(\vec{R}'') d\vec{R}'' = 0 , \tag{11}$$

where $V_{D\lambda}$ represents the direct potential from the no-exchange ($x=0$ term) contribution and K_λ represents the kernel function for the nonlocal interaction.[18] By solving the above equation subject to appropriate boundary conditions, one then obtains bound-state eigenenergies and scattering phase shifts. Further, by carefully analyzing the results, one can acquire valuable knowledge concerning the

important characteristics of exchange effects arising from antisymmetrization.

III. RESULTS

A. Determination of u

For the $n+{}^6$Li, $d+{}^6$Li, and ^{3}He + ^{6}Li systems, the values of u will be determined by using the criteria described in Refs. 15 and 16. The resultant values in the unequal-width-parameter case are

$$u=0.98,\ 1.05,\ \text{and}\ 0.95 \tag{12}$$

for these three systems, respectively. In the case of the $\alpha+{}^6$Li system, we observe that the 2.4-MeV scattering data[19] can be well fitted by the elastic-transfer model of von Oertzen.[20,21] Since the process of elastic transfer is automatically taken into account in a calculation utilizing a totally antisymmetric wave function, this means that the measured cross-section angular distribution at this particular energy should be fairly well explained also by our calculation. That this is indeed so is shown in Fig. 1 where the results obtained with $u=0.84$, 0.88, and 0.92 are exhibited. From this figure, it is seen

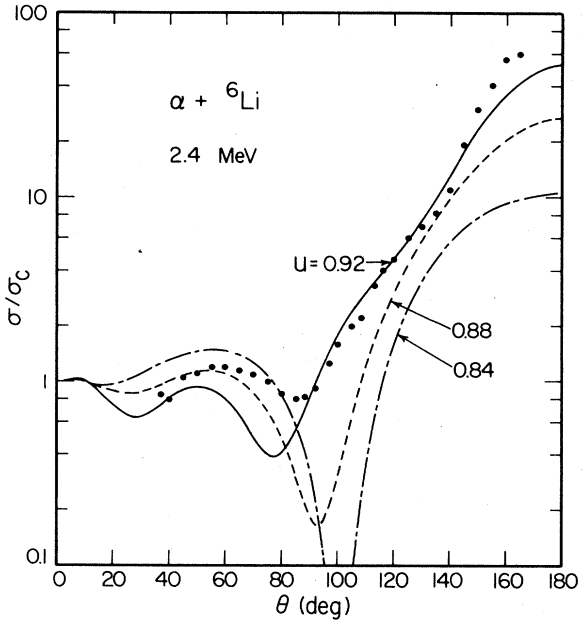

FIG. 1. Comparison of calculated and experimental differential cross sections for $\alpha+{}^6$Li scattering at 2.4 MeV. The calculated curves are obtained with $u=0.84$, 0.88, and 0.92. Experimental data shown are those of Ref. 19.

that, with

$$u=0.92\ , \tag{13}$$

there is a reasonable agreement between calculation and experiment; in particular, the strong rise in the backward direction is nicely reproduced. Therefore, this particular value of u will be adopted in all our subsequent calculations for this system.

It is interesting to point out that the variation in u among the four systems considered here is the same as that found in our previous study of light-ion scatterings by ^{16}O.[3] In particular, we note that the u values are especially large for the $d+{}^6$Li and $d+{}^{16}$O systems. This suggests strongly that the effects of specific distortion[22] are quite strong in these latter systems and such effects should be properly considered in future investigations.

The values of u have been similarly determined in the equal-width-parameter case. The results are

$$u=0.98,\ 1.01,\ 0.95,\ \text{and}\ 0.92 \tag{14}$$

for the n, d, ^{3}He, and $\alpha+{}^6$Li systems, respectively.

B. Phase shifts

Phase shifts are computed up to about 50 MeV for all four systems. As an illustration in the unequal-width-parameter case, we show by solid curves in Figs. 2 and 3 the $\alpha+{}^6$Li even l and odd l phases, respectively. From these figures, one notes that there exist a number of positive- and negative-parity bands consisting of levels having predominantly an $\alpha+{}^6$Li cluster configuration. The ground-state band, for example, contains bound 0^+ and 2^+ levels and a resonance level at about 5 MeV.

One interesting point worth mentioning is that, in the $\alpha+{}^6$Li case, sharp resonance levels occur only in the energy region below about 5 MeV/nucleon. Similar findings have also been noted in the n, d, and ^{3}He + ^{6}Li systems. In fact, this seems to be a rather general feature for levels having a large degree of nucleon clustering. From many resonating-group studies such as n, d, ^{3}He, and $\alpha+{}^{16}$O scattering, we have always reached the same conclusion.

From Fig. 3, it is also seen that there are $l=1$ and 3 spurious resonances[23] which arise from the existence of almost-forbidden states, discussed particularly by Saito.[24] These resonances occur at rather high energies and are shown as breaks in the phase-shift curves for convenience. As has been previously discussed,[1] they do not represent the existence of observable resonance states in the com-

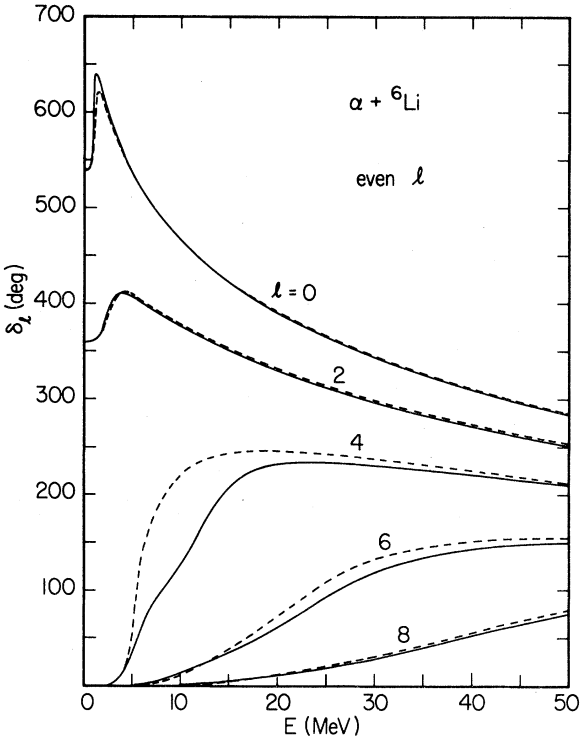

FIG. 2. Even l phase shifts for $\alpha+{}^6$Li scattering calculated with unequal (solid curves) and equal (dashed curves) width parameters.

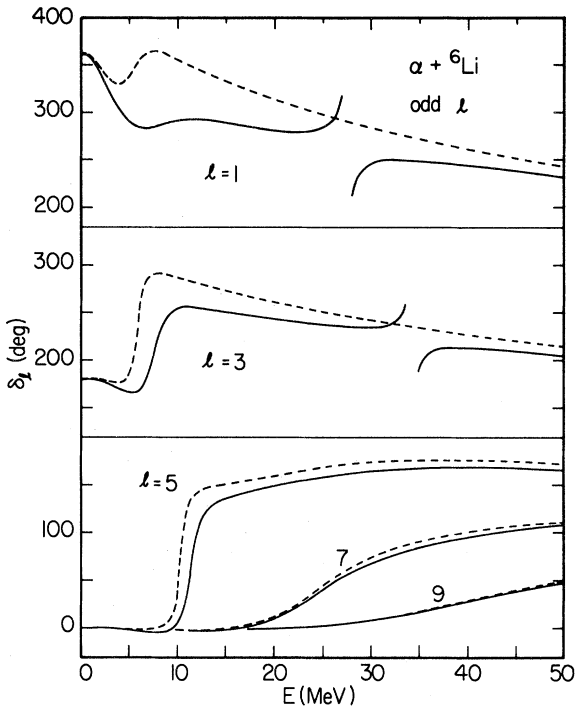

FIG. 3. Odd l phase shifts for $\alpha+{}^6$Li scattering calculated with unequal (solid curves) and equal (dashed curves) width parameters. Breaks in the solid curves represent spurious resonances.

pound system and their presence can be easily suppressed by the introduction of a phenomenological imaginary potential into the resonating-group formulation.[25]

C. Differential cross sections

In the energy region higher than about 5 MeV/nucleon, the various phase shifts vary smoothly with energy (see Sec. III B for a discussion of the phase-shift behavior near a spurious resonance), and our present calculation may be employed to explain the main features of experimental results on scattering angular distributions. For this purpose, we introduced into Eq. (11) a phenomenological imaginary-potential term $iW(\vec{R}')$ to take reaction effects crudely into account. For $W(\vec{R}')$ we assume the form

$$W(\vec{R}')=-W_I\left[\frac{1}{1+\exp[(R'-R_I)/a_I]}+\frac{4\exp[(R'-R_I)/a_I]}{\{1+\exp[(R'-R_I)/a_I]\}^2}\right], \tag{15}$$

with the diffuseness parameter a_I taken to have a reasonable value of 0.6 fm. The radius parameter R_I is chosen such that the rms radius of the volume part of $W(\vec{R}')$ is approximately equal to the rms radius of the nuclear part of the direct potential $V_{D\lambda}$. The resultant values are

$$R_I=3.0,\ 4.0,\ 3.8,\ \text{and}\ 3.6\ \text{fm} \tag{16}$$

for the n, d, ^{3}He, and $\alpha+{}^6$Li systems, respectively. With a_I and R_I thus fixed, the only adjustable quantity is the depth parameter W_I which will then be adjusted to obtain a best fit with the experimental differential cross-section result.

Differential cross sections are calculated at 12, 14.7, 18, and 27 MeV (around 10 MeV/nucleon) in the n, d, ^{3}He, and $\alpha+{}^6$Li cases, respectively. The results (solid curves) are compared with experimental data[26–30] (solid dots) in Fig. 4. To obtain the calculated curves, we have used $W_I=3.0$, 3.5, 4.3, and 3.5 MeV, respectively, in these four cases, resulting in reaction cross sections of 530, 949, 926, and 816 mb. As is seen from this figure, the calculation does explain fairly well the main features of measured angular distributions. In particular, as a result of taking exchange effects properly into ac-

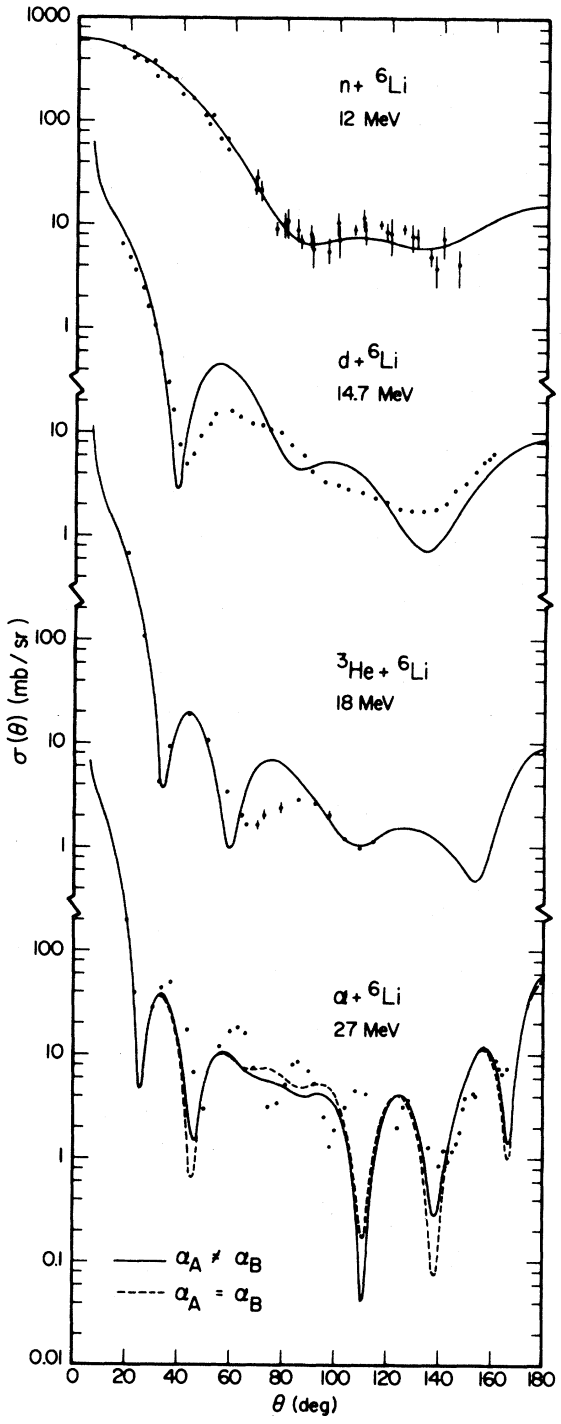

FIG. 4. Comparison of calculated and experimental differential cross sections for various systems at indicated energies. The solid curves represent results obtained with unequal width parameters, while the dashed curve represents the equal-width-parameter result. Experimental data shown are those of Refs. 26–30.

count, the cross-section rise in the backward angular region is reasonably reproduced.

The lack of close agreement between calculation and experiment can probably be attributed to two main causes: (i) specific distortion effects are not considered, and (ii) the ^{6}Li wave function used does not allow for enough $d+\alpha$ clustering. To improve the calculation in these respects will need a substantial modification of the trial wave function. The computation will be fully within the capability of the complex-generator-coordinate technique, but the amount of additional effort required will be quite large.

IV. INFLUENCE OF UNEQUAL WIDTH PARAMETERS

As has been mentioned, a resonating-group calculation using unequal cluster width parameters requires substantially more effort than one in which the simplifying assumption of equal width parameters is adopted. Thus, it is interesting to determine how different the results are when unequal and equal width parameters are used. In Figs. 2 and 3, we illustrate this by comparing in the $\alpha+^6$Li system the phase-shift values obtained in the unequal-width-parameter (solid curves) and equal-width-parameter (dashed curves) cases. By examining the behavior of these curves, one notes that the most striking difference is the absence of spurious resonances in the latter case. This is as should be,[1] but has little practical significance since, with the addition of an imaginary potential into the formulation, the real parts of the phase shifts will smoothly vary with energy even in the unequal-width-parameter calculation.

The major difference occurs at low energies where sharp resonance levels exist. This is especially apparent for odd l partial waves in this particular system. From Fig. 3, one sees that the characteristics of the $l=1$, 3, and 5 resonance states are appreciably different in these two calculations. On the other hand, it is noted that, at higher energies greater than about 10 MeV/nucleon, the difference does become much less significant, and the energy- and l-dependent behavior of the phase shifts in these two cases are essentially the same.

In the energy region above about 10 MeV/nucleon, the relative insignificance of adopting unequal width parameters can also be demonstrated by comparing the cross-section results. This is shown in Fig. 4, where the $\alpha+^6$Li scattering cross sections at 27 MeV (11.3 MeV/nucleon) calcu-

lated with unequal and equal width parameters are depicted by solid and dashed curves, respectively. In both calculations, the values of W_I are chosen as 3.5 MeV, resulting in almost exactly the same reaction cross section. As is seen, the behaviors of these curves are different only in a minor way, which results not only from the rather small differences in the real parts of the phase shifts but also from the fact that absorption effects are already quite strong at this energy.

At an even higher energy of 62.4 MeV (26 MeV/nucleon), the conclusion remains essentially unchanged. In Fig. 5, we show the values of σ/σ_C calculated with unequal (solid curve) and equal (dashed curve) width parameters, and the experimental data (solid dots) of Hauser *et al.*[31] for this quantity. For the theoretical calculations, we have adjusted W_I such that the cross-section values at forward angles become as similar as possible in these two cases. The resultant choices are $W_I=5.0$ and 5.3 MeV in the unequal- and equal-width-parameter calculations, respectively; the corresponding reaction cross sections are equal to 733 and 748 mb. From this figure, one finds that, because of weaker absorption at this higher energy, there is now a more noticeable difference. However, even in the backward angular region where

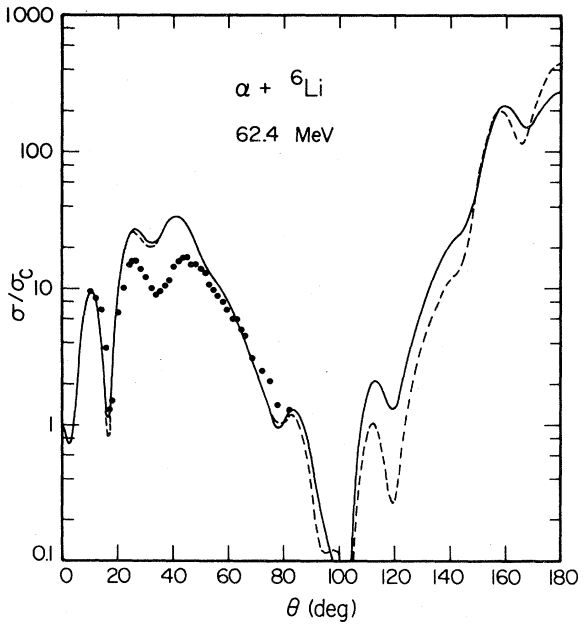

FIG. 5. Comparison of calculated and experimental values of $\sigma(\theta)/\sigma_c(\theta)$ for $\alpha+{}^6$Li scattering at 62.4 MeV. The solid and dashed curves represent results obtained with unequal and equal width parameters, respectively. Experimental data shown are those of Ref. 31.

core-exchange effects dominate, the difference is still tolerable and the two theoretical angular distributions have essentially the same oscillatory behavior.

At this stage when most resonating-group calculations are still rather crude, the choice as to whether equal or unequal width parameters should be used must, therefore, depend upon what one mostly wants to learn from the calculation. If the main purpose is to study the level structure in the low-energy region, then it may be worth the extra effort to choose unequal width parameters in the calculation. On the other hand, if one merely wishes to learn, at relatively high energies, the essential features of scattering angular distributions or the main characteristics of nucleon-exchange effects, then the assumption of equal width parameters could be reasonably adopted to simplify the computation.

V. EFFECTS OF ANTISYMMETRIZATION

A. Contributions from class-*A* and class-*B* nucleon-exchange terms

Antisymmetrization or nucleon-exchange effects are most simply studied at energies greater than about 25 MeV/nucleon.[6] At these energies, the cross-section angular-distribution curve normally exhibits a distinct V shape with its tip occurring at an angle θ_m which is a measure of the relative importance of the class-B exchange terms. When these terms make strong contributions, θ_m will be relatively small (i.e., close to about 90°), and vice versa. For $\theta<\theta_m$, the angular-distribution curve has a decreasing trend and the contribution to the cross section comes essentially from the direct and class-A terms (mainly the one-exchange terms), while for $\theta>\theta_m$, it has an increasing trend and the contribution to the cross section comes essentially from the class-B terms (mainly the core-exchange terms).

In this section, we study the importance of class-A and class-B terms in the energy region mentioned above. Since it was shown in Sec. IV that at such energies, the essential behavior of the nuclear system can be reasonably described by adopting the assumption of equal width parameters for the clusters, we shall use $\alpha_A=\alpha_B=\bar{\alpha}$ [see Eq. (8)] to simplify the discussion. Also, for clarity, we shall omit all charge effects by letting the charge of the proton to be infinitesimally small.

To qualitatively guide our analysis, we first com-

pute, by using Eqs. (7.17)–(7.20) of Ref. 1, the one-exchange characteristic range R_1 and characteristic energy E_1, and the core-exchange characteristic range R_c and characteristic energy E_c (see Refs. 1 and 6 for a detailed discussion). For the computation of these quantities, we have adopted an average value of 0.35 fm^{-2} for the two-nucleon range parameter κ to be used in Eq. (7.9) of Ref. 1. The results for the n, d, ^{3}He, and $\alpha+^6$Li systems, together with the characteristic ranges R_D of the direct potentials, are listed in Table I. From this table, one notes in particular that the value of E_c is quite large in the $\alpha+^6$Li system but becomes much smaller in the $n+^6$Li and $d+^6$Li systems. Since a previous Born-approximation study[1] shows that the depth of the equivalent local core-exchange potential contains an energy-dependent factor $\exp(-E/E_c)$, this indicates that, if one examines the resonating-group results at a rather high energy, say, 50 MeV, then one should observe for these systems a large difference in the cross-section behavior at backward angles where class-B terms make dominant contributions.

In Fig. 6, we show the differential cross sections calculated at 50 MeV for all four systems. As is seen, the qualitative prediction mentioned above based on the values of E_c is indeed borne out. The value of θ_m is 103° in the $\alpha+^6$Li system,[32] 111° in the ^{3}He + ^{6}Li system, and around 130° in both the $n+^6$Li and $d+^6$Li systems. In addition, one notes that the differential cross sections at backward angles are quite large for $\alpha+^6$Li scattering, in contrast to those for $n+^6$Li and $d+^6$Li scattering. This is again an indication that class-B terms are comparatively more important in the $\alpha+^6$Li case.

Using the fact that, at a relatively high energy, the equivalent local potentials for the class-B terms have mainly a Majorana character, one can also demonstrate the importance of the class-B contributions by examining the phase-shift behavior. This is shown in Fig. 7 for all these systems. Here one finds that the odd-even l-dependent feature is quite evident in the $\alpha+^6$Li case, somewhat less so in the ^{3}He + ^{6}Li case, but is very weak in the $n+^6$Li and $d+^6$Li cases. Furthermore, it is noted that even in the $\alpha+^6$Li and ^{3}He + ^{6}Li systems, the odd-even behavior shows up clearly only for lower values of l. This is not an unexpected feature, but is related to the fact that the characteristic ranges for the class-B equivalent potentials are shorter than the characteristic ranges of the direct potentials (see Table I).

From Fig. 7, one also observes the interesting feature that, in the ^{3}He + ^{6}Li system, the odd-even behavior in the $s=\frac{3}{2}$ channel-spin state is distinctly different from that in the $s=\frac{1}{2}$ channel-spin state. This is a result of the influence of blocking which arises because of the presence of two nucleons in the nonclosed $1p$ shell of ^{6}Li, the consequence of which is that the odd-even characteristics of the $d+^3$He subsystem[33] are also reflected in the phase-shift nature of the present system. From a qualitative viewpoint, one may try to understand this special feature in the following way. In the lowest harmonic-oscillator shell-model configuration of the compound nucleus ^{9}B, there exists an odd number of five quanta of excitation. With d and ^{3}He clusters, the energetically most-favored positive-parity cluster which can be formed is a ^{5}Li cluster characterized by relative orbital angular momentum 0 and spin angular momentum $\frac{3}{2}$. Consequently, when s is equal to $\frac{3}{2}$, the Pauli-favored states are odd l states; i.e., the effective intercluster interaction in this particular channel-spin state is expected to be stronger in odd l states than in even l states. In the case of $s=\frac{1}{2}$, the phase-shift behavior in Fig. 7 shows that the opposite is true. This is not very easy to understand even qualitatively, but is likely related to the fact that, with this value of the spin angular momentum, the d and ^{3}He clusters are energetically favored to form a negative-parity, diffuse ^{5}Li cluster having predominantly a $p+\alpha$ cluster configuration.

In the $n+^6$Li system, the situation is somewhat different. Here the lowest harmonic-oscillator shell-model configuration of the compound nucleus ^{7}Li has three quanta of excitation, and the energeti-

TABLE I. Values of characteristic quantities.

System	N_A-N_B	$\bar{\alpha}$ (fm^{-2})	R_D (fm)	R_1 (fm)	E_1 (MeV)	R_c (fm)	E_c (MeV)
$n+^6$Li	5	0.305	2.37	1.38	29.3	1.36	12.6
$d+^6$Li	4	0.261	2.82	2.20	54.3	1.73	13.8
^{3}He + ^{6}Li	3	0.327	2.73	2.11	78.5	1.82	25.0
$\alpha+^6$Li	2	0.367	2.68	2.06	98.6	2.09	47.4

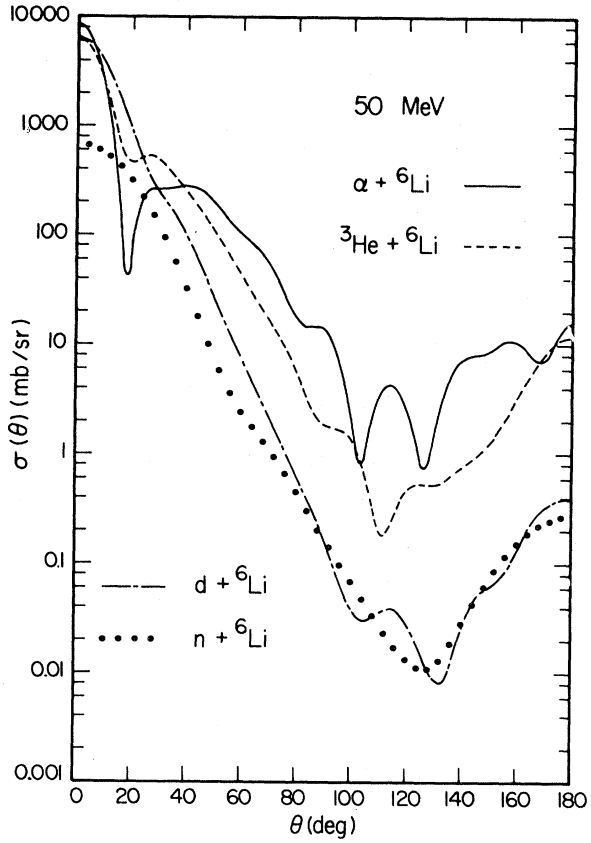

FIG. 6. Calculated differential cross sections at 50 MeV for all four systems.

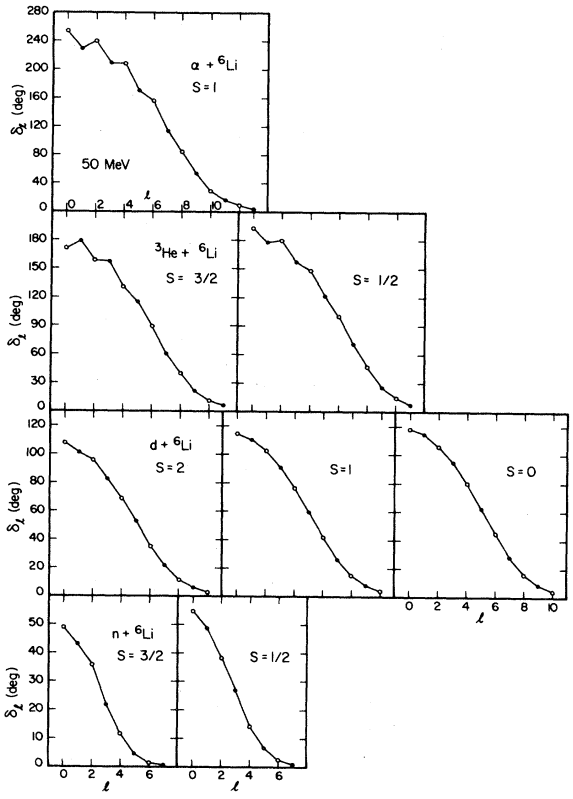

FIG. 7. Calculated phase shifts at 50 MeV as a function of the orbital angular momentum for all four systems.

cally most-favored positive-parity cluster which has an $n+d$ cluster structure is a ^{3}H cluster with relative orbital angular momentum 0 and spin angular momentum $\frac{1}{2}$. Thus, the odd l states are now Pauli-favored states when s is equal to $\frac{1}{2}$, in agreement with the finding of the resonating-group calculation.[34]

To obtain a quantitative measure of the importance of class-A and class-B exchange terms, we adopt the following procedure. We compute the phase-shift values with an effective intercluster potential of the form

$$\hat{V}_\lambda = C_\lambda V_{D\lambda} , \tag{17}$$

and compare these values with those from the corresponding resonating-group calculation. This is illustrated in the $\alpha+^6$Li case at 60 MeV in Fig. 8, where the two solid curves connect the phase-shift points computed with $C_\lambda=1.13$ and 1.21, respectively, and the solid dots represent the resonating-group result. Here one sees that, especially for lower values of l, the resonating-group even l phases can be well reproduced by using an effective potential with $C_\lambda=1.21$, while the resonating-group odd l phases can be well reproduced by using an effective potential with $C_\lambda=1.13$. The important point to note is that these two C_λ values are rather different and appreciably larger than 1. This indicates that both class-A and class-B exchange terms make significant contributions and should be carefully considered for a proper description of the behavior of this system.

B. Odd-even potential model

From the resonating-group formulation, it is well known that the intercluster interaction has a complicated nonlocal nature. In the Born approximation, however, it was shown[5] that the nonlocal terms can be represented by equivalent local, energy-dependent potentials with either a Wigner or a Majorana character. This suggests that, for a

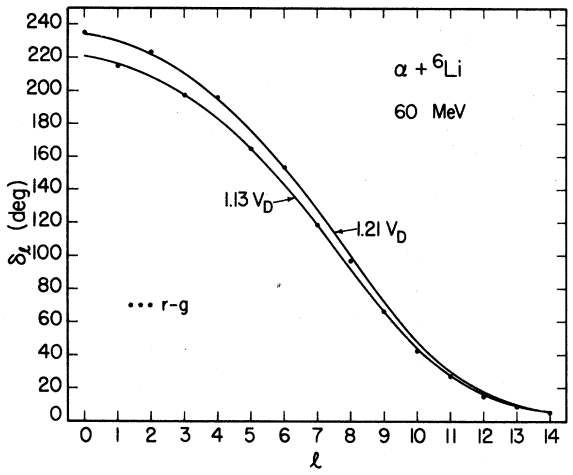

FIG. 8. Comparison of $\alpha+^6$Li phase shifts at 60 MeV obtained with the resonating-group method and with the effective potential of Eq. (17) using $C_\lambda=1.13$ and 1.21.

macroscopic description to be used at relatively high energies, one may, as a reasonable approximation, assume an effective intercluster potential of the form (the index λ is omitted for simplicity)

$$\tilde{V}(\vec{\mathrm{R}})=V_D(R)+V_W(R)+V_M(R)P^R\ , \tag{18}$$

where V_D is a direct potential obtained by a double-folding procedure, V_W is a Wigner-type potential introduced to represent the effects of class-A terms, and V_MP^R is a Majorana-type potential introduced to represent the effects of class-B terms. As has been discussed many times previously,[1] the V_MP^R term is especially important for a satisfactory description of the cross-section behavior in the backward angular region when the nucleon-number difference of the interacting nuclei is rather small.

This simple odd-even model, represented by $\tilde{V}$ of Eq. (18), is of course not expected to yield a detailed explanation of the experimental result. Rather, one should use this model to describe only the essential behavior of the nuclear system. With this viewpoint, one may, therefore, further assume for V_W and V_M the simplified forms

$$\begin{aligned} V_W(R)&=C_WV_D(R)\exp(-\beta_WR^2)\ ,\\ V_M(R)&=C_MV_D(R)\exp(-\beta_MR^2)\ , \end{aligned} \tag{19}$$

where the parameters β_W and β_M are introduced to take into consideration the fact that the characteristic range of V_D may be sufficiently different from the characteristic ranges of the dominant parts of the class-A and class-B terms.

To find if the result obtained by this potential model agrees reasonably well with that from the corresponding resonating-group calculation, we consider again the $\alpha+^6$Li system at 60 MeV. Since it is seen from Table I that the characteristic ranges R_D, R_1, and R_c have relatively similar values, we set, for simplicity, β_W and β_M to be equal to zero. Then, based on the result shown in Fig. 8, one can easily see that

$$C_W=0.17,\quad C_M=0.04 \tag{20}$$

at this energy. In Fig. 9, we compare the differential cross sections obtained with the potential model (solid curve) and with the resonating-group calculation (solid dots). As is noted, the potential model does reproduce the essential features of the resonating-group angular distribution, and the overall agreement between these two calculations is quite reasonable.

The importance of including the V_MP^R term in $\tilde{V}$ is also demonstrated in Fig. 9. In this figure, we show by the dashed curve the differential cross sections calculated by the potential model with $C_W=0.17$ and $C_M=0$. Here one sees that, with the adoption of $C_M=0$, this model now yields satisfactory results only at forward angles, and is entirely

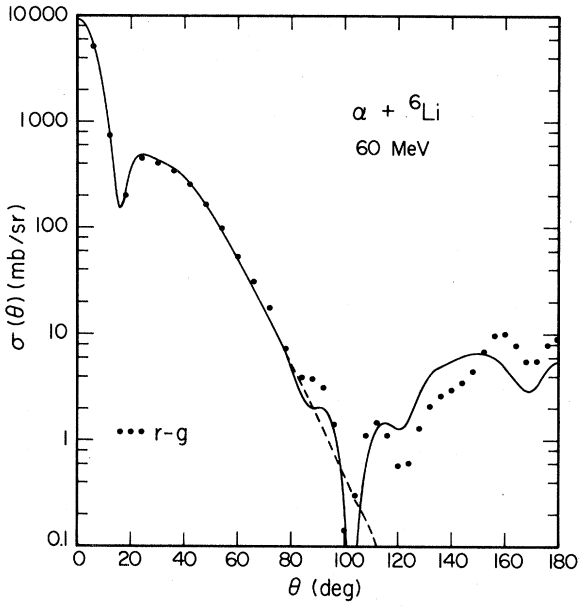

FIG. 9. Comparison of differential cross sections for $\alpha+^6$Li scattering at 60 MeV obtained with the resonating-group method (solid dots) and with the potential model of Eqs. (18) and (19) using $C_W=0.17$ and $C_M=0.04$ (solid curve). The potential-model result obtained using $C_W=0.17$ and $C_M=0$ (dashed curve) is also shown.

inadequate with regard to the cross-section behavior in the backward angular region.

The odd-even potential model has similarly been used to analyze the $\alpha+{}^6$Li resonating-group result at a lower energy of 30 MeV. Here it is found that the best values of C_W and C_M are equal to 0.20 and 0.06, respectively. These values are larger than the corresponding values at 60 MeV. This indicates that the depths of the equivalent exchange potentials become smaller as the energy increases, consistent with the finding based on the properties of the exponential factors in the exchange-kernel function.

Without a general understanding of the properties of the blocking effect at this moment, one can only hope to extract qualitative information from the C_M value obtained in the $\alpha+{}^6$Li potential-model analysis at 60 MeV (25 MeV/nucleon). For this purpose, we have further performed a brief potential-model study of the $d+\alpha$ system,[35] where blocking is not important and where the core-exchange characteristic energy per nucleon is very close to that in the $\alpha+{}^6$Li case. The result shows that, at a similar energy per nucleon (i.e., 22.5 MeV/nucleon), the value of C_M is around 0.08. The fact that this latter value is appreciably larger than the $\alpha+{}^6$Li value of 0.04 is a strong hint that the presence of two nucleons in the nonclosed $1p$ shell of ^{6}Li has resulted in a significant reduction of the importance of the core-exchange contribution in the $\alpha+{}^6$Li system.

VI. CONCLUSION

In this investigation, we have examined the scattering of light ions (n, d, ^{3}He, and α) by ^{6}Li with the single-channel resonating-group method. The cluster internal functions used are translationally-invariant shell-model functions of the lowest configurations in harmonic-oscillator wells having different width parameters. The result shows that the essential features of the cross-section angular distributions in all these four cases can be reasonably well explained. This indicates that, in spite of the adoption of a number of simplifying assumptions, the present calculation can still be used to study the main features of these nuclear systems.

The adoption of unequal width parameters for the interacting clusters results in a significant increase in computational effort, but is found to be necessary in the low-energy region where sharp resonance levels exist. In the higher-energy region above 10 MeV/nucleon, however, we note that the phase-shift and cross-section results obtained with equal and unequal width parameters are, in fact, rather similar. At these energies, if the purpose is merely to learn the essential features of scattering angular distributions or to understand the main characteristics of nucleon-exchange effects, then one can simplify the computation by appropriately choosing a common cluster width parameter to be used in the calculation.

The effects of the class-B nucleon-exchange terms, among which the core-exchange terms are very likely the most important, have particularly been carefully examined. Here the interesting finding is that the Born-approximation predictions, based largely on the exponential factors in the exchange-kernel function, can again be used to qualitatively and reliably explain many of the main features of antisymmetrization, even though in the present case a non-closed-shell nucleus is involved in the problem. In particular, it is found that the nucleon-number difference is still a dominating factor in determining the importance of the class-B exchange terms.

The phenomenon of blocking, arising from the presence of nucleons in a nonclosed shell, seems to have a rather profound influence. Because of this phenomenon, there exists generally a channel-spin dependence for the odd-even behavior of the phase shift. In addition, aided by a potential-model analysis, we have also found that it may have substantially reduced the effects of the core-exchange contributions.

In conclusion, we mention again that the present investigation is just another step in our search for a detailed understanding of the effects of antisymmetrization in nuclear systems. To achieve this aim, we have to carry through the tedious procedure of further performing other selected resonating-group calculations. Alternatively, one might proceed in a different direction by trying to determine analytically the general properties of the higher-power terms in the polynomial factors of the exchange-kernel function. However, this will also be a difficult task, and the accomplishment of which will require much mathematical expertise and computational innovations.

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*Present address: Space and Communications Group, Hughes Aircraft Company, El Segundo, California 90245.
†Present and permanent address: Department of Physics, Niigata University, Niigata, Japan.
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