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Mesonic folded diagram theory of the nucleon-nucleon potential

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We propose a general theory for deriving the nucleon-nucleon (NN) potential V from meson exchanges. Our potential preserves the half-on-shell NN T matrix, contains mesonic folded diagrams, and is energy independent. Mesonic folded diagrams are estimated to be of negligible effect for the long-range part of V but may be important for its medium- and short-range parts.

[NUCLEAR REACTIONS Nucleon-nucleon potential, meson exchanges, folded diagram, T matrix.]

A fundamental subject in nuclear physics is the nucleon-nucleon (NN) interaction. As is well known, nucleons interact with each other by exchanging elementary particles such as mesons. But in low-energy nuclear physics, it has been a long and rather successful tradition that nuclei are treated as a collection of nucleons (neutrons and protons) only, interacting with each other via a NN potential V. How do we reconcile these two apparently very different approaches? For many years, the determination or derivation of this V has been a central and active problem in theoretical nuclear physics (see, for example, Refs. 1–9). In this paper, we would like to propose a new theory for deriving the above NN potential V from meson exchanges. Our motivations are described below.

There are two rather interesting questions about the NN potential V. First, is V unique? As is well known, there exist many theories and models for V, such as the sequence of the phase-shift equivalent potentials. That there are many V's gives rise to a rather difficult situation, namely one is often not sure about which of these V's should be used in nuclear many-body calculations. Thus, it would be desirable if one could formulate a theory which leads to a unique V. The second question is about the so-called energy dependence of V. Early phenomenological models of V are of the energy independent (E-indep) type, such as the well-known Reid potential.[3] Several recent NN potentials[4–8] are, however, of the energy dependent (E-dep) type. As we will further discuss later, it is more convenient in several aspects to have an E-indep V, because, if V is E-dep, we need to use different V's for nuclear states of different energies and this causes difficulties in nuclear many-body calculations. Thus it will be very interesting as well as useful to formulate a theory where one can derive an E-indep V from meson exchanges. Johnson[9] has proposed such a theory. An essential question here is how to define an E-indep NN potential. Johnson's starting point is that such potentials should be "instantaneous." In this way, there is, however, a fundamental ambiguity[9] and, consequently, the resulting potentials are generally *not* intended to be unique. This we will discuss in more detail later on.

In this paper, we would like to propose a new theory for deriving, from meson exchanges, an essentially unique NN potential V which is E-indep and suitable for use in nuclear many-body calculations. We will show that such a theory can be readily obtained exactly, by using a physical T-matrix definition, which is basically different from that of Johnson, and by using the folded-diagram method of Kuo, Lee, and Ratcliff.[10] We will also show that the calculation of the long- and medium-range parts of our V appears to be particularly simple.

We start with the NN transition matrix $\langle f \mid T \mid i \rangle$ where f and i are each a two-nucleon state. T is the physical NN transition matrix where nucleons interact with each other by exchanging mesons and other elementary particles. Let us define an effective NN transition matrix $\langle f \mid \bar{T} \mid i \rangle$ where nucleons interact with each other only via an effective NN potential V. The physics given by $\bar{T}$ must agree with that given by T. This is an essential point, and leads to the basic equation

$$\langle f \mid T(E_i) \mid i \rangle = \langle f \mid \bar{T}(E_i) \mid i \rangle \ , \tag{1}$$

which defines our potential V. Note that we require the half-on-shell equivalence, within the NN subspace, of T and $\bar{T}$. Thus we preserve not only the phase shifts, but also the wave functions and other associated physical quantities.

To derive V from Eq. (1), it is convenient and probably essential to use the time-ordered perturbation theory (time-ordered Feynman diagrams).[1,8,9] The reason is that V is designed to act within the model space composed of nucleons only, excluding antinucleons. Nucleons and antinucleons are easily separated in the time-ordered formulation, where nucleons are denoted by upward-going lines and antinucleons by downward-going lines (see Fig. 1). In this formulation, the integral equations for T and $\bar{T}$ are, respectively,

$$\langle f \mid T(E_i) \mid i \rangle = \langle f \mid \textstyle\sum(E_i) \mid i \rangle + \langle f \mid \textstyle\sum(E_i) \mid j \rangle \times \langle j \mid G_0(E_i) T(E_i) \mid i \rangle \tag{2}$$

and

$$\langle f \mid \bar{T}(E_i) \mid i \rangle = \langle f \mid V \mid i \rangle + \langle f \mid V \mid j \rangle \times \langle j \mid G_0(E_i) \bar{T}(E_i) \mid i \rangle \,, \tag{3}$$

where f, j, and i are each a two-nucleon state with j summed over all such states. Note that E_i is the two-nucleon energy associated with the state $\mid i \rangle$, and $G_0(E_i)$ represents the free two-nucleon propagator. Note also that $\langle f \mid V \mid j \rangle$ is *independent* of E_i. The diagrammatic structures of these equations are shown in Fig. 1, where the vertex function $\sum$ is composed of irreducible diagrams such as diagrams (i)–(iv). Our purpose is then to derive V, the NN potential, which satisfies Eqs. (1)–(3).

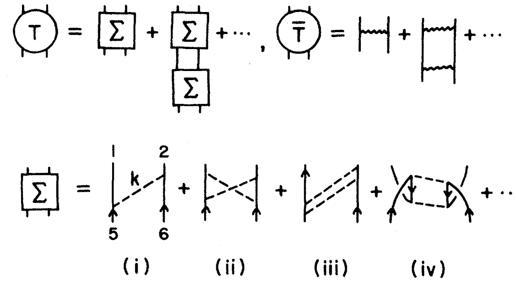

FIG. 1. Diagrammatic representations of T and $\bar{T}$. The V vertex is denoted by a wavy line and mesons by dashed lines. Down-going lines are antinucleons.

In the time-ordered formulation, we expand the T matrix in terms of the time-ordered Feynman diagrams. The rules for evaluating these diagrams are readily obtained.[9,1,8] To illustrate, let us consider the calculation of diagram (a) of Fig. 2. This diagram is a reducible T-matrix diagram having four πNN vertices, each of the form

$$H_i(t) = \int d^3x\, \mathscr{H}_i(x)$$

with

$$\mathscr{H}_i(x) = i\sqrt{4\pi}gN(\bar{\psi}\gamma_5 \vec{\tau}\cdot\phi\psi) \,,$$

located at times 0, t_1, t_2, and t_3 with the ordering

$$0 > t_1 > t_2 > t_3 > -\infty \,.$$

This diagram is given by

$$(\mathrm{a}) = (i^3) \int_{-\infty}^{0} dt_1 \int_{-\infty}^{t_1} dt_2 \int_{-\infty}^{t_2} dt_3\, I_a \tag{4}$$

with the integrand I_a given by

$$I_a = \sum_{p_3 p_4 k_1 k_2} \langle p_2 \mid H_i \mid p_4 k \rangle \langle p_1 k \mid H_i \mid p_3 \rangle \langle p_4 \mid H_i \mid p_6 k' \rangle \langle p_3 k' \mid H_i \mid p_5 \rangle (4\omega_k \omega_{k'})^{-1} \times \exp\{ i(E_1 + \omega_k - E_3)t_1 + i(E_4 - \omega_{k'} - E_6)t_2 + i(E_3 + \omega_{k'} - E_5)t_3 \} \,, \tag{5}$$

where $E_j = \sqrt{p_j^2 + m^2}$ and $\omega_k = \sqrt{k^2 + \mu^2}$. m and μ are, respectively, the nucleon and meson rest mass. The vertex matrix elements are given simply. For example, we have

$$\langle p' \mid H_i \mid pk \rangle = i\sqrt{4\pi}g(2\pi)^3 \delta(\vec{p}\,' - \vec{p} - \vec{k}) \times \bar{u}_{\vec{p}\,'\sigma'} \gamma_5 u_{\vec{p}\sigma} (m/E_{p'})^{1/2} (m/E_p)^{1/2} \,,$$

for the case of nucleon π^0-meson coupling. Note that at each vertex only the three momentum is conserved, but not the energy. This is a special feature of the time ordered diagrams, due to $\int d^3x\, \mathscr{H}_i(x)$. Therefore, in general, we can have $E_f \neq E_i$ for $\langle f \mid T \mid i \rangle$.

Having expressed the T matrix in terms of the time-ordered diagrams, the folded diagram method of Kuo, Lee, and Ratcliff,[10,11] can now be conveniently applied to derive the NN potential V. The integrands for diagrams (a), (b), and (c) of Fig. 2 are all the same, i.e, I_a of Eq. (5). But they have different time integration limits. They are

$$\int_{-\infty}^{0} dt_1 \int_{-\infty}^{0} dt_2 \int_{-\infty}^{t_2} dt_3$$

and

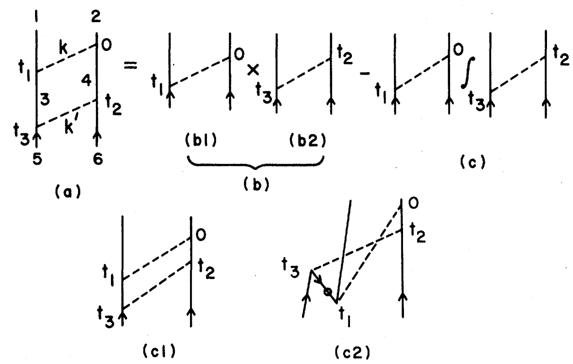

FIG. 2. Folded diagram factorization of the time-ordered T-matrix diagrams.

$$\int_{-\infty}^{0} dt_1 \int_{t_1}^{0} dt_2 \int_{-\infty}^{t_2} dt_3$$

for (b) and (c), respectively. Note that for (b), the upper integration limit for t_2 is 0. In this sense, diagram (b) is factorized into two parts, denoted as (b1)$\times$(b2). Clearly (a)$\neq$(b). But (a)=(b)−(c), with (c) defined as the mesonic *folded* diagram. The symbol $\int$ is to denote a generalized folding. Thus (c) is in fact a generalized folded diagram, meaning that it is the sum of several time-ordered folded diagrams. For example, we have (c)=(c1) + (c2) where (c1) has the time ordering

$$0 > t_2 > t_1 > t_3 > -\infty \ ,$$

which is in fact just the nonfolded diagram (iii) of Fig. 1, while (c2) has

$$0 > t_2 > t_3 > t_1 > -\infty \ .$$

Note that the propagator from t_3 to t_1 of diagram (c2) is *not* an antinucleon line, although it has a downward arrow. Instead, it is a folded nucleon line, and we have labeled it by a circle to emphasize this point.

The above procedure can be readily generalized[10,11] to factorize the general T-matrix expansion of Fig. 1, expressed in terms of the $\sum$ boxes. This is shown in Fig. 3. Clearly, the entire T matrix is given by diagrams (1) + (2) + (3) + $\cdots$. Diagram (2) is factorized into $\alpha\times A-\beta$ where β is a generalized once-folded diagram. Diagram (3) can be factorized into $\alpha\times B-\beta\times A+\gamma$, where γ is a generalized twice-folded diagram. Continuing this process, we see clearly

$$T=(\alpha-\beta+\gamma-\cdots)+(\alpha-\beta+\gamma-\cdots)$$
$$\times(A+B+\cdots)\ .$$

This leads to the central result that the T matrix of

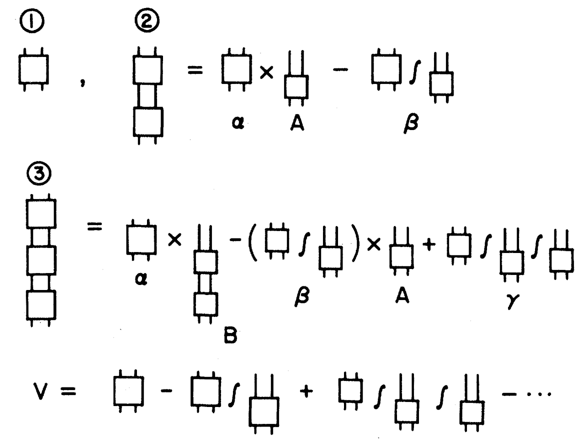

FIG. 3. Derivation of the NN potential V.

Eq. (2) is transformed into

$$\langle f\,|\,T(E_i)\,|\,i\rangle=\langle f\,|\,V\,|\,i\rangle + \sum_j \langle f\,|\,V\,|\,j\rangle \times\langle j\,|\,G_0(E_i)T(E_i)\,|\,i\rangle \qquad (6)$$

with

$$\langle f\,|\,V\,|\,j\rangle=\langle f\,|\,\textstyle\sum(E_j)-\sum\int\sum(E_j) +\sum\int\sum\int\sum(E_j)-\cdots\,|\,j\rangle \qquad (7)$$

as indicated by the diagrammatic equation in the third line of Fig. 3. Comparing Eqs. (1)−(3) with Eqs. (6) and (7), we see that V of Eq. (7) is exactly the V we needed for Eqs. (1)−(3). Thus we have completed the derivation of the NN potential V. We now discuss some special features of our theory.

1. Energy dependence. For the T matrix of Eq. (2), knowing the indices f and j alone is not sufficient in determining the value of $\langle f\,|\,\sum(E_i)\,|\,j\rangle$. In addition, we need to know the energy variable E_i which is independent of f and j. Hence we cannot define a potential V by equating $\langle f\,|\,\sum(E_i)\,|\,j\rangle$ to $\langle f\,|\,V\,|\,j\rangle$, if V is E-indep. But an E-dep potential V_E (Refs. 4−8) may, however, be defined by requiring

$$\langle f\,|\,\textstyle\sum(E_i)\,|\,j\rangle=\langle f\,|\,V_E(E_i)\,|\,j\rangle\ ,$$

where E_i is independent of f and j.

The NN potential V of the present work is, however, E-indep, as it is easily seen from Eq. (7) that $\langle f\,|\,V\,|\,j\rangle$ is only dependent on the indices f and j, but *not* E_i. This is convenient for many-body calculations. Note well that the folded diagrams contained in our V can also be derived from familiar perturbation theories, where one expands $T=\sum_n T^{(n)}$ and $V=\sum_n V^{(n)}$, (n) denoting the number of vertices contained in $T^{(n)}$ and $V^{(n)}$. Then from Eqs. (1)−(3), we readily have

$$\langle f \mid V^{(2)} \mid i\rangle = \langle f \mid T^{(2)}(E_i) \mid i\rangle ,$$

and

$$\begin{aligned}\langle f \mid V^{(4)} \mid i\rangle = &\langle f \mid T^{(4)}(E_i) \mid i\rangle \\ &-\langle f \mid T^{(2)}(E_j) \mid j\rangle \\ &\times\langle j \mid G_0(E_i)T^{(2)}(E_i) \mid i\rangle . \end{aligned} \quad (8)$$

Similarly we can expand the energy dependent potential V_E as $\sum_n V_E^{(n)}$. A delicate point should be pointed out. We have $V_E^{(2)}=V^{(2)}$. But $V_E^{(4)}\neq V^{(4)}$. $\langle f \mid V_E^{(4)} \mid i\rangle$ is given by Eq. (8) with $T^{(2)}(E_j)$ replaced by $T^{(2)}(E_i)$. This is a subtle yet crucial point. Diagram (a) of Fig. 2 is of the form

$$\langle f \mid T^{(2)}(E_i)G_0(E_i)T^{(2)}(E_i) \mid i\rangle .$$

But the factorized diagram (b) of Fig. 2 is of the form

$$\langle f \mid T^{(2)}(E_j) \mid j\rangle\langle j \mid G_0(E_i)T^{(2)}(E_i) \mid i\rangle .$$

Their difference is just the folded diagrams. It may be mentioned that V can be shown to be an energy average[11] of V_E.

2. Uniqueness. As mentioned earlier, it would be desirable and of interest to have a unique E-indep NN potential V. We have not succeeded in this regard, although we have made some progress toward this goal. Let us first examine the definition of V, which is of basic importance in discussing the uniqueness question. We have defined our V by the T-matrix conditions of Eqs. (1)−(3). Suppose that Eq. (1) is replaced by the on-shell condition

$$\langle i \mid T(E_i) \mid i\rangle = \langle i \mid \bar{T}(E_i) \mid i\rangle , \quad (9)$$

and we denote the potential defined by Eqs. (9), (2), and (3) as V'. Clearly one will have many more solutions for V' than for V, as the latter conditions provide a much weaker set of constraints on the NN potential than that provided by the former. In fact, the many solutions of V' are just the familiar phase-shift equivalent potential, as is obvious from Eq. (9). Another possible way to define the NN potential is to replace Eq. (1) by the left-on-shell condition

$$\langle f \mid T(E_f) \mid i\rangle = \langle f \mid \bar{T}(E_f) \mid i\rangle .$$

This and Eqs. (2) and (3) can lead to NN potentials which are, in general, different from V and V'.

So, it is clear that the question of uniqueness about V is intrinsically related to how we define the NN potential. In other words, it is related to what physical information one would like the potential to reproduce. In the present work, we have chosen V to reproduce not only the phase shifts (i.e., the totally on-shell T matrix) but also the projection of the physical eigen*ket* vectors onto the nucleonic model space. Within the framework of this definition, we have obtained a unique perturbation expansion for V as indicated by Eqs. (8) and (7). If this expansion is convergent, then it would lead to a unique nucleon-nucleon potential. As will be discussed in Part 3, higher order terms of this perturbation expansion appear to correspond to shorter ranges of the NN potential. Thus it is likely that our expansion is convergent for the long- and medium-range parts of the NN potential and therefore provides an essentially unique determination for these parts of the NN potential.

It has been a long tradition that one uses microscopic nuclear structure calculations, such as the calculation of the nuclear matter binding energy, to test which of the many phase-shift equivalent NN potentials is "best" for nuclear many-body calculations. We feel that it may be necessary for the NN potential V to satisfy the half-on-shell condition of Eq. (1), if it is to be used in nuclear many-body nuclear structure calculations. [For a general nuclear system with A nucleons, we still define V by Eqs. (1)−(3), except replacing the states f, j, and i by the corresponding A-nucleon states A_f, A_j, and A_i. V now has many-body components, but its general structure is still the same as that of Eq. (7). It can be shown that the two-body part of this V is the same as that for the $A=2$ system. Note that our V is generally non-Hermitian as shown by Eq. (7), but by Eq. (1) the energy eigenvalues given by it are still the same as given by the original mesonic Hamiltonian.] For example, the ground-state energy shift of nuclear matter can be written as

$$\Delta E_0 = \langle \Phi \mid \bar{T} \mid \Phi\rangle / \langle \Phi \mid (1+G_0\bar{T}) \mid \Phi\rangle ,$$

where Φ is the unperturbed ground state wave function in the nucleonic model space and G_0 the unperturbed Green's function. Thus to preserve ΔE_0, we need V to satisfy the condition specified by Eq. (1). It follows that among the many phase-shift equivalent potentials, only those who can satisfy Eq. (1) are suitable for use in the ΔE_0 many-body calculation mentioned above. It will be very useful as well as of much interest to further investigate the consistency between the definition and derivation of the NN potential and its subsequent use in nuclear many-body calculations.

It may be of interest to compare the present theory for the NN potential V with that of Johnson,[9] especially in the context of its degree of uniqueness. There are mainly two areas of difference. First, in Johnson's work the potential is defined to preserve the two-nucleon phase shifts, corresponding to the definition of Eq. (9). Thus Johnson's theory explicitly leads to many phase-shift equivalent potentials. Clearly, we have employed a different and much more restricted definition as discussed earlier. The second area of difference is the derivation of V. A basic requirement in Johnson's theory is that the NN potentials must be instantaneous, and this requires the shrinking of the finite-time irreducible $\sum$ boxes (see, for example, Fig. 1) to those of zero time duration.[12] There are many ways to perform this shrinking, thus leading to the introduction of free parameters for time averaging and consequently giving rise to a multiplicity of NN potentials.[9] Clearly, our procedure of deriving V has not employed such shrinking procedures. Instead, our procedures may be considered as elongating each $\sum$ box to that of infinite time duration. We have demonstrated that there is only *one* way to perform this elongation and thus we have obtained a unique expansion for V, as given by Eq. (7). In short, the constraint imposed on V by the present theory is considerably stronger than that by Johnson's theory, and this strongly limits the number of solutions for the NN potentials. In fact, Johnson's potentials, especially those obtained by the time averaging procedures advocated by him, do not in general—and are not intended—to satisfy Eq. (1), the constraint employed in the present theory. The diagrammatic and algebraic structure of Johnson's potentials are both quite different and somewhat more complicated than those of our V. For example, his potential contains one-meson folded diagrams while in our theory the lowest-order folded diagram has at least the exchange of two mesons. It will be of interest to further study the connection of these two theories.

3. Effect of mesonic folded diagrams. To estimate the effect of the mesonic folded diagrams, we have calculated the pion-fold-pion ($\pi\int\pi$) diagrams (c1) and (c2) of Fig. 2, for the case of neutral scalar nucleons and mesons. Using the same nonrelativistic approximation which leads to the one-pion-exchange potential $e^{-\mu r}/r$, we have found that these folded diagrams contribute a potential

$$(\mathrm{c1}+\mathrm{c2})=\frac{e^{-\mu r}}{r}\int_0^\infty dq\frac{\cos qr}{(\mu^2+q^2)^{1/2}} \simeq\frac{e^{-2\mu r}}{r}\left[a_0+\frac{a_1}{\mu r}+\frac{a_2}{(\mu r)^2}\right], \qquad (10)$$

where μ is the pion rest mass, $a_0\simeq 0.32$, $a_1\simeq 1.21$, and $a_2\simeq -0.36$. It is interesting to note that the range of the $\pi\int\pi$ diagram is $\sim\frac{1}{2}\mu$, i.e., half the one-pion-exchange range. Based on the uncertainty principle ($\Delta E\Delta t\sim\hbar$), we expect the above to be a general result and consequently diagrams of more folds will be of even shorter ranges. For example, we expect the range of the $\pi\int\pi\int\pi$ diagram to have a range of $\frac{1}{3}\mu$. Comparing with the E-dep NN potentials,[4–8] our potential has a new type of diagram—the mesonic folded diagram. The above observations indicate that the calculation of the E-indep potential, especially its long- and medium-range parts, is not more complicated than that of the E-dep potential. Folded diagrams will have only a negligible effect on the long range part of V. For its medium-range part, we probably only need to calculate the lowest order folded diagram $\pi\int\pi$, in addition to the usual nonfolded diagrams. Diagrams with more folds will influence the short-range parts of V, but we should probably not calculate them because the short range parts of V are not yet all well understood and had better be treated phenomenologically.[6] For the case of neutral scalar nucleons and mesons, we have found a strong cancellation between the folded diagrams (c1) and (c2) and the nonfolded two-pion diagrams (ii) and (iii) of Fig. 1. Note that diagram (iii) is canceled by diagram (c1) exactly.

In conclusion, we have proposed a new and exact theory for deriving the E-indep NN potential V. Because it is E-indep [or half-on-shell, as indicated by Eq. (7)], this V may be more convenient, in some important aspects, for use in nuclear many-body calculations than the E-dep potentials. For example, if V is E-dep, there are the well known off-shell difficulties in defining an average nuclear field for nucleons in a nucleus. But for the E-indep V, these difficulties are simply removed. It will be very interesting to further investigate the $\pi\int\pi$ diagram, as it may have a significant effect on the medium range part of V. Investigations along this line are in progress.

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[1]G. E. Brown and A. D. Jackson, *The Nucleon-Nucleon Interaction* (North-Holland, Amsterdam, 1976).
[2]C. W. Wong and K. F. Liu, in *Lecture Notes in Physics,* edited by T. T. S. Kuo and S. S. M. Wong (Springer, Berlin, 1981), Vol. 144.
[3]R. V. Reid, Ann. Phys. (N.Y.) 50, 411 (1968).
[4]M. Chemtob, J. W. Durso, and R. O. Riska, Nucl. Phys. B38, 141 (1972).
[5]E. L. Lomon and M. H. Partovi, Phys. Rev. D 2, 1999 (1970); 8, 2307 (1973).
[6]M. Lacomb *et al.,* Phys. Rev. C 21, 861 (1980).
[7]A. D. Jackson, J. W. Durso, and B. VerWest, Nucl. Phys. A249, 397 (1975).
[8]H. Müther, A. Faessler, M. R. Anastasio, K. Hollinde, and R. Machleidt, Phys. Rev. C 22, 1744 (1980).
[9]M. B. Johnson, Ann. Phys. (N.Y.) 97, 400 (1976).
[10]T. T. S. Kuo, S. Y. Lee, and K. F. Ratcliff, Nucl. Phys. A176, 65 (1971).
[11]T. T. S. Kuo, F. Osterfeld, and S. Y. Lee, Phys. Rev. Lett. 45, 786 (1980).
[12]M. B. Johnson and M. Baranger, Ann. Phys. (N.Y.) 62, 172 (1971).