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Intermediate structure of $^{12}C+^{12}C$ system

S. Y. Lee and H. W. Wilschut
Department of Physics, State University of New York at Stony Brook, Stony Brook, New York 11794

R. Ledoux
Department of Physics and Laboratory for Nuclear Science, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139

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We show that the $^{12}C + ^{12}C$ elastic angular distribution can be described very well with an S matrix obtained from a deep optical potential to which resonance is added. We interpret these resonances as bound states embedded in a continuum. Their effect on the total reaction cross section is also discussed.

[NUCLEAR REACTIONS Resonances in heavy ion scattering.]

Intermediate structure ($\Gamma \approx 400$ keV) in the excitation function of systems like $^{12}C + ^{12}C$, $^{12}C + ^{16}O$, and $^{16}O + ^{16}O$ has been known for many years.[1,2] It is most prominently present in the $^{12}C + ^{12}C$ system and therefore most experimental and theoretical studies concerning this phenomenon have been focused on this reaction. Explanations range from statistical fluctuation[2] to existence of rotational bands in the ^{24}Mg nucleus [e.g., the band crossing model (BCM) (Ref. 3)]. Shapiro *et al.*[2] have shown that the single angle excitation functions are indeed consistent with the statistical theory, which implies that the S matrix fluctuates statistically around the optical model value. In the BCM,[3] one argues that the shape resonances of the entrance and exit channels form crossing rotational bands which give rise to the intermediate structure observed in the experimental data. (Note that the recent effort in BCM is to understand the gross structure.) This mechanism requires an l- or J-dependent absorptive potential to simulate a sharp resonance in the heavy ion system. As of now there is no clear evidence for the necessity of the l- or J-dependent absorption. In addition, the origin of these rotational bands is not clearly understood. All these models are able to fit to some degree the experimental excitation functions.[4–7] However, the energy dependence of the S matrix differs in all models. Thus a phase shift analysis is therefore a sensitive tool to distinguish various models (see also Ref. 8).

Recently, Ledoux *et al.*[9] measured many more sets of elastic angular distributions for the $^{12}C + ^{12}C$ system. Their phase shift analysis shows strong evidence for the resonant character of the intermediate structure with $J_{res} \cong l_{grazing}$. The large number of free parameters, however, leads to possible ambiguities in the background phase shifts,[10] which, obtained in Ref. 9, do not agree with any realistic optical potential.

We believe that a mean field or optical potential exists for the heavy ion system. Phenomenological studies over the past few years indicate that a deep optical potential is preferred.[11] Because of the large Coulomb interaction one does not need to know the exact shape of the potential.[12] One can use, for example, the double folding G-matrix interaction for which a detailed prescription has been given by Satchler *et al.*[13] Such a mean field will produce molecular resonances that are very broad and, therefore, give rise to gross structure only ($\Gamma_{gross} \gtrsim 3$ MeV).[14,15]

In this report, a partial phase-shift analysis is performed to fit the $^{12}C + ^{12}C$ elastic angular distributions. We shall present evidence that the origin of the intermediate structure in the excitation function is a result of the existence of bound states embedded in the continuum (BSEC), where the continuum is composed of either underlying states in the compound nucleus or of direct channels. For example, it is known that (sd) shell nuclei exhibit a considerable amount of cluster structure.[16] Since clus-

ter states couple only weakly to the direct reaction channels (including the elastic channel) and the complicated states in the compound nucleus, they may be considered as BSEC states.

The elastic S matrix in the presence of sharp resonances (e.g., BSEC) is given by[17]

$$S_{cc}=\langle S_{cc}\rangle\left[1-i\sum_{\mu}\frac{\Gamma_{\mu,\mathrm{el}}}{E-E_{\mu}+i\frac{1}{2}\Gamma_{\mu}}\right]$$

$$=\langle S_{cc}\rangle[1+a(E)+ib(E)]\,, \tag{1}$$

where $\langle S_{cc}\rangle$ is the S matrix of the optical potential. The depths of the optical potential are rather important to describe the refractive scattering.[14] We use the double folding optical potential[13] with the $M3Y$ force.[18] The density of the ^{12}C nucleus is taken from electron scattering analysis.[19] The imaginary part is chosen to have the same form factor as that of the real part. The strength of the imaginary potential is $W_0(E)=5.0+0.5E_{\mathrm{c.m.}}$, which fits the magnitude of the cross section at 90° reasonably well.

Equation (1) manifests itself in that the effect of the resonance on the scattering is most important near the grazing angular momentum L_{gr}. The reason is simple: For angular momentum with $l\gg L_{\mathrm{gr}}$, the partial width $\Gamma_{\mu,\mathrm{el}}$ is small owing to small penetrability, while for $l\ll L_{\mathrm{gr}}$, $\langle S_{cc}\rangle$ is itself small. Therefore, the scattering process is affected mainly by resonances near the grazing wave. Thus, the term $a(E)+ib(E)$ was added to the elastic S matrix of the optical potential for the grazing partial wave. Note that $a(E)$ and $b(E)$ are free parameters at each energy without an *a priori* assumption for the energy dependence. At the transition region, where the grazing angular momentum lies in between two even integer values, we used two such terms, i.e., four parameters. This happens at $E_{\mathrm{c.m.}}\simeq 15$ and 19.5 MeV, where we have used parameters for partial waves 10, 12 and 12, 14, respectively. Figure 1 is a sample of the results of our analysis from $E_{\mathrm{c.m.}}=14.6$ MeV to $E_{\mathrm{c.m.}}=23$ MeV. We choose to work in this energy range because the data there are obtained in smaller energy steps. The χ^2/N was about 20–50 in the two-(sometimes four)-parameter fit. The data can be fitted almost perfectly at every point with six parameters (three partial waves). We feel, however, that the accuracy of the data is not able to support this degree of freedom as is evident from the rapid increase of the error in the fitted parameters.

From Eq. (1), the real and imaginary parts of the

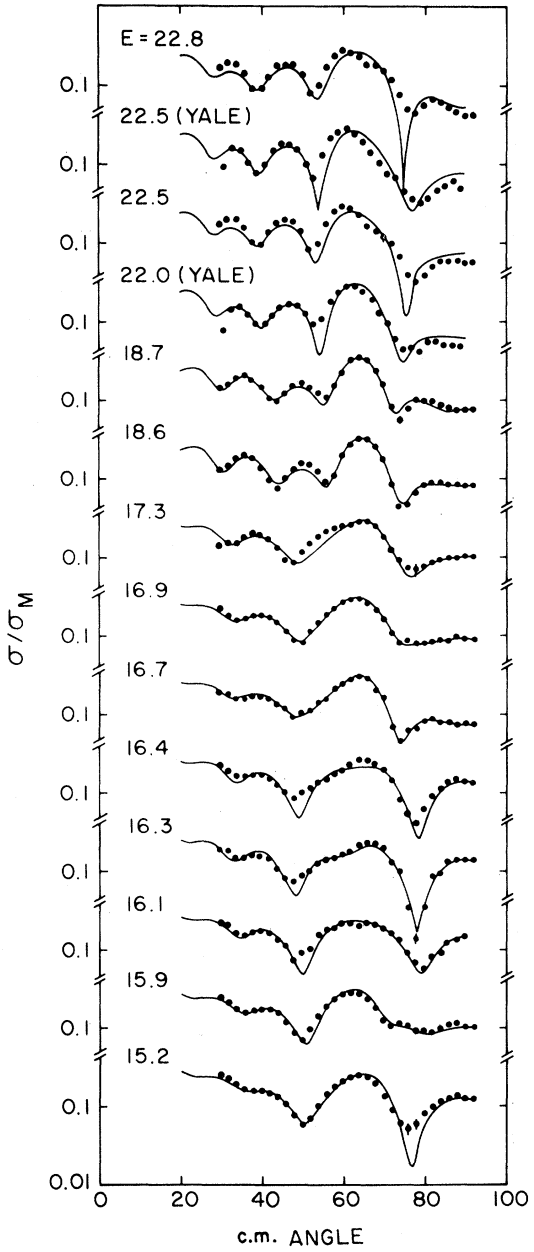

FIG. 1. Sample of fitted angular distributions. The data is taken from Ref. 9. Yale data in Fig. 1 is taken from Wieland in Ref. 1..

$$a(E)=-\sum_{\mu}\frac{\Gamma_{\mu}\Gamma_{\mu,\mathrm{el}}}{(E-E_{\mu})^2+(\frac{1}{2}\Gamma_{\mu})^2} \tag{2}$$

and

$$b(E)=-\sum_{\mu}\frac{\Gamma_{\mu,\mathrm{el}}(E-E_{\mu})}{(E-E_{\mu})^2+(\frac{1}{2}\Gamma_{\mu})^2}\,. \tag{3}$$

added term in S matrix should have the form
Figure 2 shows $a(E)$ and $b(E)$ for $l=12$ and 14 obtained in fitting these angular distributions. $a(E)$ shows clearly a Lorentzian shape around resonance energies and $b(E)$ also shows the dispersive feature. Note, however, that $b(E)$ is much more sensitive to the interference of resonances than $a(E)$. One can obtain resonance parameters, E_μ, Γ_μ, and $\Gamma_{\mu,\mathrm{el}}$, from χ^2 fit with Eqs. (2) and (3) to the fitted parameters $a(E)$ and $b(E)$. To this end, one needs more angular distribution data from $E_{\mathrm{c.m.}}=15$ MeV to $E_{\mathrm{c.m.}}=18$ MeV. Figure 2 shows, however, that these resonances are not overlapping. In this limit, E_μ and Γ_μ can be obtained from the peak position and width of the Lorentzian shape of $a(E)$ and the partial width can be estimated by $\Gamma_{\mu,\mathrm{el}}/\Gamma_\mu \cong \frac{1}{2}a(E_\mu)$. Note that $\langle \Gamma_\mu(l=12)\rangle \cong 450$ keV is almost independent of the excitation energy and $\Gamma_{\mu,\mathrm{el}}= 2\cdot P(l)\gamma_{\mu,\mathrm{el}}{}^2$ will increase with the penetrability $P(l)$. For $l=12$, $\Gamma_{\mu,\mathrm{el}}/\Gamma_\mu \cong 0.1\sim 0.3$ (Fig. 2).

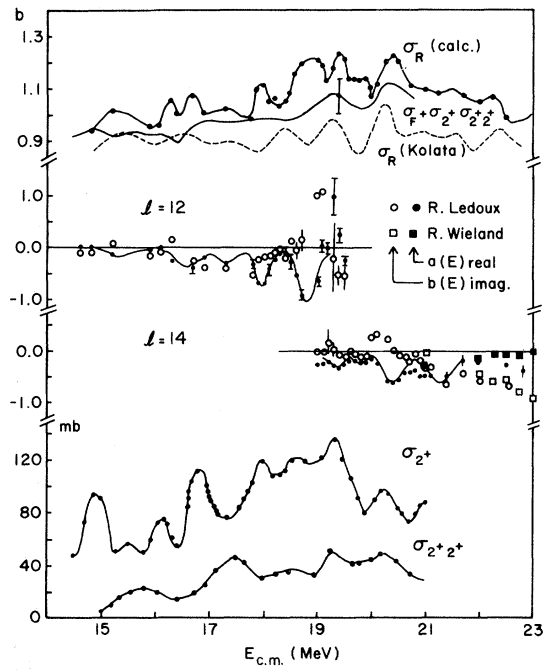

FIG. 2. The lower part of the figure shows the total angle integrated 2^+ and natural $(2^+,2^+)$ inelastic cross sections (Ref. 5). In the middle of the figure, we show the parameters $a(E)$ and $b(E)$ from the χ^2 fit. The solid curve drawn through $a(E)$ is only to guide the eye. Note that there is a difference between the fitted parameters $a(E)$ and $b(E)$ for Yale and MIT data. On the upper part, we compare the calculated total reaction cross section with that of Ref. 4 and the sum of total fusion cross section, single and mutual 2^+ inelastic cross sections.

The average spacing D between BSEC states is about 750 keV. Thus we have $\Gamma/D \cong 0.6$, which indicates that the statistical explanation[2] of these intermediate structures is unlikely and at best marginal. On the other hand, there will be great difficulty in trying to reproduce the spacing of these resonances by the band crossing models.[3] (The spacing of the molecular resonance of the C + C system is of the order of 3 MeV.) Note also that the angular momentum of $E_{\mathrm{c.m.}}=19.3$, 19.8, and 20.4 MeV states is $14\hbar$, which was previously[9] determined to be $12\hbar$.

Since $a(E)$ in Fig. 2 has a very nice Lorentzian shape, the mixing phase between the resonance term and the background S matrix is small. Equations (2) and (3) assume specifically zero mixing phase. The fact that the mixing phase is small supports the BSEC interpretation. Presumably there are two possible interpretations for these BSEC states: (1) They may be states near the yrast line of the composite system. The width and level density of such states are expected to follow the level density formula. Our analysis indicates that the width and level density of BSEC states are essentially constant 15 MeV $< E_{\mathrm{c.m.}} <$ 20 MeV for $l=12$. (2) They could be the cluster states. Spacing of these states depends on the cluster configuration. The width of these states is $\Gamma_\mu=\Gamma_\mu^\downarrow+\Gamma_{\mu,\mathrm{el}}+\Gamma_{\mu,\mathrm{inel}}+\Gamma_{\mu,\alpha}$. $\Gamma_\mu^\downarrow$ is the partial width of coupling to the compound nuclear states, which could lead to one-nucleon emission. The escape width $\Gamma_\mu^\uparrow=\Gamma_{\mu,\mathrm{el}}+\Gamma_{\mu,\mathrm{inel}}+\Gamma_{\mu,\alpha}$ will depend on the reduced width and penetrability of each channel, which depend on energy. Since the observed Γ_μ (see Fig. 2) is rather energy independent, it will be interesting to investigate the energy dependence of the partial widths of various channels. Investigation of neutron and proton emission is also interesting to obtain information about the partial width $\Gamma_\mu^\downarrow$ of BSEC states.

Before closing this section, it is worth pointing out that the angular distribution cannot be fitted well with a "normal" shallow optical potential (e.g., $V_0=20$ MeV), where normal means no surface transparency, and no l dependent absorption is added to the optical potential. We emphasize in our analysis the importance of the refractive effect in the mean field (or the background S matrix), which has been thoroughly discussed in Ref. 14.

To study the effect of the BSEC on the total reactions, we show the calculated total reaction cross section from the fitted elastic S matrix on the upper part of Fig. 2. The experimental reaction cross section of Kolata *et al.*[4] and the cross section of

$\sigma_F+\sigma_{2^+}+\sigma_{2^+,2^+}$ (Ref. 5) are also shown for comparison. If these oscillatory structures in the total cross section come from the resonances in Eq. (1), their contribution to the total reaction cross section is given by a sum of Lorentzian forms added to the reaction cross section of the mean field

$$\sigma_R=\langle\sigma_R\rangle+(2J+1)\frac{\pi}{k^2}\times\sum_\mu\frac{(\Gamma_\mu-\Gamma_{\mu,\mathrm{el}})\Gamma_{\mu,\mathrm{el}}}{(E-E_\mu)^2+(\frac{1}{2}\Gamma_\mu)^2}\ , \tag{4}$$

where we have made an approximation that these resonances do not overlap with each other. $\langle\sigma_R\rangle$ is the total reaction cross section calculated from the optical model potential. If these resonances overlap with each other, one should also consider the interference contribution to Eq. (4). Note that our calculated result is systematically larger than the measured cross section, which is consistent with our spin assignment being two units higher than in previous studies. For example, we found the 20.4 resonance to be 14^+ instead of 12^+. The oscillatory structure in our calculated total reaction cross section does, however, agree reasonably well with that of the measured reaction cross sections.

In conclusion, we have found that the angular distribution can be satisfactorily fitted by adding resonance terms (BSEC) to the S matrix calculated from the folding potential with the two nucleon G-matrix interaction ($M3Y$). Owing to the weak coupling and small mixing phase, these states give rise to sharp structure on top of the cross section calculated from the mean field. In our analysis, we found that $a(E)$ and $b(E)$ do not possess the property that the statistical theory predicts. Therefore, the statistical explanation of the intermediate structure seems unlikely. Interpretation of the nature of these BSEC states is, however, remaining open. We, however, argued in favor of the cluster explanation. To investigate this important question, one should analyze elastic and inelastic channels simultaneously in detail and obtain a set of consistent resonance parameters. Theoretical study (based on some models) of the structure of these BSEC states can then be compared with the partial widths deduced from the experimental data.

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