Particle-hole ambiguity in the parameters of the microscopic interacting boson model

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The parameters of the microscopic interacting boson model that emerge from a multi-nondegenerate-orbit theory are shown to depend on whether they are generated in terms of correlated pairs of particles or correlated pairs of holes. It is suggested that the ambiguity results from the shell-model truncation inherent in the interacting boson model and that it can be alleviated by including renormalization effects from outside the interacting boson model subspace.

[NUCLEAR STRUCTURE Microscopic interacting boson model; particle-hole ambiguity in model parameters.]

The purpose of this paper is to report an apparent particle-hole ambiguity in the *microscopic derivation* of the parameters of the interacting boson model (IBM).[1] More specifically, we have found that when the parameters of the IBM are derived from the underlying shell model, they depend sensitively on whether the fundamental "boson" building blocks are correlated pairs of particles or correlated pairs of holes.[2] By considering the origin of the ambiguity, we have concluded that it can be removed by "accurately" incorporating renormalization effects that arise from configurations outside the standard IBM space.

We first discovered this ambiguity while carrying out calculations for the parameters κ_ν and χ_ν appropriate to the tungsten isotopes using a recently developed procedure which permits the exact calculation of low-generalized-seniority[3] matrix elements for several nondegenerate orbits. The basis of our procedure is to expand the nth power of the $J=0^+$ correlated pair creation operator in terms of $J=0^+$ pair creation operators for individual orbits. In this way, matrix elements for the multiorbit problem can be related to matrix elements for individual orbits, for which the usual seniority-reduction formulas apply.[4] The procedure will be discussed in detail in a subsequent article.[5]

In our calculations for the tungsten isotopes, we assumed that the valence neutrons were distributed over the single-particle levels listed in Table I.[6] They were then allowed to interact via a residual surface delta interaction (SDI)[7] with isovector strength parameter $A_1=0.1$ MeV. The structures of the correlated S and D (favored)[8] pairs were obtained by diagonalizing the Hamiltonian $H_{\nu\nu}$ in the space of two-neutron configurations and retaining the energetically-lowest $J=0^+$ and $J=2^+$ eigenvectors. The neutron parameters κ_ν and χ_ν were then obtained by equating the matrix elements of the fermion quadrupole operator,

$$Q^F=\frac{1}{\sqrt{5}}\sum_{j_1,j_2}\langle j_1||r^2Y_2||j_2\rangle\left[\nu^\dagger_{j_1}\tilde{\nu}_{j_2}\right]^{(2)}, \quad (1)$$

between states with generalized seniorities 0 and 2 with those of the boson quadrupole operator

$$Q^B=\kappa_\nu\left[\left[d^\dagger_\nu\tilde{s}_\nu+s^\dagger_\nu\tilde{d}_\nu\right]+\chi_\nu\left[d^\dagger_\nu\tilde{d}_\nu\right]\right]^{(2)}, \quad (2)$$

between the corresponding boson states. In Eq. (1)

TABLE I. Single-particle and single-hole energies used in calculations of correlated-particle and correlated-hole pairs for the tungsten isotopes.

Valence orbit	Single-particle energy (in MeV)	Single-hole energy (in MeV)
$2f_{7/2}$	0	2.50
$1h_{9/2}$	0.20	2.30
$3p_{3/2}$	1.60	0.90
$1i_{13/2}$	1.80	0.70
$2f_{5/2}$	2.25	0.25
$3p_{1/2}$	2.50	0

$v_j^\dagger(\tilde{v}_j)$ is the *neutron* creation (annihilation) operator for valence orbit j. Similarly, in Eq. (2) $s_\nu^\dagger$ and $d_\nu^\dagger$ ($\tilde{s}_\nu$ and $\tilde{d}_\nu$) are the creation (annihilation) operators for s and d *neutron bosons*. The resulting κ_ν and χ_ν parameters are given by the solid curves in Figs. 1(a) and (b), respectively.

We then considered the possibility of deriving the parameters of the boson Hamiltonian in terms of correlated hole pairs. In this case, we assumed that the valence orbits in Table I are filled and determined the operators that create the energetically-lowest 0^+ and 2^+ *two-hole* states. The operators were obtained using the same SDI and the analogous single-hole spectrum, also given in Table I. Applying the same procedure of equating fermion quadrupole matrix elements to boson quadrupole matrix elements, but now for holes, leads to the parameters given by the dashed curves in Figs. 1(a) and (b).

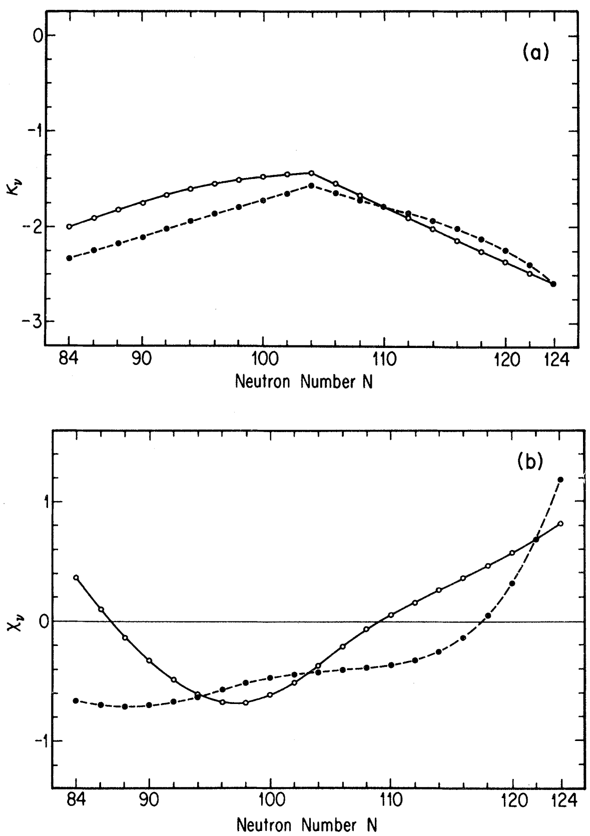

FIG. 1. Multiorbit predictions for the IBM parameters (a) κ_ν and (b) χ_ν appropriate to the tungsten isotopes vs neutron number N. The results of calculations based on correlated S and D particle pairs are indicated by open circles and are connected (to guide the eye) by solid lines. The results of calculations based on correlated hole pairs are indicated by solid dots and are connected by dashed lines. The magnitude of κ_ν is given in arbitrary units.

In both figures, the solid and dashed curves disagree so that the parameters κ_ν and χ_ν depend on whether they are generated in terms of correlated pairs of particles or correlated pairs of holes.

The procedure that is usually adopted in the phenomenological IBM (and most relevantly in its neutron-proton version)[9] is to introduce particle bosons up to the middle of the shell and hole bosons from the middle to the end. In the $N=82-126$ valence shell under discussion, the midshell nucleus with $N=104$ can be treated either as 11 neutron particle bosons or as 11 neutron hole bosons. The phenomenological midshell parameters are of course independent of the choice. The analogous microscopic IBM prescription would be to use correlated particle pairs to generate the parameters in the first half of the shell and correlated hole pairs to generate the parameters in the second half. However, as is evident from Fig. 1, the midshell $N=104$ parameters that are derived microscopically do depend on whether they are generated from particle pairs or from hole pairs.

The midshell ambiguity is also evident in the neutron occupation numbers for the generalized-seniority-zero states, as can be seen in Table II. The first row in the table gives the distribution of valence neutrons in the 22-neutron generalized-seniority-zero state generated in terms of particle pairs. The second row gives the distribution for the analogous state generated in terms of hole pairs. The results were obtained by evaluating the expectation value of the operator $\hat{N}_j=-\sqrt{2j+1}[v_j^\dagger\tilde{v}_j]^{(0)}$ which can also be done exactly using our formalism. As for the quadrupole parameters κ_ν and χ_ν, the particle and hole predictions for the neutron occupation numbers differ.

Although we have only discussed the neutron parameters in the $N=82-126$ major shell, similar results have been found for the neutron parameters appropriate to the $N=50-82$ major shell and also for the proton parameters appropriate to the $Z=50-82$ major shell.

The ambiguity we have reported is a consequence of the shell-model truncation inherent in the IBM. Complete shell-model calculations for a given major shell yield results that are independent of whether they use particles or holes, as long as the same single-particle spectra and interactions are used. This is not to say that the results of an n-particle calculation are identical to those of an n-hole calcu-

TABLE II. Distribution of valence neutrons in the 22-valence-particle and 22-valence-hole (midshell) states with generalized seniority zero.

	$f_{7/2}$	$h_{9/2}$	$p_{3/2}$	$i_{13/2}$	$f_{5/2}$	$p_{1/2}$
Particle state	7.18	8.18	1.25	3.81	1.23	0.35
Hole state	6.55	7.95	1.74	4.84	0.82	0.10

lation, but rather that the results of an n-particle calculation are identical to those of a $(2\Omega - n)$-hole calculation of the same nucleus, where 2Ω is the degeneracy of the shell. However, once the particle and hole spaces are truncated, the equivalence need not be maintained. In particular, despite their apparent similarity, a truncation to the lowest S- and D-particle pairs is not necessarily equivalent to a truncation to the lowest S- and D-hole pairs.[2]

The above remarks suggest that in order to regain the desired particle-hole independence in the microscopic IBM, it is crucial to include renormalization effects from outside the S-D subspace. The most important renormalization effects are expected from the lowest $J = 4^+$ pair (the so-called g boson) and from excited $J = 0^+$ and 2^+ pairs (the s' and d' bosons). Such renormalization effects will also not be the same for the parameters of the particle IBM and for those of the hole IBM (Ref. 10); however, if included with sufficient accuracy they should restore the desired particle-hole independence. The crucial question still to be addressed is whether sufficient accuracy can be achieved either in low orders of perturbation theory or via some nonperturbative approximation scheme.

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[2]It should be noted that this statement is only true for nondegenerate-orbit calculations. Calculations for a single orbit or for degenerate orbits do not show an ambiguity between the results for particle pairs and for hole pairs.

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