Some properties of projected three-body theories of deuteron-nucleus collisions

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Three-body theories of deuteron-nucleus collisions, based on limited sets of relative angular momenta, are discussed as projected components within a more complete theory. It is argued that the extension to the more complete theory is well behaved and that the previous truncations are reasonable. Possible tests are considered.

[NUCLEAR REACTIONS Scattering theory, deuteron-nucleus, angular-momentum projected three-body theory.]

Several alternative decompositions of the quantum-mechanical three-body problem are known,[1-3] which preserve the mathematical reliability of the Faddeev theory, but which put different aspects of the dynamics into their definitions of component wave functions. The present report reviews another formal decomposition into coupled component wave functions, which embeds and extends certain heuristic truncated three-body theories of deuteron-nucleus scattering.[4-9] This extension of the heuristic theories to a complete coupled formalism is seen to be appropriate, a result that enhances the interest of these theories and provides encouragement for their further development.

The target "nucleus" is assumed to be structureless and at rest, so that the system is described by a Hamiltonian

$$H=K+U_n(r_n)+U_p(r_p)+V(r)\ , \tag{1}$$

where K is the kinetic energy operator for the neutron and proton and V is the neutron-proton interaction. The single-particle potentials U_n and U_p may be real or complex, as required for particular applications. (Complex single-particle potentials are known to complicate the description of rearrangement channels.[6]) Relative and center of mass coordinates are $\vec{r}=\vec{r}_p-\vec{r}_n$ and $\vec{R}=\frac{1}{2}(\vec{r}_p+\vec{r}_n)$.

The truncated three-body theories are constructed in terms of a projection operator P_N that selects the lowest N partial waves of angular momentum in the relative coordinate $\vec{r}$. Thus we define

$$\Psi=\Psi_1+\Psi_2\ , \tag{2}$$

with

$$\Psi_1\equiv P_N\Psi,\quad \Psi_2\equiv(1-P_N)\Psi\ . \tag{3}$$

Applications treated thus far[4-9] have emphasized projection operators for s waves, or for s and d waves. However, the essential structure of the projected theory does not depend on the number of partial waves selected by P_N.

It is characteristic[5,6] of the projected theory that distinct asymptotic arrangements of the particles into two-body channels are separated uniquely between Ψ_1 and Ψ_2. To see this, we recognize first that the deuteron channel has a definite relative angular momentum $l=0$ and therefore it must be in Ψ_1 and not in Ψ_2. On the other hand, a stripping rearrangement channel has an asymptotic form such as

$$\phi(r_n)e^{ikr_p}/r_p\ , \tag{4}$$

[where for simplicity the bound wave function $\phi(r_n)$ has been chosen to have angular momentum zero]. The application of P_N to obtain the Ψ_1 component of Eq. (4) then leads to integrals with the structure

$$\int d\hat{r}\,Y_{lm}(\hat{r})\phi(r_n)(e^{ikr_p}/r_p)\ . \tag{5}$$

Such integrals decrease asymptotically as $(rR^2)^{-1}$, essentially because as r,R increase the finite-range function $\phi(|\vec{R}-\frac{1}{2}\vec{r}|)$ subtends a decreasing part of the angular range of $\hat{r}$. This analysis holds for *any finite number* of partial waves that may be selected by P_N, and it allows the conclusion that Eq. (4) makes a vanishing asymptotic contribution to Ψ_1. Thus all channels that have one nucleon bound

to the target nucleus appear in Ψ_2.

Of course, the component wave functions can also contain three-body breakup channels, in which three free particles appear asymptotically. We recall that the wave functions in three-body channels[10] decrease asymptotically as $\mathscr{R}^{-5/2}$, where $\mathscr{R}=(r_n^2+r_p^2)^{1/2}$.

Suitable coupled equations for Ψ_1 and Ψ_2 are

$$P_N H(\Psi_1+\Psi_2)=0\ , \tag{6}$$

$$(1-P_N)H(\Psi_1+\Psi_2)=0\ , \tag{7}$$

which become

$$[E-K_R-\bar{U}-(K_r+V)]\Psi_1 = P_N(U_n+U_p)\Psi_2\ , \tag{8}$$

$$[E-K_R-\hat{U}-(K_R+V)]\Psi_2 = (1-P_N)(U_n+U_p)\Psi_1\ , \tag{9}$$

where

$$\bar{U}\equiv P_N(U_n+U_p)P_N\ , \tag{10}$$

$$\hat{U}\equiv(1-P_N)(U_n+U_p)(1-P_N)\ . \tag{11}$$

The diagonal potentials $\bar{U}$, $\hat{U}$ have a weak long range tail,[6] as in Eq. (5), which decreases asymptotically as $(rR)^{-1}$. This long range tail is only relevant to the open three-body parts of Ψ_1 and Ψ_2, and in this application its effects in matrix elements are reduced further[11] by averaging over the variety of energy sharing between the $\vec{r},\vec{R}$ degrees of freedom in the three-body channels. The net effect is that the necessary matrix elements of $\bar{U},\hat{U}$ decrease asymptotically as $R^{-5/2}$.

Equations (8) and (9) are solvable in principle if the coupling terms on their right hand sides are sufficiently short ranged. There are three sets of terms to consider: (1) Contributions to the coupling terms from open three-body channels have ranges controlled by the $\mathscr{R}^{-5/2}$ decrease of those parts of Ψ_1 and Ψ_2. This behavior is typical of the configuration space Faddeev theory,[12] where it is considered acceptable. (2) The two-body channel in Ψ_1 in Eq. (9) contains a bound function of $\vec{r}$ and it is multiplied by short-ranged functions of r_n and r_p, therefore these contributions to the coupling term in Eq. (9) are reliably short ranged, again as in the Faddeev theory. (3) Only the two-body channels in the coupling term in Eq. (8) require more care, because in this case wave functions such as Eq. (4) are multiplied by potentials that contain the same variables. Fortunately, the P_N projection operator introduces an additional $(rR)^{-1}$ decrease of the coupling term, as in Eq. (5). We conclude that the coupling terms are sufficiently short ranged for usual solution procedures.

Thus, in principle Eqs. (8) and (9) can be solved for Ψ_1 and Ψ_2, perhaps by suitable iterative or variational methods. In general $\vec{r},\vec{R}$ coordinates should be used in Ψ_1 and $\vec{r}_n$, $\vec{r}_p$ coordinates in Ψ_2, to correspond with the asymptotic structure of the two-body channels in the respective component wave functions.

Most truncated versions of the projected formalism[4–7,9] carry only the left hand side of Eq. (8), and entirely omit Ψ_2. It is known that under such angular momentum truncation the kernel of the three-body Lippmann-Schwinger equation has well-behaved, Hilbert-Schmidt properties.[13] The present formalism suggests systematic study of the errors of the truncated theories, by means of more complete calculations that include Ψ_2.

On the other hand, the present formalism also provides a setting that allows the truncations to seem more reasonable in themselves. Thus, even the simplest version of Ψ_1 incorporates several principal physical features of the complete solution. Additional physics can be transferred from Ψ_2 into Ψ_1 by adjusting the definition of the projector P_N to include more partial waves. This flexibility of definition of P_N suggests that it may be sufficient to continue to ignore Ψ_2 and to seek improvements of Ψ_1 by including more partial waves. We would only need to be concerned whether there may be some characteristic errors of Ψ_1 that cannot be corrected in finite terms without explicit use of Ψ_2.

One characteristic error is that the omission of Ψ_2 gives zero asymptotic flux in stripping rearrangment channels, no matter how large the (finite) number of partial waves chosen in P_N; for this reason the asymptotic parts of rearrangement channels cannot be represented in Ψ without explicit use of Ψ_2. On the other hand, Ψ_1 has significant overlap at *finite* r_p with rearrangement wave functions like Eq. (4). Therefore, the outgoing flux in Ψ_1 tends to contribute outgoing flux in the rearrangement channels at finite r_p or r_n, even though the explicit asymptotic rearrangement boundary conditions of Ψ_2 are not in the truncated theory. This may partially compensate for the absence of Ψ_2.

(It is also interesting that the outgoing boundary condition for the continuum part of Ψ_1 is normally only imposed explicitly on the $\vec{R}$ coordinate, in the context of an expansion in standing wave eigenfunctions of $\vec{r}$. Integration over the continuum of breakup energies leads to a stationary phase condition that selects outgoing waves in both coordinates.[14] Of course some terms in the expansion of

Ψ_1 associate negative energy with the $\vec{R}$ coordinate; by stationary phase, these terms give no asymptotic flux in any coordinate.)

The omission of Ψ_2 not only suggests questions about boundary conditions, it also leads to possible concern that important influences on Ψ_1 at small radii may be lost. But we see in Eq. (8) that the high order angular momenta in Ψ_2 are coupled directly to Ψ_1 only through high order multipoles of $U_n + U_p$. At small radii these multipoles vanish and they do not affect Ψ_1.

In conclusion, the complete angular momentum decomposition of Eq. (2) provides an orderly three-body theory. Provided we avoid excessive emphasis on the asymptotic regions in rearrangement channels, truncation on the relative angular momentum leads to a heuristically-reasonable approximate theory that is subject to systematic improvement. Practical experience regarding the relations between the Ψ_1 and Ψ_2 components of the truncated theory is limited. Kawai *et al.* did find[8] that stripping terms of type Ψ_2 sensitively affect the results of a variational calculation in which Ψ_1 is truncated to $l=0$. However, they did not check whether this sensitivity to Ψ_2 could be reduced by extending the angular momentum content of Ψ_1.

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