Meson fields with vanishing expectation values

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A method for treating meson fields whose expectation values must vanish because of symmetry is presented in terms of a simplified model of pions, nucleons, and deltas constituting a model alpha particle. Self-consistent equations for the wave functions are derived from the variational principle. The relation to the mean-field procedure, which can be used for fields with expectation values, is discussed.

[NUCLEAR STRUCTURE Self-consistent pion field.]

INTRODUCTION

The idea that nuclear interactions are due to the exchange of virtual mesons has been implemented in two distinct ways in the theory of nuclear systems. Most frequently, the meson exchange interaction is converted into a nucleon-nucleon potential, which is then used in treating nuclear systems as consisting of nucleons interacting through two-body potentials. The framework for this paper is the alternate formulation, in which a nuclear system is considered to consist of nucleons and meson fields with interactions of the Yukawa type $\Psi^\dagger\Psi\Phi$, where Ψ is the nucleon field operator and Φ is a meson field operator; generally, several meson fields are involved. The Yukawa interaction can also be thought of as describing the emission and absorption of virtual mesons by the nucleon current. There are various techniques that have been applied to nuclear systems in this meson-field framework; all of them have in common the use of a scalar meson field to provide attraction, a vector meson field to provide repulsion, and a velocity or momentum dependence in the fermion-scalar vertex that weakens the effects of the scalar field at high density so as to give saturation. Both the scalar field and the vector field have nonzero expectation values in the nuclear ground state; these expectation values or mean fields act as potentials in which the nucleons move. The source of the meson mean field is the nucleon current in the mean-field potential, so that there are coupled equations for the nucleons and the meson mean field that must be solved self-consistently.[1]

In such mean-field procedures, the pion field can have no effect, since there can be no pion mean field in any system with isospin zero or in a system with spin zero and definite parity. That a field that is known to interact so strongly with nuclei is ignored is a serious defect of the mean-field treatments of meson-nucleon systems. It might be argued that the scalar and vector fields, which are typically taken to have masses of the order of two to four pion masses, in some way represent the effects that arise from the exchange of virtual pion pairs. Such an argument would be consistent with the nature of the nucleon-nucleon potential that has been derived from ideas about boson exchange. In this "Paris" potential,[2] the intermediate-range part of the potential does actually come from two-pion effects, but there is neither a proof nor even a good heuristic argument that shows that the same intermediate-range potential would come from the scalar and vector meson exchanges that are used in the mean-field treatments of nuclear systems.

What is needed is an extension of the mean-field procedure that will provide for the treatment of a field whose expectation value is zero. If the coupling of such a field were small, perturbation theory would provide a satisfactory way of treating its effects; however, the pion field is not weakly coupled. In this paper, a general method for treating meson fields whose expectation values are zero is presented. The method can be applied to nuclear boson fields with nonzero isospin or nonzero spin or odd parity, as well as to other boson fields of interest, such as, for example, the gluon field in quark systems. The presentation is in terms of a particular model, but the focus is not on the specific features of the model; rather it is the technique of treating the meson field that is of interest. The calculation of numerical properties of the model, insofar as they may be of interest, is reserved for a separate description; because of the subordinate role of the model, it is presented in a simpler form than a model with more realistic properties would have.

DESCRIPTION OF THE SIMPLIFIED MODEL

In order to emphasize the vanishing mean field, the model is chosen to be a highly simplified alpha particle, with four nucleons all in 0*s* states; the 0*s* states all have the same radial wave function and differ only in the spin-isospin parts of the wave function. For such a system, the Hartree-Fock procedure would give an equation to determine the form of the 0*s* radial wave function. The aim here is a corresponding determination of the form of the radial wave function that arises from the interaction due to virtual pions. The virtual pions are taken to be *p*-wave pions because these are known to have the strongest interaction with nucleons. The emission or absorption of a virtual pion by a nucleon is assumed here to be accompanied by the transformation of the nucleon into a delta particle with spin $\frac{3}{2}$ and isospin $\frac{3}{2}$; pion-nucleon-nucleon and pion-delta-delta vertices are omitted in the simplified model. Parity and angular momentum conservation allow the delta to be in an *s* state; there is therefore a 0*s* state for the delta, and its form must be determined as well as the form of the nucleon 0*s* state. The *p*-wave radial wave function is taken to be the same for all the pions; it also must be determined. With these restrictions on the state vector of the system, the effective Hamiltonian is evidently

$$H_{\text{eff}}=F_N\Sigma_m b^\dagger_{Nm}b_{Nm}+F_\Delta\Sigma_m b^\dagger_{\Delta m}b_{\Delta m}+W\Sigma_{i\lambda}A^\dagger_{i\lambda}A_{i\lambda} -\Sigma_{i\lambda}(Kb^\dagger_\Delta S_i T_\lambda b_N+K^*b^\dagger_N S^\dagger_i T^\dagger_\lambda b_\Delta)(A_{i\lambda}+A^\dagger_{i\lambda}) \; . \tag{1}$$

Here b_{Nm} is the annihilation operator for a nucleon in the 0*s* state with spin and isospin projections specified by the subscript *m*, and $b_{\Delta m}$ is the corresponding operator for deltas; there are four spin-isospin possibilities for the nucleon and sixteen for the delta. The operator $A_{i\lambda}$ is the annihilation operator for a *p*-state pion with angular momentum projection *i* and isospin projection λ. The *S* and *T* operators are the generalized vector spin and isospin operators[3] that transform a state with (iso)spin $\frac{1}{2}$ into a state with (iso)spin $\frac{3}{2}$. All of the information about the nucleon and delta 0*s* wave functions and the pion 0*p* wave function is contained in the constants F_N, F_Δ, W, and K that appear in H_{eff}; these constants are functionals of the wave functions. The energies *F* of the nucleon and delta are obviously

$$F_N=\int (p^2/2M_N)\,|\,\tilde{\psi}_N(\vec{p})\,|^2 d\vec{p} \; ,$$
$$F_\Delta=M_\Delta-M_N+\int (p^2/2M_\Delta)\,|\,\tilde{\psi}_\Delta(\vec{p})\,|^2 d\vec{p} \; , \tag{2}$$

where ψ is the 0*s* space wave function for the corresponding fermion. For the pion it is convenient to use a wave function with the angular part removed; for arbitrary orbital angular momentum projection *m* the pion wave function is

$$\tilde{\varphi}_\pi(k)Y_{1m}(\hat{k})/k \; , \tag{3}$$

so that the pion energy *W* is given by

$$W=\int_0^\infty \omega(k)\tilde{\varphi}_\pi{}^2(k)dk \; , \quad \omega=(k^2+m_\pi{}^2)^{1/2} \; , \tag{4}$$

with $\tilde{\varphi}_\pi(k)$ real. Then if the Yukawa interaction term in the field-theoretic Hamiltonian has a nucleon current of the form

$$\Psi^\dagger_\Delta T_\lambda \vec{S}\cdot\vec{J}\Psi_N \; , \tag{5}$$

it follows that *K* is given by

$$K=\frac{g}{\pi}\int \frac{\tilde{\varphi}_\pi(k)}{[12\omega(k)]^{1/2}}\tilde{\psi}^*_\Delta(\vec{p})\frac{\vec{k}\cdot\vec{J}(\vec{p},\vec{q})}{k^2} \times\tilde{\psi}_N(\vec{q})\delta(\vec{p}-\vec{q}-\vec{k})d\vec{p}\,d\vec{q}\,d\vec{k} \; , \tag{6}$$

although for the present discussion the exact details of the constants and form factors are irrelevant. The three wave functions are assumed normalized.

SELF-CONSISTENT EQUATIONS

The appropriate self-consistent procedure is now obvious. The ground-state energy of H_{eff} depends on the four constants F_N, F_Δ, W, and K and through them on the three wave functions. It is only necessary to minimize the ground-state energy of H_{eff} subject to the normalization constraints and the constraint that the baryon number of the system be four. Let $E_4(F,W,K)$ be the four-baryon ground-state energy of H_{eff} as a function of its parameters. Then the self-consistent equations are

$$\frac{\delta E_4}{\delta\tilde{\psi}_N^*(\vec{p})}=\frac{\partial E_4}{\partial F_N}\frac{p^2}{2M_N}\tilde{\psi}_N(\vec{p})+\frac{\partial E_4}{\partial K^*}\int V_{N\Delta}(\vec{p},\vec{q})\tilde{\psi}_\Delta(\vec{q})\,d\vec{q}=\mu_N\tilde{\psi}_N(\vec{p})\,,$$
$$\frac{\delta E_4}{\delta\tilde{\psi}_\Delta^*(\vec{p})}=\frac{\partial E_4}{\partial F_\Delta}\left[M_\Delta-M_N+\frac{p^2}{2M_\Delta}\right]\tilde{\psi}_\Delta(\vec{p})+\frac{\partial E_4}{\partial K}\int V_{\Delta N}(\vec{p},\vec{q})\tilde{\psi}_N(\vec{q})\,d\vec{q}=\mu_\Delta\tilde{\psi}_\Delta(\vec{p})\,, \quad (7)$$
$$\frac{\delta E_4}{\delta\tilde{\varphi}_\pi(k)}=\frac{\partial E_4}{\partial W}2\omega(k)\tilde{\varphi}_\pi(k)+\frac{\partial E_4}{\partial k}\rho_{\Delta N}(k)+\frac{\partial E_4}{\partial K^*}\rho_{N\Delta}(k)=2\mu_\pi\tilde{\varphi}_\pi(k)\,,$$

where the potentials V and sources ρ are given by

$$V_{\Delta N}(\vec{p},\vec{q})=V_{N\Delta}^*(\vec{q},\vec{p})=\frac{g}{\pi}\int\frac{\tilde{\varphi}_\pi(k)}{[12\omega(k)]^{1/2}}\frac{\vec{k}\cdot\vec{J}(\vec{p},\vec{q})}{k^2}\delta(\vec{p}-\vec{q}-\vec{k})\,d\vec{k}\,,$$
$$\rho_{\Delta N}(k)=\rho_{N\Delta}^*(k)=\frac{g}{\pi[12\omega(k)]^{1/2}}\int\tilde{\psi}_\Delta^*(\vec{p})\frac{\vec{k}\cdot\vec{J}(\vec{p},\vec{q})}{k^2}\tilde{\psi}_N(\vec{q})\delta(\vec{p}-\vec{q}-\vec{k})d\vec{p}\,d\vec{q}\,. \quad (8)$$

Some information about the nature of the solutions is an immediate consequence of the form of Eqs. (7) and (8). Clearly the Lagrange parameters μ_N and μ_Δ are equal and are given by the lowest eigenvalue of the coupled single-particle equations for ψ_N and ψ_Δ; this is the usual result for self-consistent fermion wave functions. The normalization of ψ_N and ψ_Δ is independent of μ_N and μ_Δ. The third part of Eq. (7) gives the form of the pion wave function $\tilde{\varphi}_\pi(k)$; the Lagrange parameter μ_π must be chosen so that the pion wave function is normalized. Results from the static model of the nucleon-pions system show that μ_π is small or zero in that case.[4] Other relations that follow from the form of the equations are useful in reducing the computational complexity of the self-consistent equations.

For present purposes, the essential feature is the existence of a set of equations for the nucleon, delta, and pion wave functions for this case where the expectation value of the pion field is zero. The extra complications from the vanishing mean field appear at the stage of determining $E_4(F,W,K)$, the lowest four-baryon eigenvalue of $H_{\rm eff}$. The states that make up the ground state of $H_{\rm eff}$ are of the form

$$\{N^4\}^{00}$$
$$\{A^+,\{N^3\Delta\}^{11}\}^{00}\,,\quad\{\{A^+,A^+\}^{ts},\{N^3\Delta\}^{ts}\}^{00}\,,$$
$$\{\{(A^+)^n\}^{ts},\{N^3\Delta\}^{ts}\}^{00}$$
$$\{\{(A^+)^n\}^{ts},\{N^2\Delta^2\}^{ts}\}^{00},\ \text{etc.}\,, \quad (9)$$

where the { } indicate vector coupling and the rest of the notation should be self-evident. The vanishing mean field means that at least two distinct states of the nucleons-deltas system must be involved in the ground-state vector, in contrast to the case of nonvanishing mean field, where a single fermion state vector can be used. This is just another way of saying that a pion emission or absorption is accompanied in the simplified model by the transformation of a baryon from nucleon to delta or vice versa; when a meson has nonvanishing mean field, it can be emitted without changing the baryon quantum numbers. In addition, an accurate calculation of E_4 must take into account the effects of pion pairs and other multipion states. Some methods for handling pion pairs have been previously developed and applied to the static model of the pion-nucleon system.[5] As was the case in Ref. 5, here also the number of state components, which are of the types shown in (9), that are involved when there is no mean field is expected to be large, so that it is clear that, just as was done in Ref. 5, the symmetry properties of the Hamiltonian must be thoroughly exploited in order to reduce the dimensions of the problem to manageable size.

RELATION TO THE MEAN-FIELD PROCEDURE

In the case of a meson field that can have an expectation value, the indices on the meson annihilation operator A are absent. In such a case, it is also possible to use a ground state that has only a single fermion wave function component or single Slater determinant (SSD). In a SSD fermion wave function, every fermion operator is equivalent to a c number, and $H_{\rm eff}$ is equivalent to a Hamiltonian of the form

$$H_{MF}=F+WA^+A-K(A^++A)\,,\qquad K=K^*\,, \quad (10)$$

which is trivially solved to give

$$H_{MF}=F-K^2/W+W(A-K/W)^{\dagger}(A-K/W)\ , \tag{11}$$

with ground-state energy $F-K^2/W$. The essential point is that not only does the operator A have the expectation value K/W in the ground state, but the ground state is an eigenstate of the operator A with eigenvalue K/W. This information is sufficient to permit the usual mean-field equations to be recovered.

Additionally, it is clear that it is possible to extend the procedure leading to Eqs. (7) and (8) to the case when there is a mean field. Several states of the fermions can be used to model cases where the field creation operator A is expected to couple strongly to certain fermion particle-hole excitations. Then a result like Eqs. (7) and (8) is obtained. The difference is that more than one matrix element K is needed; in particular, there are K matrix elements that are diagonal in the fermion space in cases with mean field, as in Eq. (10).

MESON-NUCLEON SHELL PICTURE

The form of the self-consistent problem is that of a generalized shell model in which not only the nucleon field operator is expanded in terms of a set of single-particle orbitals, but also the delta field operator and the pion field operator. In the static model of the nucleon-pions system, the use of a single preferred mode for the pion field was suggested by Tomonaga.[6] The use of the boson field creation and annihilation operators $A_{i\lambda}^{\dagger}$ and $A_{i\lambda}$ in H_{eff} allows the pion orbitals to be occupied by arbitrary numbers of virtual mesons. In the simplified model used above, only one pion orbital is used, but it is clear that the model can be generalized to include various pion orbitals, such as $0s$, $2s$, $0d$, etc. Moreover, similar orbitals can be used to include effects of other mesons; the scalar and vector mesons often used in meson-theoretic treatments of nuclear systems can be treated in the same way.

Formally, the meson-nucleon shell picture can be defined by expanding the meson and baryon field in terms of orthonormal functions. For simplicity of notation, it is assumed that there is a single baryon field operator $\Psi(\vec{x})$ and a single meson field operator $\Phi_\lambda(\vec{x})$ with isospin or color or other symmetry index λ. For the meson field, it is not the field operator $\Phi_\lambda(\vec{x})$ that is expanded, but rather the momentum-space annihilation operator $a_\lambda(\vec{k})$, whose relation to $\Phi_\lambda(\vec{x})$ depends on the nature of the meson field. Now choose a finite set of expansion functions for each field operator and write

$$a_\lambda(\vec{k})=\sum_1^N A_{\lambda\alpha}\varphi_\alpha(\vec{k})+a_{\lambda\perp}(\vec{k})\ ,$$
$$\Psi(\vec{x})=\sum_1^P B_i\psi_i(\vec{x})+\Psi_\perp(\vec{x})\ . \tag{12}$$

Then within the subspace of the Hilbert space that is generated by the operators $A_{\lambda\alpha}^{\dagger}$ and $B_i^{\dagger}$ acting on the vacuum, each of the field operators is equivalent to the finite part of the sum; moreover, the Hamiltonian is equivalent within this subspace to the original field-theoretic Hamiltonian with each field operator replaced by its equivalent finite sum. The result is an effective Hamiltonian written entirely in terms of the set of operators $A_{\lambda\alpha}$ and B_i and their adjoints. If the field-theoretic Hamiltonian is of the Yukawa form, then the effective Hamiltonian is

$$H_{\text{eff}}=\sum_{ij}F_{ij}B_i^{\dagger}B_j+\sum_{\alpha\beta}W_{\alpha\beta}A_\alpha^{\dagger}A_\beta - \sum_{ij\alpha}\left[B_i^{\dagger}B_jA_\alpha+K_{ij;\alpha}^{*}A_\alpha^{\dagger}B_j^{\dagger}B_i\right]\ . \tag{13}$$

A significant point is that the number of distinct F, W, and K coefficients is much smaller than it might seem; many of the coefficients are related by Clebsch-Gordan algebra, so that the number of distinct coefficients is only the number of distinct reduced matrix elements.

As with the effective Hamiltonian of Eq. (1), it seems useful to break the treatment of the effective Hamiltonian of Eq. (13) into two steps. First it is necessary to find the N-baryon ground-state energy of H_{eff} as a function of the parameters F_{ij}, $W_{\alpha\beta}$, and $K_{ij;\alpha}$. The coupled equations like Eqs. (7) and (8) for the baryon and meson wave functions can be derived from the assumed form of E as a function of the parameters.

EXCHANGE

When meson fields are treated in the mean-field approximation, the result is an effective baryon energy functional that contains only the direct or Hartree terms that would arise from a corresponding one-meson-exchange potential in a baryon-baryon interaction Hamiltonian. It has previously been shown that the effect of the Fock exchange terms is carried by the baryon self-energy terms.[7] The baryon in the N-baryon system has lowered energy because of its interaction with the meson field shared by the N baryons. On the other hand, a free baryon has a self-energy due to its interaction with its own meson field, and this latter energy must be considered when comparing a system of N bound baryons with the same N free baryons. A similar situation can be expected to occur when there is no mean field; the single-baryon self-energies must be included in the calculations in order to produce the effects that would usually be given by Fock exchange terms.

It is interesting to note that the same separation of Fock from Hartree terms occurs also in the path-integral formulation of theories with only baryon-baryon interaction.[8] In that case it is fluctuation effects that produce the Fock exchange term.

SUMMARY

A simple model has been used to show how to treat meson fields that have no expectation value in a variational procedure. The usual mean-field procedure for treating meson fields with expectation values can be obtained as a special case of the more general variational procedure.

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