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Inelastic scattering of kaons on ^{12}C

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The cross sections for the reactions $^{12}C(K^{\pm},K^{\pm\prime})^{12}C^*$ ($J^{\pi}=2^+$, $E_x=4.44$ MeV; $J^{\pi}=0^+$, $E_x=7.65$ MeV, and $J^{\pi}=3^-$, $E_x=9.64$ MeV) at incident momentum $p_{\text{lab}}=800$ MeV/c are calculated in the distorted wave impulse approximation. Multiple scattering effects are taken into account by means of the $K^{\pm}$-nucleus elastic scattering amplitudes. The results are compared with recent data for the reactions leading to the 2^+ and 3^- states. The difference between the angular patterns of the cross sections for the K^- and K^+ scattering is mainly due to the fact that the K^-N scattering amplitude depends strongly on the momentum transfer compared with the K^+N amplitude.

[NUCLEAR REACTIONS $^{12}C(K^{\pm},K^{\pm\prime})^{12}C^*$ (2^+, 4.44 MeV; 0^+, 7.65 MeV; 3^-, 9.64 MeV), $p_{\text{lab}}=800$ MeV/c, $d\sigma/d\Omega$ calculated in DWIA, distortion effects through $K^{\pm}$-nucleus elastic scattering amplitudes.]

I. INTRODUCTION

In the energy region of 0 to 1 GeV, the K^- meson whose strangeness is $S=-1$ forms resonances with nucleons. The K^+ meson having $S=+1$ interacts with nucleons weakly without making any well-established resonance. The K^+-nucleus interaction is also weak and slowly varying with energies. The K^+ meson is less absorptive than the K^- meson in the nucleus. Together with these features, the advantages of kaons as a probe of the nucleus have been extensively[1,2] discussed. The predictions[3,4] in the distorted wave impulse approximation (DWIA) have been also made with K-nucleus optical potentials constructed from KN scattering amplitudes. Now, experimental data[5,6] are available. The agreement of the predictions with data seems not to be satisfactory, especially in the shape of the angular distributions of the cross sections.

Before trying to improve the agreement, it is noted that data for hadron-nucleus scattering are well reproduced[7,8] with the Glauber model even at low energies where the validity of the model seems not to be clearly justified. With the Glauber model it is very convenient to directly relate hadron-nucleon scattering amplitudes with hadron-nucleus scattering ones. It is worthwhile to calculate the cross sections for the $K^{\pm}$-nucleus scattering in the framework of the Glauber model and to see how the very different characters of K^+ and K^- mesons appear in the $K^{\pm}$-nucleus scattering.

Although the kaons form four members of the octet of pseudoscalar mesons, the K^+ and K^- mesons are not members of the same isospin multiplet. The K^-- and K^+-nuclear interactions are not related by crossing symmetry. For learning about the kaon reaction mechanism and the nuclear response, such as the formation of hypernuclei, it is indispensable to compare the results for the K^-- and K^+-nucleus scattering at the same energies.

In the present paper the parameter-free calculations based on the Glauber model are performed for the reactions $^{12}C(K^{\pm},K^{\pm\prime})^{12}C^*$ exciting the $J^{\pi}=2^+$ state at $E_x=4.44$ MeV, the 0^+ state at 7.65 MeV, and the 3^- state at 9.64 MeV for the incident kaon momentum of $p_{\text{lab}}=800$ MeV/c ($T_{\text{lab}}=446$ MeV), to extract the main features of kaon probes. Distortion effects owing to multiple scattering are taken into account by means of the $K^{\pm}$-nucleus elastic scattering amplitudes.

II. CALCULATIONS

The amplitude for the transition of a nucleus with the mass number A from a state i to a state f as the results of collision with a projectile of its wave vector $\vec{k}$ is

$$F_{fi}(\vec{q})=\frac{ik}{2\pi}\int e^{i\vec{q}\cdot\vec{b}}\left[\psi_f(A),\sum_j^A\Gamma_j(\vec{b}-\vec{s}_j)\prod_{k=1}^{A-1}\{1-\Gamma_k(\vec{b}-\vec{s}_k)\}\psi_i(A)\right]d^2\vec{b}\ , \tag{1}$$

in the DWIA based on the Glauber model.[9] Here, $\vec{q}$ is the momentum transfer; $\vec{b}$ is the impact parameter; $k=|\vec{k}|$; ψ_i and ψ_f are the nuclear wave functions of the initial and final states, and $\Gamma_j(\vec{b}-\vec{s}_j)$ is the profile function which is the Fourier transform of the amplitude $f_j(\vec{q})$ for the scattering of the incident particle on the jth particle in the scattering system, $\vec{s}_j$ being the projection of the radius vector of the jth particle $\vec{r}_j$ on the plane perpendicular to the direction of $\vec{k}$ chosen as the z axis.

The amplitude for the elastic scattering by the A-particle system is

$$F_{ii}(\vec{q})=\frac{ik}{2\pi}\int e^{i\vec{q}\cdot\vec{b}}\left[\psi_i(A),\left\{1-\prod_{k=1}^{A}[1-\Gamma_k(\vec{b}-\vec{s}_k)]\right\}\psi_i(A)\right]d^2\vec{b}\ , \tag{2}$$

in the Glauber model. The Fourier transform of Eq. (2) is

$$\left[\psi_i(A),\prod_{k=1}^{A}[1-\Gamma_k(\vec{b}-\vec{s}_k)]\psi_i(A)\right]=1-\frac{1}{2\pi ik}\int e^{-i\vec{q}\cdot\vec{b}}F_{ii}(\vec{q})d^2\vec{q}\ . \tag{3}$$

The substitution of Eq. (3) into Eq. (1) gives the inelastic scattering amplitude of the form[10]

$$F_{fi}(\vec{q})=Af(\vec{q})S_{fi}(\vec{q})-\frac{A}{2\pi ik}\int f(\vec{q}\,')S_{fi}(\vec{q}\,')F_{ii}(\vec{q}\,'')\delta(\vec{q}-\vec{q}\,'-\vec{q}\,'')d^2\vec{q}\,'d^2\vec{q}\,''\ , \tag{4}$$

where the vectors $\vec{q}$, $\vec{q}\,'$, and $\vec{q}\,''$ lie in the plane perpendicular to the direction of $\vec{k}$. In Eq. (4) the nuclear wave functions for the initial and final states are assumed to be the product of the single-particle functions. The $F_{ii}(\vec{q}\,'')$ is the amplitude for the elastic scattering on the $(A-1)$ system; the particle associated with the inelastic collision being singled out. The form factor S_{fi} of the transition is

$$S_{fi}(\vec{q})=\frac{1}{A}\left[\psi_f(A),\sum_{j}^{A}e^{i\vec{q}\cdot\vec{r}_j}\psi_i(A)\right]\ , \tag{5}$$

which is explicitly expressed as, for the final state with $f=JM$ and the initial state with $J=0$,

$$S_{JM,0}(\vec{q})=\left[\frac{4\pi}{2J+1}\right]^{1/2}Y^*_{JM}(\vec{q}/q)S_J(q)\ . \tag{6}$$

The reduced inelastic form factor $S_J(q)$ is related to the cross section for the electron-nucleus inelastic scattering.

The amplitude (4) is valid for the transition caused by exchange of vacuum quantum number, that is, of state with natural parity in the t channel. Hadron-nucleon form factor effects are taken into account by the momentum transfer dependence of the scattering amplitudes $f(\vec{q})$ and $f(\vec{q}\,')$. The amplitude (4) is separated into two terms. The first term is the amplitude in the plane wave impulse approximation. The second term is concerned with distortion effects through the elastic scattering amplitude. Equation (4) is convenient to see how distortion effects play a role in the inelastic scattering.

III. RESULTS AND DISCUSSION

The K^-N scattering amplitudes given by Gopal *et al.*[11] and by Alston-Garnjost *et al.*[12] are used to calculate the K^--nucleus scattering amplitudes. The amplitudes of Martin[13] and of $BGRT(i)C$ (solution C of Ref. 14 for isospin zero, solution i of Ref. 15 for isospin one) are used for the K^+ scattering. The Fermi motion of individual nucleons is considered by averaging each partial wave amplitude for KN scattering in the forward direction over the momentum distribution of nucleons in the nucleus.

To see how $F_{ii}(\vec{q}\,'')$ used in Eq. (4) reproduces the cross sections for the elastic scattering, the cross sections are calculated with Eq. (2) in the Glauber model. The results are shown in Figs. 1 and 2 for the K^- and K^+ scattering on ^{12}C, respectively, together with data[6] at $p_{lab}=800$ MeV/c. The nuclear wave function of ^{12}C is taken to be a harmonic-oscillator one with the length parameter $a=1.61$ fm for the s- and p-shell nucleons, which reproduce the form factor found by an analysis of electron scattering data. In the results shown in Figs. 1 and 2, the elastic scattering amplitude (2) is multiplied by the factor $\exp[a^2q^2/(4A)]$ to take account of the center of mass correction. The amplitude $F_{ii}(\vec{q}\,'')$ in Eq. (4) is that for the scattering on the system of $A-1$ particles. This is estimated from the amplitude for the scattering on the system of A particles.

The inelastic nuclear transition operator is written as a sum of single-target-nucleon operators in

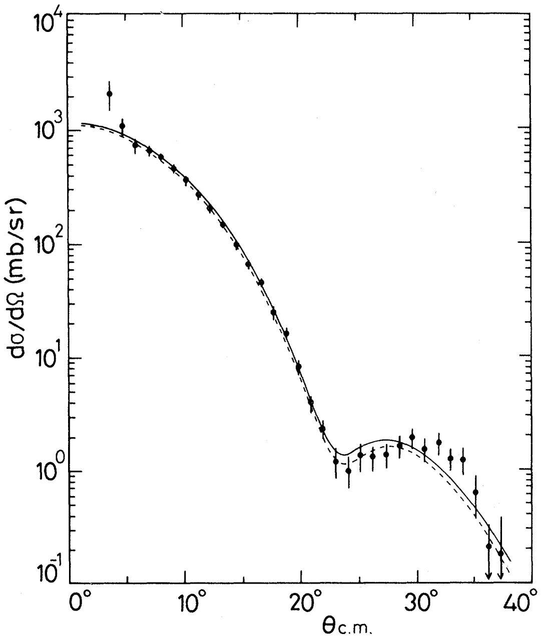

FIG. 1. The K^--^{12}C elastic scattering cross sections at $p_{\rm lab}=800$ MeV/c. The curves are calculated in the Glauber model with the K^-N scattering amplitudes given by Gopal *et al.* (solid) and by Alston-Garnjost *et al.* (dashed). The data are taken from Ref. 6.

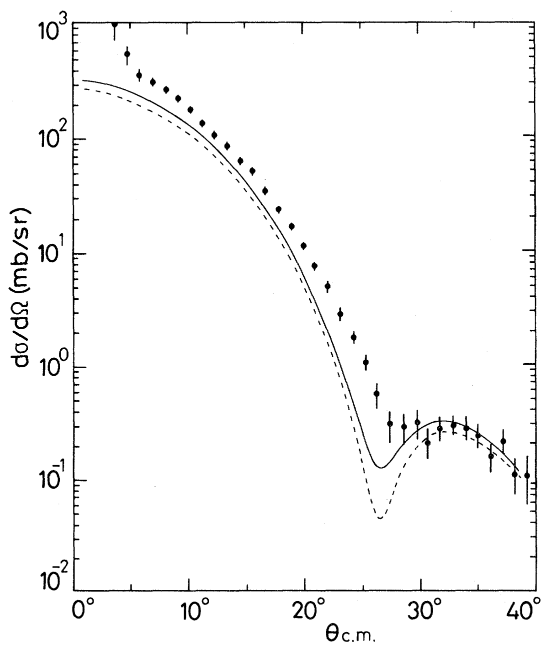

FIG. 2. The K^+-^{12}C elastic scattering cross sections at $p_{\rm lab}=800$ MeV/c. The curves are calculated in the Glauber model with the K^+N scattering amplitudes of Martin (solid) and of $BGRT(i)C$ (dashed). The data are taken from Ref. 6.

the DWIA. The final nuclear excited state, linked to the ground state (g.s.) by the transition operator, can be obtained by particle-hole creation or annihilation operator on the g.s. The random phase approximation is often used for obtaining the nuclear wave functions which differ from the g.s. However, instead of calculating the wave functions with the approximation, the inelastic form factors found directly by fitting electron scattering data are used. The reduced form factors have the form

$$S_J(q)=B_J q^J(1-C_J q^2)e^{-\alpha_J q^2} .$$

For the transition of the g.s. to the $J^\pi=2^+$ state at $E_x=4.44$ MeV in ^{12}C the parameters $B_2=0.24$ fm^2, $C_2=0.13$ fm^2, and $\alpha_2=0.57$ fm^2 are suited to reproduce[16] electron scattering data. The parameters $B_3=0.134$ fm^3, $C_3=0$, and $\alpha_3=0.77$ fm^2 are taken[16] for the transition to the 3^- state at $E_x=9.64$ MeV in ^{12}C.

The vectors $\vec{q}\,'$, $\vec{q}\,''$, and $\vec{q}$ in Eq. (4) form a closed triangle. The configuration of q' and q'' in which q'' is small contributes mainly to the integral, since $F_{ii}(\vec{q}\,'')$ falls off with increasing q''. When $F_{ii}(\vec{q}\,'')$ is forward peaked or $S_J(q')$ falls off rapidly after reaching its maximum, the integrand becomes practically small for appropriate q' and q'' even if $\vec{q}\,'$ and $\vec{q}\,''$ are parallel to $\vec{q}$, and hence the integral converges rapidly. In the present case, both for the K^+ and K^- scattering at $p_{\rm lab}=800$ MeV/c the integrand with q' (q'') larger than 500 MeV/c does not appreciably contribute to the integral. The value of $q=500$ MeV/c corresponds to about 40° in the K-^{12}C c.m. scattering angle for $p_{\rm lab}=800$ MeV/c. It is also noted that the inelastic form factors $S_J(q)$ reproduce electron scattering data until q of around 600 MeV/c. The amplitude (4) is multiplied by $\exp(\alpha_J q^2/A)$, which can account for the c.m. correction for the inelastic scattering within the accuracy of the calculation.

The differential cross sections for the reactions ^{12}C($K^\pm$,$K^{\pm\prime}$)^{12}C* ($J^\pi=2^+$, $E_x=4.44$ MeV) are shown in Figs. 3 and 4. Figures 5 and 6 show the cross sections for the reactions ^{12}C($K^\pm$,$K^{\pm\prime}$)^{12}C* ($J^\pi=3^-$, $E_x=9.64$ MeV) at $p_{\rm lab}=800$ MeV/c. The predictions with the K^-N amplitude given by Alston-Garnjost *et al.*[12] are very similar to those with the amplitude given by Gopal *et al.*,[11] both in the shapes and absolute magnitudes for the K^--^{12}C elastic and inelastic scattering, although the two amplitudes do not lead to the identical total cross sections for the K^-N scattering around $p_{\rm lab}=800$ MeV/c. The cross sections for the K^+ inelastic scattering on ^{12}C predicted with the K^+N ampli-

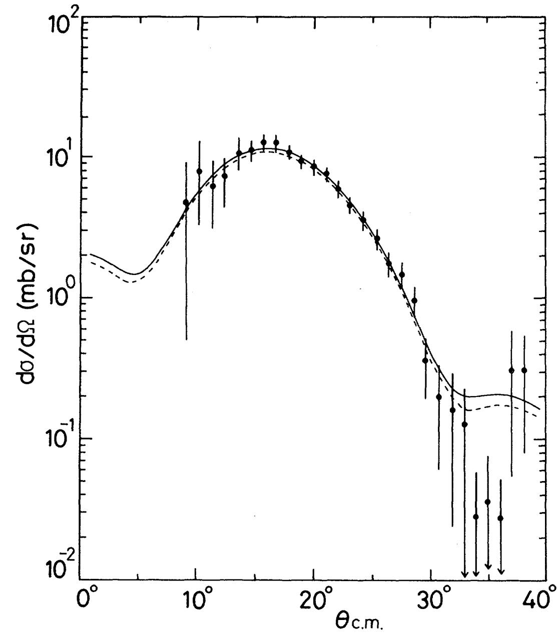

FIG. 3. The cross sections for the reaction $^{12}C(K^-,K^{-\prime})^{12}C^*$ (2^+, 4.44 MeV) at $p_{lab}=800$ MeV/c. The curves are calculated in the DWIA with the K^-N amplitudes given by Gopal *et al.* (solid) and by Alston-Garnjost *et al.* (dashed). The data are taken from Ref. 6.

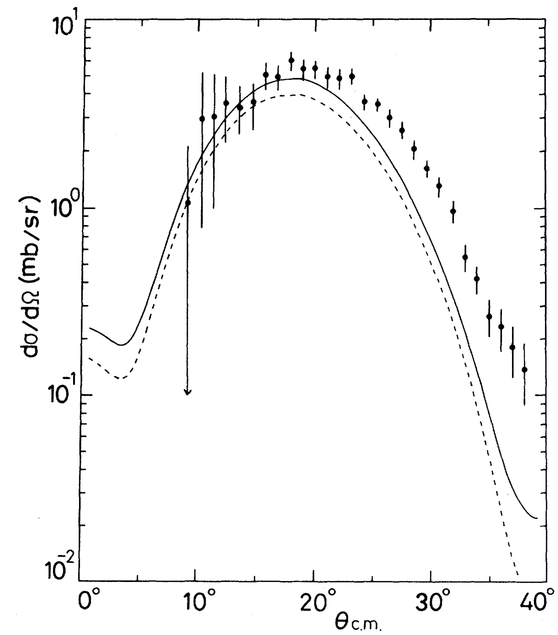

FIG. 4. The cross sections for the reaction $^{12}C(K^+,K^{+\prime})^{12}C^*$ (2^+, 4.44 MeV) at $p_{lab}=800$ MeV/c. The curves are calculated in the DWIA with the K^+N amplitudes of Martin (solid) and of $BGRT(i)C$ (dashed). The data are taken from Ref. 6.

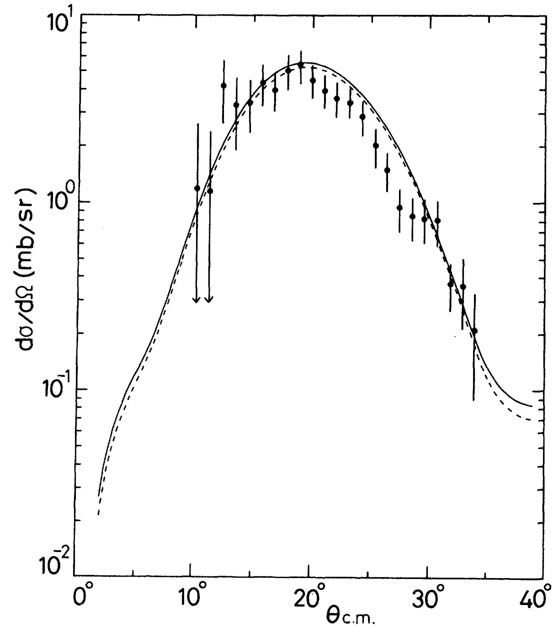

FIG. 5. The cross sections for the reaction $^{12}C(K^-,K^{-\prime})^{12}C^*$ (3^-, 9.64 MeV) at $p_{lab}=800$ MeV/c. The curves correspond to those in Fig. 3. The data are taken from Ref. 6.

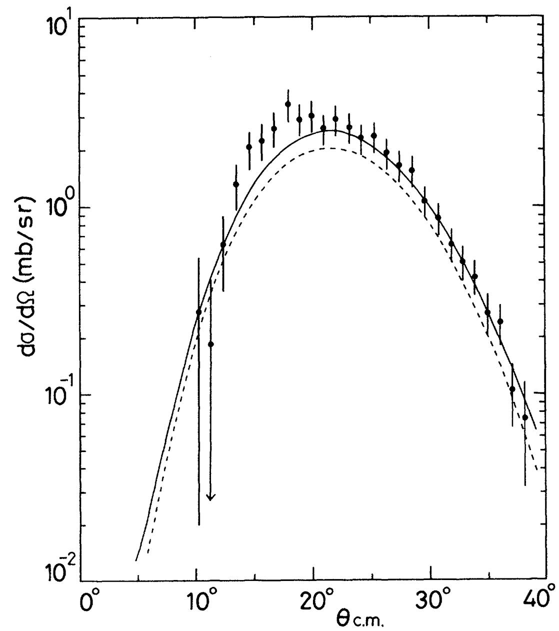

FIG. 6. The cross sections for the reaction $^{12}C(K^+,K^{+\prime})^{12}C^*$ (3^-, 9.64 MeV) at $p_{lab}=800$ MeV/c. The curves correspond to those in Fig. 4. The data are taken from Ref. 6

tude of $BGRT(i)C$ (Refs. 14 and 15) are smaller than those with the amplitude of Martin[13] by about 15%. However, the shapes of the angular distributions predicted with these amplitudes are very similar in the K-^{12}C c.m. scattering angles between 10° and 30°. The cross sections for the K^+-^{12}C elastic scattering predicted with the $BGRT(i)C$ amplitude[14,15] are also smaller than those with the Martin amplitude[13] by about 15% in the angular region.

The shape of the cross sections in the peak around 15° for the inelastic scattering reflects the shape of $S_J(q)$. The value of q which makes $S_J(q)$ maximum is a function of incident momentum and scattering angle. The angle becomes smaller as the incident momentum increases. The peak of the cross sections versus scattering angles shifts to forward with increasing energies.

The differential cross sections for the K^--^{12}C elastic scattering fall off more rapidly with increasing q or scattering angles, compared with the K^+ scattering. The angular width of the cross sections for the K^+-^{12}C inelastic scattering is wider than that for K^- scattering around the peak in the region of 10° to 30° for the same excited state, as seen in comparing, respectively, the results in Figs. 3 and 5 with those in Figs. 4 and 6. This is mainly caused by the fact that the momentum transfer dependence of the K^-N scattering amplitude differs from that of K^+N amplitude. Higher partial waves contribute to the K^-N scattering compared with the K^+N scattering at $p_{\rm lab}=800$ MeV/c. The K^-N amplitude depends on the momentum transfer stronger than the K^+N amplitude. Hence, the K^-N form factor effects appear more sensitively in the K-nucleus scattering.

The K^-N interaction is stronger than the K^+N one and hence the cross sections for the K^--nucleus elastic and inelastic scattering are larger compared with those for the K^+ scattering. The integral in Eq. (4) gives rise to distortion effects in the DWIA. The integral is large when the cross sections for the elastic scattering are large. The K^--nucleus inelastic scattering undergoes distortion effects more strongly compared with the K^+ scattering.

The inelastic scattering amplitude (4) for the transition of the state $J=0^+$ to the state of $J=L$ and M is identically zero when $J+M=$odd in the excited state. Moreover, the second term in Eq. (4) is zero for $q=0$ when $M\neq 0$. Hence, the cross sections for the reactions ^{12}C($K^\pm$,$K^{\pm\prime}$)^{12}C* (2^+, 4.44 MeV) do not fall off in the small angular region, in contrast with the case for the excitation of the 3^- state at 9.64 MeV. The integral with the transition form factor $S_{20,0}$ in Eq. (4) remains finite even when $q=0$, although $S_2(q)$ in the first term is small for small q or forward angles. The cross sections for the forward angles are dominated by distortion effects in the transition of $J^\pi=0^+$ to 2^+ with $M=0$. Thus, the measurement of the cross sections for the reactions ^{12}C($K^\pm$,$K^{\pm\prime}$)^{12}C* (2^+, 4.44 MeV) in the small angular region can clarify distortion effects in DWIA.

Distortion effects are clearly seen in the differential cross section for the reactions ^{12}C($K^\pm$,$K^{\pm\prime}$)^{12}C* (0^+, 7.65 MeV). In Fig. 7 the cross sections at $p_{\rm lab}=800$ MeV/c are predicted. The inelastic form factor used[16] is $S_0(q)=B_0q^2e^{-\alpha_0 q^2}$ with $B=0.167$ fm^2 and $\alpha_0=0.99$ fm^2. The angular distributions predicted are forward peaked. The forward peak is caused by distortion effects. When distortion effects are large the forward peak is high relative to the next peak in the angular distribution. The differential cross sections predicted in Fig. 7 reflect the fact that the K^--nucleus interaction is absorptive more than the K^+.

With the form factors found by fitting electron scattering data, the predictions in the DWIA based

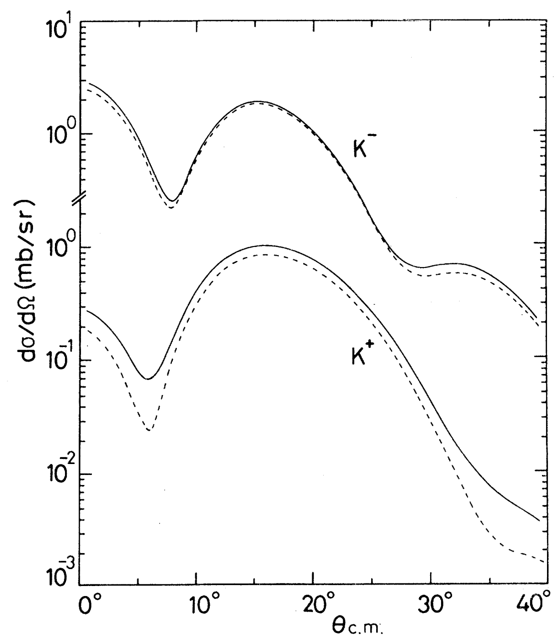

FIG. 7. The cross sections predicted for the reactions ^{12}C($K^\pm$,$K^{\pm\prime}$)^{12}C* (0^+, 7.65 MeV) at $p_{\rm lab}=800$ MeV/c. The upper curves are the results predicted for the K^- inelastic scattering. The curves correspond to those in Fig. 3. The lower curves are the results predicted for the K^+ inelastic scattering. The curves correspond to those in Fig. 4.

on the Glauber model reproduce the cross sections measured for the K-^{12}C inelastic scattering. This indicates that the same wave functions which explain the data of electroexcitations yield satisfactory agreement for the K-nucleus inelastic scattering data. It is noted that the predictions reproduce[17] also the cross sections measured for the π-^{12}C inelastic scattering. However, the agreement is not satisfactory, especially for the K^+-^{12}C elastic scattering. This cannot be improved even if the momentum transfer dependence is ignored in the K^+N amplitudes. The disregard of the dependence, that is, the use of KN forward amplitudes, makes the falloff of the differential cross sections versus the K-nucleus scattering angles the slowest. This may suggest the possibility of improving the parametrization for the K^+N scattering amplitudes.

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