Pion-deuteron scattering and the optical potential

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We have calculated the pion-deuteron elastic, total, and differential cross sections for energies between 100 and 300 MeV, using two forms of the optical potential similar to those used in the theory of pion-nucleus scattering. We have studied the accuracy of the optical potential descriptions, by comparing these cross sections with the exact ones that are obtained by solving the relativistic Faddeev equations. We found that in all cases the optical potential overestimates the multiple scattering corrections. We interpret this result as meaning that the optical potential series may not converge for pion-nucleus scattering.

[NUCLEAR REACTIONS π-d optical potentials; $E=100-300$ MeV; calculated $\sigma_{\rm tot}$, $\sigma_{\rm el}$, and $d\sigma/d\Omega$.]

I. INTRODUCTION

One of the most popular tools used in the study of pion-nucleus scattering is the optical potential, which is defined in the standard version as being proportional to the matrix elements of the free pion-nucleon T matrix between initial and final ground-state wave functions of the nucleus and plane-wave states of the pion. Two alternative forms of the optical potential can be obtained, by simplifying or complicating somewhat the standard version. Thus, if one neglects the recoil of the nucleon in the pion-nucleon T matrix, one obtains the so-called fixed-nucleon optical potential, which is proportional to the Fourier transform of the nuclear density. On the other hand, if instead of using the free pion-nucleon T matrix one includes into its Green's function the average single-particle potential of the nucleon with the rest of the nucleus, one gets the so-called three-body model of the optical potential which has been proposed by Revai[1] and by Tandy, Redish, and Bollé.[2]

In order to understand how well an optical potential succeeds in describing the scattering process, one has to compare it with the exact solution which is available, however, only in the case of very simple systems, such as that of a pion and a deuteron.[3-7] Thus, using this system as a test case, Woloshyn, Moniz, and Aaron[3] studied the fixed-nucleon optical potential, while Afnan and Stelbovics[7] have used it to study the three-body model of Revai and Tandy, Redish, and Bollé. In this paper, we will use the pion-deuteron system to examine the standard optical potential which has been used extensively for energies around the 3,3 resonance.[8] In order to compare with the exact solution, we will solve the relativistic Faddeev equations taking into account the six S and P-wave pion-nucleon channels and the two S-wave nucleon-nucleon channels, as well as the relativistic effects due to the spin of the nucleons.[6]

II. THE PION-DEUTERON OPTICAL POTENTIAL

The standard optical potential that has been used extensively in the study of pion-nucleus scattering[8] can be written in the case of the pion-deuteron system, as

$$V_{M'M}(\vec{q}\,',\vec{q})=A\langle\vec{q}\,'\phi_{M'}\,|\,t_{\pi N}\,|\,\vec{q}\phi_M\rangle, \tag{1}$$

where A is the number of nucleons which in this case is two, $t_{\pi N}$ is the free pion-nucleon T matrix, and $\vec{q}$ and $\vec{q}\,'$ are the initial and final pion-deuteron relative momenta, while ϕ_M and $\phi_{M'}$ are the initial and final wave functions of the deuteron with helicities M and M', respectively. The differential cross section in the center of mass system is given by

$$d\sigma/d\Omega=\pi^4/S\tfrac{1}{3}\sum_{MM'}|\,T_{M'M}(\vec{q}\,',\vec{q})\,|^2, \tag{2}$$

where S is the invariant mass squared of the system, and $T_{M'M}$ is the solution of the Blankenbecler-Sugar equation

$$T_{M'M}(\vec{q}\,',\vec{q})=V_{M'M}(\vec{q}\,',\vec{q})+\sum_{M''}\int d\vec{q}\,''V_{M'M''}(\vec{q}\,',\vec{q}\,'')\frac{(m_\pi{}^2+q''^2)^{1/2}+(m_d{}^2+q''^2)^{1/2}}{2(m_\pi{}^2+q''^2)^{1/2}(m_d{}^2+q''^2)^{1/2}}$$
$$\times\frac{1}{S-[(m_\pi{}^2+q''^2)^{1/2}+(m_d{}^2+q''^2)^{1/2}]^2+i\epsilon}T_{M''M}(\vec{q}\,'',\vec{q})\ . \qquad (3)$$

The optical potential (1) and wave equation (3), are straightforward generalizations, for the case of a nucleus with spin one, of the standard expressions used in pion-nucleus scattering.[8]

In order to evaluate the optical potential (1) in the helicity basis, we will use the Blankenbecler-Sugar reduction of the relativistic three-body problem, which in this case is equivalent to putting the three particles on their mass shells,[6] so that we can write Eq. (1) as

$$V_{M'M}(\vec{q}\,',\vec{q})=A\sum_{\lambda_2\lambda_3\lambda_2'\lambda_3'}\int\frac{d\vec{k}_1'}{2\omega_1(k_1')}\frac{d\vec{k}_2'}{2\omega_2(k_2')}\frac{d\vec{k}_3'}{2\omega_3(k_3')}\frac{d\vec{k}_1}{2\omega_1(k_1)}\frac{d\vec{k}_2}{2\omega_2(k_2)}\frac{d\vec{k}_3}{2\omega_3(k_3)}$$
$$\times\langle\vec{q}\,'\phi_{M'}\,|\,\vec{k}_1'\vec{k}_2'\vec{k}_3';\lambda_2'\lambda_3'\rangle\langle\vec{k}_1'\vec{k}_2'\vec{k}_3';\lambda_2'\lambda_3'\,|\,t_{12}\,|\,\vec{k}_1\vec{k}_2\vec{k}_3;\lambda_2\lambda_3\rangle$$
$$\times\langle\vec{k}_1\vec{k}_2\vec{k}_3;\lambda_2\lambda_3\,|\,\vec{q}\phi_M\rangle, \qquad (4)$$

where

$$\omega_i(k_i)=(k_i{}^2+m_i{}^2)^{1/2},$$

and we have taken the pion as particle 1 and the nucleons as particles 2 and 3 (by using, in addition, the isospin quantum numbers, particles 2 and 3 become identical), while λ_2,λ_2' and λ_3,λ_3' are helicity quantum numbers for particles 2 and 3, respectively. In the three-body c.m. frame, the pion-nucleon T matrix t_{12} is given by

$$\langle\vec{k}_1'\vec{k}_2'\vec{k}_3';\lambda_2'\lambda_3'\,|\,t_{12}(S)\,|\,\vec{k}_1\vec{k}_2\vec{k}_3;\lambda_2\lambda_3\rangle$$
$$=2\omega_3(k_3)\delta(\vec{k}_3'-\vec{k}_3)2\delta(\vec{k}_1'+k_2'-\vec{k}_1-\vec{k}_2)$$
$$\times\delta_{\lambda_3'\lambda_3}\sum_{\mu_2'\mu_2}d_{\lambda_2'\mu_2'}{}^{1/2}(\beta_2')\langle\vec{k}_{12}';\mu_2'\,|\,t_{12}[S+m_3{}^2-2S^{1/2}(m_3{}^2+k_3{}^2)^{1/2}]\,|\,\vec{k}_{12};\mu_2\rangle\,d^{1/2}_{\mu_2\lambda_2}(\beta_2), \qquad (5)$$

where μ_2,μ_2' and $\vec{k}_{12},\vec{k}_{12}'$ are the initial and final helicities of particle 2 and relative momenta between particles 1 and 2, respectively, which are all measured in the c.m. frame of the pair, while $d^{1/2}_{\lambda_2'\mu_2'}(\beta_2')$ and $d^{1/2}_{\mu_2\lambda_2}(\beta_2)$ are the matrix elements of the unitary transformation that transforms the nucleon spinors from the three-body c.m. frame to the two-body frame, with β_2 and β_2' being the angles in the Wick triangle.[9] With similar definitions, the initial and final pion-deuteron wave functions are given by

$$\langle\vec{k}_1\vec{k}_2\vec{k}_3;\lambda_2\lambda_3\,|\,\vec{q}\phi_M\rangle=2\omega_1(k_1)\delta(\vec{k}_1-\vec{q})2[\omega_2(k_2)+\omega_3(k_3)]\delta(\vec{k}_1+\vec{k}_2+\vec{k}_3)(m_2{}^2+k_{23}{}^2)^{1/4}$$
$$\times\sum_{\nu_2\nu_3}d^{1/2}_{\lambda_2\nu_2}(\alpha_2)d^{1/2}_{\lambda_3\nu_3}(\alpha_3)\langle\vec{k}_{23};\nu_2\nu_3\,|\,\phi_M\rangle, \qquad (6)$$

where $\langle\vec{k}_{23};\nu_2\nu_3\,|\,\phi_M\rangle$ is the deuteron wave function in its own c.m. frame, which in the helicity basis is given by

$$\langle\vec{k}_{23};\nu_2\nu_3\,|\,\phi_M\rangle=\langle k_{23};\nu_2\nu_3\,|\,\phi\rangle(\tfrac{3}{4}\pi)^{1/2}\mathscr{D}_{M,\nu_2-\nu_3}{}^1(\hat{k}_{23}), \qquad (7)$$

$$\langle k_{23};\tfrac{1}{2}\tfrac{1}{2}\,|\,\phi\rangle=\langle k_{23};-\tfrac{1}{2}-\tfrac{1}{2}\,|\,\phi\rangle=(\tfrac{1}{6})^{1/2}\phi_0(k_{23})-(\tfrac{1}{3})^{1/2}\phi_2(k_{23}), \qquad (8)$$

$$\langle k_{23};\tfrac{1}{2}-\tfrac{1}{2}\,|\,\phi\rangle=\langle k_{23};-\tfrac{1}{2}\tfrac{1}{2}\,|\,\phi\rangle=(\tfrac{1}{3})^{1/2}\phi_0(k_{23})+(\tfrac{1}{6})^{1/2}\phi_2(k_{23}), \qquad (9)$$

with ϕ_0 and ϕ_2 the usual S- and D-wave components of the deuteron. Substituting Eqs. (5) and (6) into Eq. (4), and summing over the intermediate nucleon helicities, we find for the optical potential the expression

$$V_{M'M}(\vec{q}\,',\vec{q})=A\sum_{\substack{\mu_2\nu_2\nu_3\\ \mu_2'\nu_2'\nu_3'}}\int\frac{d\vec{k}_2'}{\omega_2(k_2')}\frac{d\vec{k}_2}{\omega_2(k_2)}\frac{d\vec{k}_3}{\omega_3(k_3)}\delta(\vec{q}\,'+\vec{k}_2'+\vec{k}_3)\delta(\vec{q}+k_2+\vec{k}_3)$$

$$\times[\omega_2(k_2')+\omega_3(k_3)][\omega_2(k_2)+\omega_3(k_3)](m_2{}^2+k_{23}'{}^2)^{1/4}(m_2{}^2+k_{23}{}^2)^{1/4}$$

$$\times\langle\phi_{M'}\,|\,\vec{k}_{23}';\nu_2'\nu_3'\rangle d_{\nu_2'\mu_2'}{}^{1/2}(\alpha_2'+\beta_2')d^{1/2}_{\nu_3'\nu_3}(\alpha_3'+\alpha_3)d^{1/2}_{\mu_2\nu_2}(\beta_2+\alpha_2)$$

$$\times\langle\vec{k}_{12}';\mu_2'\,|\,t_{12}[S+m_3{}^2-2S^{1/2}(m_3{}^2+k_3{}^2)^{1/2}]\,|\,\vec{k}_{12};\mu_2\rangle\langle\vec{k}_{23};\nu_2\nu_3\,|\,\phi_M\rangle\ . \quad (10)$$

III. THE KMT FORMALISM

A slightly different formulation of the standard optical potential has been given by Kerman, McManus, and Thaler (KMT),[10] in which instead of the optical potential (1), one uses

$$V_{M'M}(\vec{q}\,',\vec{q})=(A-1)\langle\vec{q}\,'\phi_{M'}\,|\,t_{\pi N}\,|\,\vec{q}\phi_M\rangle, \quad (11)$$

while the differential cross section, instead of being given by Eq. (2), is in this case

$$d\sigma/d\Omega=\pi^4/S\tfrac{1}{3}\sum_{M'M}|A/(A-1)T_{M'M}(\vec{q}\,',\vec{q})|^2, \quad (12)$$

where $T_{M'M}$ is the solution of the Blankenbecler-Sugar equation (3), with the optical potential (11).

The factors $(A-1)/A$ and $[A/(A-1)]^2$ by which Eqs. (11) and (12) differ from Eqs. (1) and (2), respectively, are usually not important when one deals with medium or large nuclei, since they are very close to one. However, in the case of the deuteron where $A=2$, one may expect to see appreciable differences depending on which formulation is used.

IV. RESULTS

We have partial-wave decomposed Eqs. (3) and (10), using the three-body helicity states constructed by Wick[9] normalized as in Ref. 6, and solved Eq. (3) for the angular momentum states with $J<6$, using the Born approximation for those with $J\geq 6$.

In order to construct the exact results, we solved the relativistic Faddeev equations as described in Ref. 6 using as input the six S- and P-wave pion-nucleon channels and the two S-wave nucleon-nucleon channels by means of separable T matrices. In the case of the nucleon-nucleon channels, we used the separable T matrices and deuteron wave function obtained from Yamaguchi potentials,[11] which were made to be solutions of the Blankenbecler-Sugar equation, by using the concept of minimal relativity.[12] For the pion-nucleon channels, we used the separable T matrices constructed in Ref. 6.

We should point out that the Born approximation

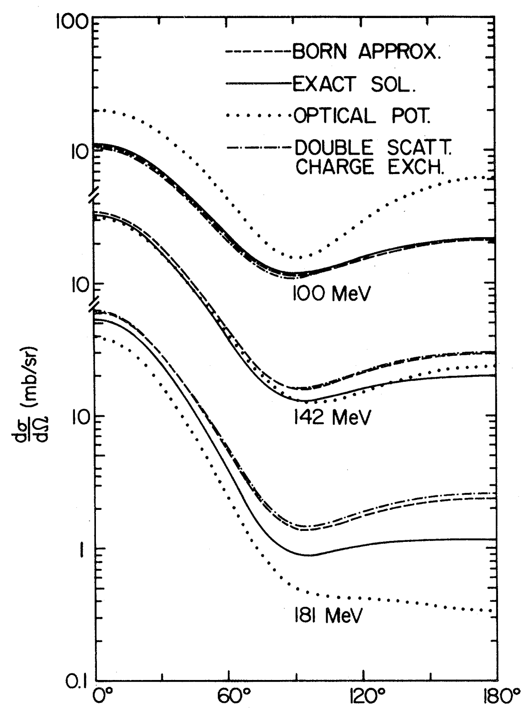

FIG. 1. Differential cross sections in the c.m. system, for three different laboratory kinetic energies of the pion. The solid lines are the exact results, the dashed lines the results of the Born approximation, and the dotted lines the results of the optical potential as defined by Eqs. (1)–(3). The dotted-dashed line is the correction that results from two consecutive single charge exchanges of the pion.

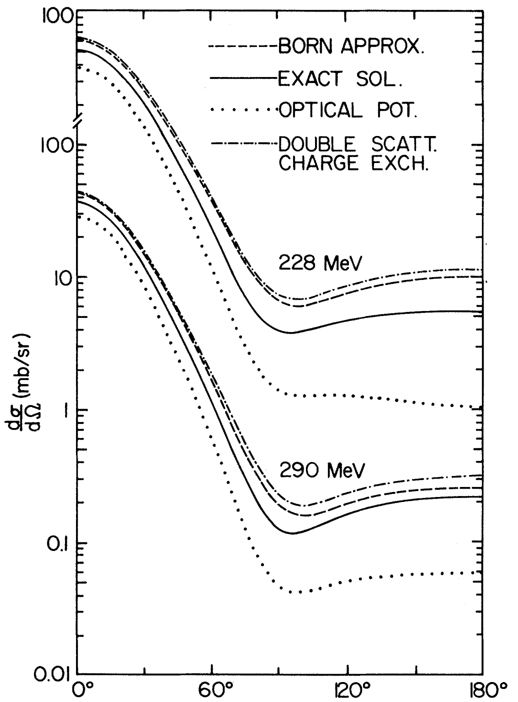

FIG. 2. Same as Fig. 1.

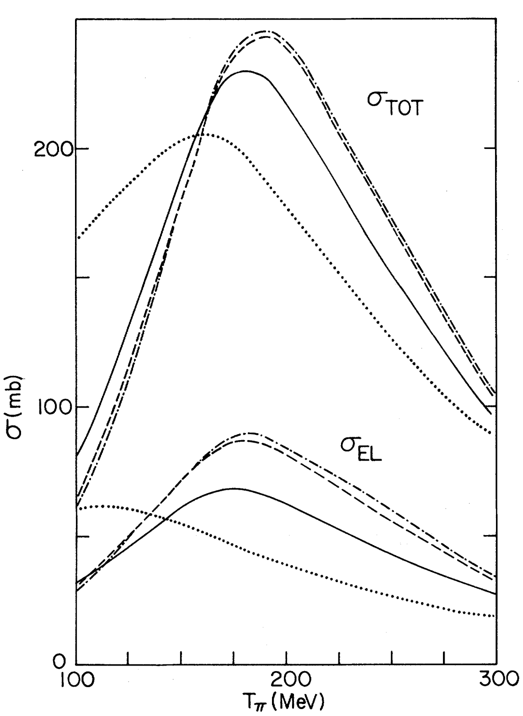

FIG. 3. Pion-deuteron total and integrated elastic cross sections. The meaning of the four curves is the same as those of Fig. 1.

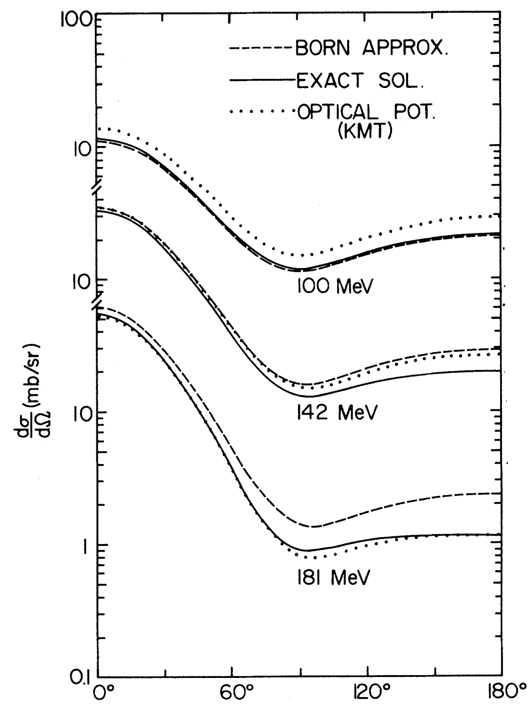

FIG. 4. Differential cross sections in the c.m. system, for three different laboratory kinetic energies of the pion. The solid lines are the exact results, the dashed lines the results of the Born approximation, and the dotted lines the results of the optical potential as defined by Eqs. (11), (12), and (3).

to the Faddeev equations and to the two optical potential approaches are identical, so that our calculations will measure the behavior of the multiple scattering corrections.

We show in Figs. 1—3 the results for the differential, total, and elastic cross sections, using the optical potential as defined by Eqs. (1)—(3). We see that the optical potential in all cases overestimates the multiple scattering corrections, so that even the Born approximation is a better approximation than the optical potential. We see, however, that the optical potential does reproduce the tendency to increase or decrease the cross sections as one goes from the Born approximation to the exact solution, except that it tends to go too far in all cases. We also show for comparison the effect of considering the double scattering term in which the pion undergoes two consecutive single charge exchanges, which as we see is not a very important correction to the differential cross sections, except perhaps at the last two energies which are above the 3,3 resonance. This process, which represents an anomaly of the pion-deuteron system, is very much

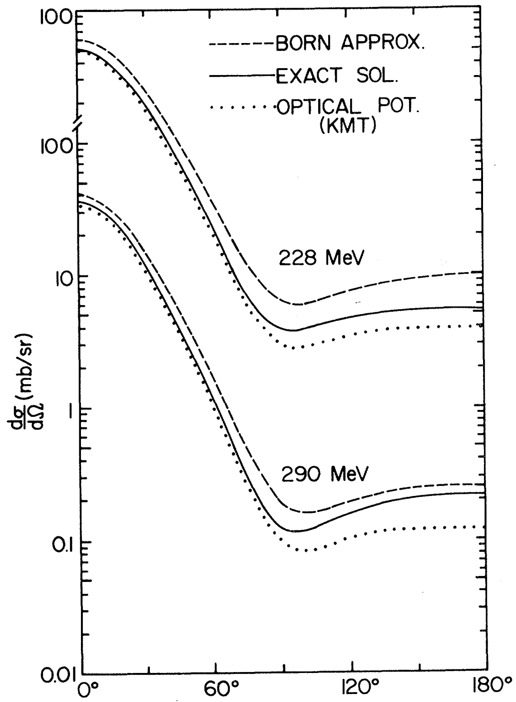

FIG. 5. Same as Fig. 4.

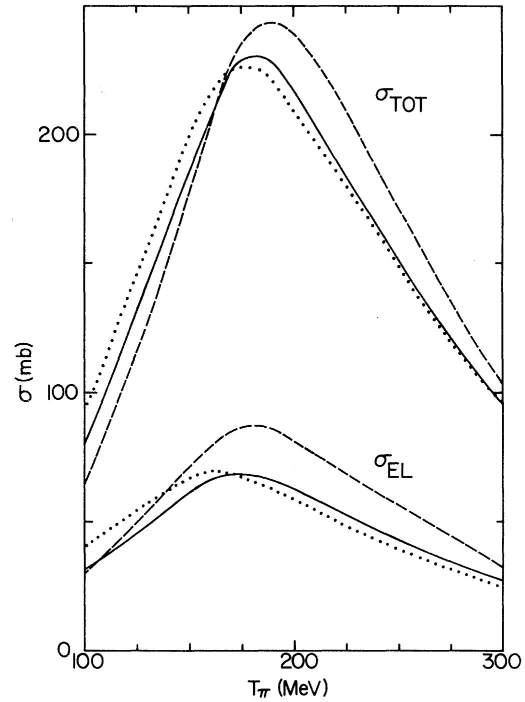

FIG. 6. Pion-deuteron total and integrated elastic cross sections. The meaning of the three curves is the same as those of Fig. 4.

suppressed in heavier nuclei, so that it never enters in the optical potential description. We see, however, that even in the case of the deuteron, the charge-exchange double-scattering term is not an important correction for energies at and below the 3,3 resonance.

As we mentioned before, we expect to see appreciable differences in the case of the deuteron, depending on whether one uses the standard optical potential, or that of the KMT theory, so that we have plotted in Figs. 4–6, the corresponding results for the KMT formalism, where again we see the tendency of the optical potential to overcorrect the solution although not as strong as in the previous case. Our results in Figs. 3 and 6 for the elastic cross section are in good agreement with those obtained by Afnan and Stelbovics, [7] who considered only the $J^{\pi}=2^{+}$ channel and used relativistic kinematics only for the pion.

The result that the optical potential overcorrects the solution is somewhat surprising, since Eqs. (1) and (11) are the first terms of infinite series for the full optical potential, so that one would have expected these potentials to account for only a fraction of the multiple scattering corrections, with the higher-order terms accounting for the rest. Moreover, if the optical potential overcorrects in the case $A=2$, where the potentials (1) and (11) are weak, we can expect it to overcorrect even more in the case of medium or large nuclei where A is large and the potentials (1) and (11) become very strong.

We would like to conclude by noticing that our results are very much in line with the long-standing puzzle of the charge-exchange reaction to the isobaric analog state in ^{13}C, where all the calculations[13,14] show that the optical potential is too strong such that the multiple scattering corrections in the initial and final states give rise to a much larger suppression of the cross section than is observed experimentally. Thus, it may be that the optical potential series does not converge in the case of pion-nucleus scattering.

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