# Separable Bethe-Salpeter equation kernels for pion-nucleon scattering

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In a previous paper we solved the Bethe-Salpeter equation with separable interactions and demonstrated the applicability of the method for the most important nucleon-nucleon partial waves. In this paper we use the Bethe-Salpeter equation for pion nucleon partial waves including the necessary two potential ansatz for the $P_{11}$ wave. A fully relativistic treatment of the $\pi$-$d$ system can be performed with these amplitudes.

NUCLEAR REACTIONS Separable Bethe-Salpeter kernels applied to pion-nucleon scattering; $E=0-300$ MeV; $l=0$, 1; $P_{11}$ wave with two potential ansatz.

## I. INTRODUCTION

In recent years there has been increasing interest in investigations of the $\pi$-$d$ system.[1] A relativistic description is called for; Giraud *et al.*[2] invoke the Blankenbecler-Sugar reduction[3] of the Bethe-Salpeter (BS) equation to describe the $\pi N$ subsystem. However, it is felt[4] that the BS equation itself should be used, and so it is the goal of this paper to present a separable approach to the BS equation for the pion nucleon system, which is suitable for the three body calculations mentioned above. The same approach was used for the nucleon-nucleon system in an earlier paper.[5] The separable kernels should be viewed as an approximation of the full BS kernel, not of the ladder approximation for which reduction techniques may have certain advantages.

A particular feature of the $\pi$-$N$ system is the occurrence of the nucleon pole in the $P_{11}$ amplitude. The necessity of identifying the contributions from the pole term and from the one particle irreducible "background," respectively, has been emphasized by Mizutani *et al.*[6] The results of such a decomposition within the formalism of the BS equation is given in Sec. II. Our numerical results are presented and discussed in the last section.

## II. SOLUTION OF THE BS EQUATION WITH SEPARABLE INTERACTIONS

The partial wave decomposed BS equation in momentum space is given by[5]

$$T_l(q_0,q,q_0',q';s)=V_l(q_0,q,q_0',q') + \frac{i}{4\pi^3}\int_{-\infty}^{\infty}dk_0\int_0^{\infty}k^2dk\, V_l(q_0,q,k_0,k)G_0(k_0,k;s)T_l(k_0,k,q_0',q';s)\ , \tag{1}$$

where $G_0$ is the relativistic free two particle Green's function and $V_l$ is the kernel ("potential") of the integral equation. $q\,(k,q')$ are the absolute values of the three dimensional vectors of the initial (intermediate, final) relative momenta, respectively; the index 0 denotes the zeroth component and $s$ is the total energy squared in the center of mass (c.m.) system.

With all external lines on the mass and energy shell we have

$$T_l(p_0,p,p_0,p;s)\equiv T_l(p) = -\frac{8\pi\sqrt{s}}{p}e^{i\delta_l(p)}\sin\delta_l(p)\ . \tag{2}$$

Except for the $P_{11}$ wave we solve Eq. (1) with the rank one separable ansatz

$$V_l(q_0,q,q_0',q')=g_l(q_0,q)\lambda g_l(q_0',q') \tag{3}$$

in closed form, where $g_l(q_0,q)$ is a relativistic generalization[7] of the well-known Yamaguchi form factor:

$l=0$:

$$g_0(q_0,q)=\frac{1}{[(q_0{}^2-q^2-\beta^2)^2+\gamma^4]^{1/2}}\ , \tag{4a}$$

$l=1$:

$$g_1(q_0,q)=\frac{q}{(q_0{}^2-q^2-\beta^2)^2+\gamma^4}\ . \tag{4b}$$

We have used "magic vectors" for a covariant treatment of the threshold behavior.[8] $\lambda$, $\beta$, and $\gamma$ are free parameters which were fitted to the phases $\delta_l(p)$ in Eq. (2).

The peculiarities of the $P_{11}$ channel have been discussed in great detail by Mizutani *et al.*[6] They emphasize that in pion-nucleus collisions the effect of the $P_{11}$ channel is not negligible: While the phase shift starts out small (Fig. 2), its sign change is indicative of a compensation between attractive and repulsive forces. Correspondingly, on the level of Feynman diagrams one has the exchange of $\sigma$ and $\rho$ mesons in the $t$ channel on the one hand, and pion absorption, i.e., the $s$-channel nucleon pole, on the other.

To single out these two effects, we do not solve the BS equation

$$T_{\text{tot}}=V+VG_0T_{\text{tot}} \tag{5}$$

directly, but first decompose the total $T$ operator (omitting the angular momentum index $l=1$)

$$T_{\text{tot}}=T_P+T_{\text{NP}}\ . \tag{6}$$

Here $T_P$ contains the $s$-channel nucleon pole; correspondingly, $T_{\text{NP}}$ is $s$-channel one particle irreducible.

We choose a separable ansatz for $T_{\text{NP}}$

$$T_{\text{NP}}(q_0,q,q_0',q';s)=\frac{g(q_0,q)g(q_0',q')}{\tau_{\text{NP}}(s)}\ , \tag{7}$$

with $g$ as in Eq. (4b). The one particle irreducible part $T_{\text{NP}}$ has to be unitary.[9] This is ensured by

$$\tau_{\text{NP}}(s)=\lambda^{-1}-\frac{i}{4\pi^3}\times\int_{-\infty}^{\infty}dk_0\int_0^{\infty}k^2dk\,g^2(k_0,k)G_0(k_0,k;s)\ , \tag{8}$$

and $T_{\text{NP}}$ solves the BS equation (5) with $V$ replaced by

$$V_{\text{NP}}(q_0,q,q_0',q';s)=g(q_0,q)\lambda g(q_0',q')\ . \tag{9}$$

The second term in

$$V=V_{\text{NP}}+V_P \tag{10}$$

will generate the nucleon pole term $T_P$ through

$$T_P=(1+T_{\text{NP}}G_0)\hat{T}_P(1+G_0T_{\text{NP}}) \tag{11}$$

with

$$\hat{T}_P=V_P+V_P(G_0+G_0T_{\text{NP}}G_0)\hat{T}_P\ . \tag{12}$$

This relativistic Gell-Mann-Goldberger formula is readily obtained from the formal iteration solution of the BS equation (5) with (10) by reordering the terms of the perturbation series.

For the BS kernel $V_P$ we propose the separable ansatz

$$V_P(q_0,q,q_0',q';s)\equiv v(q_0,q)\Lambda(s)v(q_0',q')\ . \tag{13}$$

The vertex function $v$ is chosen similar to Eq. (4b):

$$v(q_0,q)=\frac{q}{(q_0{}^2-q^2-\beta_p{}^2)^2+\gamma_p{}^4}\ . \tag{14}$$

Equations (11)–(13) imply that $T_p$ is also separable

$$T_P(q_0,q,q_0',q';s)\equiv\frac{h(q_0,q;s)h(q_0',q';s)}{\tau_p(s)}\ , \tag{15}$$

with

$$\tau_P(s)=\Lambda^{-1}(s)+\frac{i}{4\pi^3}\int_{-\infty}^{\infty}dk_0\int_0^{\infty}k^2dk\,v(k_0,k)G_0(k_0,k;s)h(k_0,k;s) \tag{16}$$

and

$$\begin{aligned}h(q_0,q;s)&=v(q_0,q)+\frac{i}{4\pi^3}\int_{-\infty}^{\infty}dk_0'\int_0^{\infty}k'^2dk'\,T_{\text{NP}}(q_0,q,k_0',k';s)\,G_0(k_0',k';s)v(k_0',k')\\&=v(q_0,q)+\frac{i}{4\pi^3}g(q_0,q)\tau_{\text{NP}}{}^{-1}(s)\int_{-\infty}^{\infty}dk_0'\int_0^{\infty}k'^2dk'\,g(k_0',k')\,G_0(k_0',k';s)v(k_0',k')\ .\end{aligned} \tag{17}$$

$h(q_0,q;s)$ describes the absorption vertex of the pion; a similar formula is given for the emission vertex $h(q_0',q';s)$.

The computation of these integrals is straightforward. In contrast to investigations which use reduced equations such as, e.g., Ref. 10, we do the $k_0$ integrations. With our choice of form factors (4) and (14) these integrations can be done analytically.

For the coupling $\Lambda$ in Eq. (13) we use

$$\Lambda(s)=(s-m_0^2)^{-1}\Lambda_P \tag{18}$$

to model a bare nucleon of mass $m_0$ in the direct channel, chosen such that $T_P$ exhibits the desired singularity when $s$ approaches the squared mass of the physical nucleon:

$$\tau_P(m_N^2)=0\ . \tag{19}$$

## III. RESULTS

All partial waves are fitted to the phase shift analysis of Koch and Pietarinen,[11] with $\beta$, $\gamma$, and $\lambda$ as free parameters.

### A. Small phases

For the so-called small phases in the partial waves $S_{11}, S_{31}, P_{13}$, and $P_{31}$, the potential parameters are given in Tables I and II. The quality of our fits can be seen in the scattering lengths or volumes (Tables I and II), and particularly in the phase shifts (Fig. 1).

TABLE I. Potential parameters and scattering lengths for the pion-nucleon $S$ waves ($S_{11}, S_{31}$).

| | |
|---|---|
| | $S_{11}$ |
| $\lambda$ | $-3647.086\ \text{fm}^{-4}$ |
| $\beta$ | $2.151\ \text{fm}^{-1}$ |
| $\gamma$ | $2.916\ \text{fm}^{-1}$ |
| $a_{\text{th}}$ | $0.166 m\pi^{-1}$ |
| $a_{\text{exp}}$ | $0.173 m\pi^{-1}$ |
| | $S_{31}$ |
| $\lambda$ | $931.819\ \text{fm}^{-4}$ |
| $\beta$ | $0.610\ \text{fm}^{-1}$ |
| $\gamma$ | $2.298\ \text{fm}^{-1}$ |
| $a_{\text{th}}$ | $-0.140 m\pi^{-1}$ |
| $a_{\text{exp}}$ | $-0.101 m\pi^{-1}$ |

TABLE II. Potential parameters and scattering volumes for the pion-nucleon small $P$ waves ($P_{13}, P_{31}$).

| | |
|---|---|
| | $P_{13}$ |
| $\lambda$ | $11914.030\ \text{fm}^{-6}$ |
| $\beta$ | $1.304\ \text{fm}^{-1}$ |
| $\gamma$ | $2.872\ \text{fm}^{-1}$ |
| $a_{\text{th}}$ | $-0.040 m\pi^{-3}$ |
| $a_{\text{exp}}$ | $-0.030 m\pi^{-3}$ |
| | $P_{31}$ |
| $\lambda$ | $2468.943\ \text{fm}^{-6}$ |
| $\beta$ | $0.547\ \text{fm}^{-1}$ |
| $\gamma$ | $2.006\ \text{fm}^{-1}$ |
| $a_{\text{th}}$ | $-0.050 m\pi^{-3}$ |
| $a_{\text{exp}}$ | $-0.045 m\pi^{-3}$ |

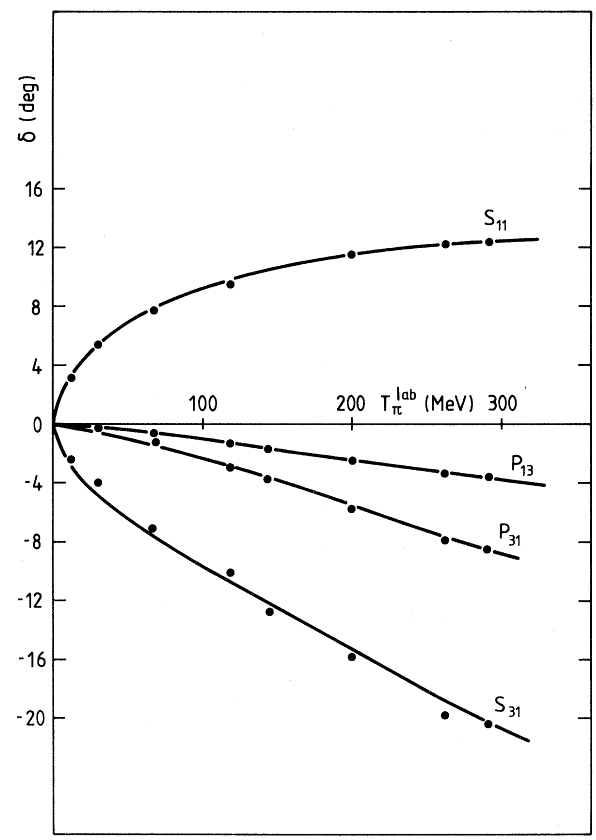


FIG. 1. Pion-nucleon "small" phases in the partial waves $S_{11}, S_{31}, P_{13}$, and $P_{33}$. The experimental values are taken from Ref. 11.

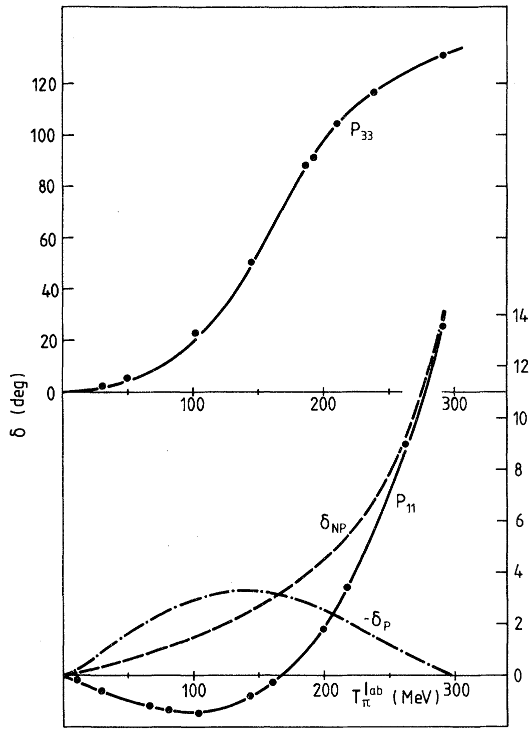


FIG. 2. Pion-nucleon phases in the partial waves $P_{33}$ and $P_{11}$. $\delta_{NP}$ and $\delta_p$ denote the phase shifts, resulting from the nonpole part and from the effect of the direct channel pole of the $P_{11}$ wave, respectively. The experimental values are taken from Ref. 11.

TABLE III. Potential parameters and scattering volumes for the pion-nucleon $P$ waves ($P_{33}$,$P_{11}$).

| | | | |
|---|---|---|---|
| | $P_{33}$ | | |
| $\lambda$ | 3864.839 fm$^{-6}$ | | |
| $\beta$ | 1.256 fm$^{-1}$ | | |
| $\gamma$ | 1.670 fm$^{-1}$ | | |
| $a_{\text{th}}$ | $0.103 m\pi^{-3}$ | | |
| $a_{\text{exp}}$ | $0.214 m\pi^{-3}$ | | |
| | $P_{11}$ | | |
| $\lambda$ | 148.020 fm$^{-6}$ | $\Lambda_p$ | 3682.701 fm$^{-4}$ |
| $\beta$ | 1.317 fm$^{-1}$ | $\beta_p$ | 0.870 fm$^{-1}$ |
| $\gamma$ | 0.697 fm$^{-1}$ | $\gamma_p$ | 1.370 fm$^{-1}$ |
| $a_{\text{th}}$ | $-0.066 m\pi^{-3}$ | $f_{\text{th}}^2$ | 0.079 |
| $a_{\text{exp}}$ | $-0.081 m\pi^{-3}$ | $f_{\text{exp}}^2$ | 0.079 |

### B. The $P_{33}$ phase

Parameters and scattering lengths are given in Table III and the phase shift can be found in Fig. 2. It is worth pointing out that a rank one separable kernel in the BS equation is quite adequate to describe a resonant phase such as the $P_{33}$ without the use of an energy dependent coupling constant.[2,12] Before turning to the two potential description of the $P_{11}$ we note in passing that the parameter $\gamma$ is of the same order in all rank one kernels, in agreement with the same observation in the nucleon-nucleon case.[5]

### C. The $P_{11}$ phase

(1) The parameters in Table III give an excellent fit to the experimental phase shift. This can be seen in Fig. 2. We also display phase shift $\delta_{\text{NP}}$ resulting from the "nonpole" part of the interaction $V_{\text{NP}}$ as well as the effect of the direct channel pole on the phase shift

$$\delta_P(p)=\delta_{\text{tot}}(p)-\delta_{\text{NP}}(p) \ . \tag{20}$$

(2) Comparing with the two sets proposed in Ref. 6 we find smaller $\delta_p$ and $\delta_{NP}$.

(3) Agreement with the experimental scattering volume is comparatively good.

(4) The $\pi NN$ coupling constant which we did not introduce as an input parameter is determined by our model at its accepted value 0.08.

(5) The Roper resonance appears in our calculation at the appropriate value of 400 MeV and originates essentially from $\delta_{\text{NP}}$ since $\delta_P$ is small there. So, in particular, our model does not predict a $\delta_{\text{NP}}$ resonance effect in $\pi$-nucleon scattering away from the Roper resonance as would the Blankenbecler-Sugar treatment of Ref. 6 (model $B$).

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[1]T. V. Bugg and A. W. Thomas, invited papers in the Proceedings of 9th International Conference on High Energy Physics and Nuclear Structure, Versailles, 1981 (unpublished).

[2]N. Giraud, C. Fayard, and G. H. Lamot, Phys. Rev. C 21, 1959 (1980).

[3]R. Blankenbecler and R. Sugar, Phys. Rev. 142, 1052 (1966); K. Schwarz, H. F. K. Zingl, and L. Mathelitsch, Phys. Lett. 83B, 297 (1979).

[4]See, for instance, C. E. Carlson in *The Few Body Problem,* edited by F. S. Levin (North-Holland, Amsterdam, 1981), p. 257c. BS investigations of the *NN* system can be found in: C. Schwarz and C. Zemach, Phys. Rev. 141, 1454 (1966); M. J. Levine, J. Wright, and J. A. Tjon, *ibid.* 154, 1433 (1967); J. L. Gammel, M. T. Menzel, and W. R. Wortman, Phys. Rev. D 3, 2175 (1971); J. Fleischer and J. A. Tjon, Nucl. Phys. B84, 375 (1975).

[5]K. Schwarz, J. Fröhlich, H. F. K. Zingl, and L. Streit, Acta Phys. Austr. 53, 191 (1981).

[6]T. Mizutani, C. Fayard, G. H. Lamot, and S. Nahabetian, Phys Rev C 24, 2633 (1981); C. Fayard, G. H. Lamot, and T. Mizutani, Phys. Rev. Lett. 45, 524 (1980); I. R. Afnan and B. Blankleider, Phys. Lett. 93B, 367 (1980).

[7]J. Fröhlich and K. Schwarz, 6$^e$ Session D'Etudes Biennale De Physique Nucleaire, Universite Claúde Bernard, Lyon, LYCEN 8102, 1981 (unpublished).

[8]See, for instance, W. M. Kloet and R. R. Silbar, Nucl. Phys. A338, 281 (1980).

[9]M. Ida, Phys. Rev. 135B, 499 (1964).

[10]Y. Avishai and T. Mizutani, Nucl. Phys. A338, 377 (1980).

[11]R. Koch and E. Pietarinen, Nucl. Phys. A336, 331 (1980).

[12]R. M. Woloshyn, E. J. Moniz, and R. Aaron, Phys. Rev. C 13, 312 (1976); A. S. Rinat and A. W. Thomas, Nucl. Phys. A282, 365 (1977).