Differential cross section for the $\pi^+ d \to pp$ reaction from 80 to 417 MeV

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Differential cross sections were measured for the reaction, $\pi^+ d \to pp$, at seven energies, 80, 100, 140, 182, 230, 323, and 417 MeV. Sufficient data were taken for a 1% statistical uncertainty at eleven different angles for most of these energies. The elastic $\pi^+ p$ reaction was used to normalize the $\pi^+ d \to pp$ data. Legendre polynomial fits of the data are presented. The increased accuracy and energy of these data require an addition of a $\cos^6\theta^*$ term in the traditional low energy form of the differential cross section: $C(A + \cos^2\theta^* + B\cos^4\theta^*)$. The coefficients, A and B, are used to compare the present data with a survey of previous experiments and the theoretical work of Niskanen. Agreement is usually within uncertainties, but some diagreement remains even among recent experiments.

[NUCLEAR REACTIONS $d(\pi,pp)$, $E = 80-417$ MeV; measured $\sigma(\theta)$; calculated Legendre polynomial fits; compared to earlier work.]

INTRODUCTION

Nonradiative pion absorption in deuterium has been studied with increasing accuracy since its first observation in 1951.[1] Originally used to obtain the spin of the pion, the reaction has since served as an important test of theories of absorption of free pions on nucleons. Its simplicity makes detailed microscopic models possible, and several such models have recently achieved moderate quantitative success.[2–5] The reaction and its inverse also provide a test of detailed balance, while isorotations test isospin invariance of the strong force. Unfortunately, existing evidence is so inconsistent that few conclusions are possible. While most experiments agree within uncertainties on the total cross section, the differential cross section measurements remain contradictory, even with recent data.

In this paper we wish to report new measurements of the $\pi^+ d \to pp$ reaction over the energy region from 80 to 417 MeV. The statistical accuracy in the measurements of the relative differential cross section approaches 1%. The absolute normalizations are more uncertain, so that our analysis has been directed primarily toward an accurate parametrization of the shape of the differential cross section at each energy. We have found that a $\cos^6\theta^*$ term is necessary to fit the differential cross sections above a pion energy of 180 MeV.

EXPERIMENTAL METHOD

The experiment was performed at the Clinton P. Anderson Meson Physics Facility (LAMPF). A diagram of the experimental layout in the high energy pion channel ($P3$) at LAMPF is shown in Fig. 1. The target was a cylindrical stainless steel flask of liquid deuterium 7.6 cm in diameter, positioned with its axis perpendicular to the scattering plane. Reaction products passed through 0.0076 cm of stainless steel, 0.005 cm of aluminum, 0.029 cm of Mylar, and approximately 60 cm of air before entering a time-of-flight spectrometer. The spectrometer consisted of two detector arms which measured the velocities and angles of the two protons in coincidence. Each of the arms consisted of three plastic scintillation detectors, with three helically-wound proportional chambers[6] located between the first and second scintillators. To reduce multiple scattering, helium bags were placed between all detectors except for the final two scintillators on

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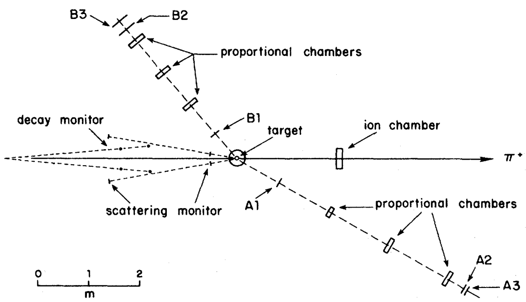

FIG. 1. Experimental layout of time-of-flight spectrometer. $A1$, $A2$, $A3$, $B1$, $B2$, and $B3$ represent plastic scintillation detectors.

each arm. Timing signals from the scintillators were used to measure the time of flight of the protons. Arm A, which defined the 0.865 msr solid angle, was 5.2 m long and used three scintillators 15.2×15.2 cm in area. Arm B was 3.5 m long and its last two scintillators were 35.6×35.6 cm in area. The time resolution of the pulses from these scintillators was typically 250 ps for the smaller ones and 500 ps for the larger ones.

The beam was monitored with an ionization chamber, two decay telescopes, and two scattering telescopes. The decay telescopes were designed to detect muons from pion decay in flight along the beam,[7] while the scattering telescopes were set to detect particles scattered from the target at 170°. These beam monitors agreed with one another to a typical precision of 0.5%. Typical fluxes were 10^6 π/sec at 80 MeV, 5×10^6 π/sec at 182 MeV, and $10^7\pi$/sec at 417 MeV. The beam was tuned for a momentum spread of 1% FWHM at 182, 230, and 323 MeV, and 2% elsewhere.

Timing information and amplitudes of the pulses from the scintillators and wire chambers were recorded along with a variety of scaled counting data on magnetic tape. The event trigger logic required a coincidence of signals from either of the front scintillation detectors, $A1$ or $B1$, and a signal from the end of each arm, either $A2$ or $A3$, and either $B2$ or $B3$. In symbols, an event was defined as $\langle A1\vee B1\rangle\wedge\langle A2\vee A3\rangle\wedge\langle B2\vee B3\rangle$. Because of the looseness of this trigger it was possible to measure the efficiency of all detectors during operation. Triggering efficiency exceeded 99%. The wire chambers were not included in the event trigger or directly in the analysis, so their inefficiency is not a factor in determining cross sections.

DATA REDUCTION

Many combinations of the scintillator timing signals could have been used to separate the reaction from background. One procedure was to use the front and back counters on each arm to separately calculate the time of flight of each proton. In most cases this worked very well, but with the foward arm at small angles, the counters close to the target tended to have such a high counting rate that the time digitizers would occasionally be set by a random pulse occurring ahead of the event pulse. Although a correction could be made, it proved to be more satisfactory to compare the times of the signals at the ends of the two arms. We used the time difference $A3$-$B2$ as our primary timing signal. This combination minimizes a solid angle correction (described below). A histogram of a typical raw time distribution is shown in Fig. 2, where the pion absorption signal stands out very clearly above the background. Empty target runs were analyzed in the same way to obtain the background, generally less than 1%, from the steel target flask. After sub-

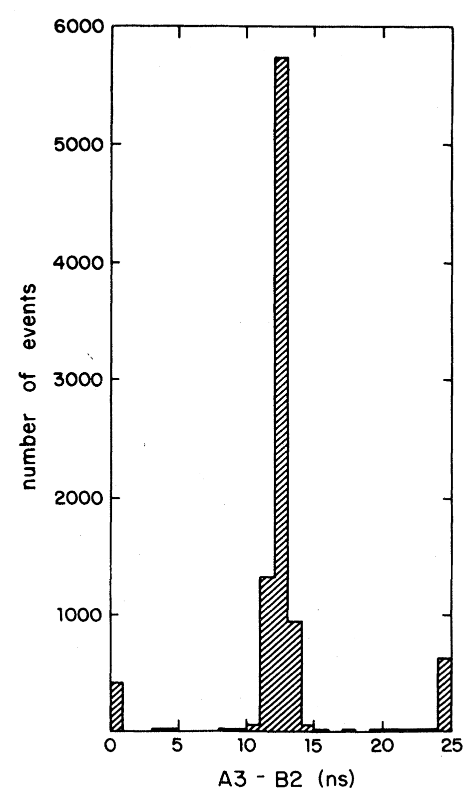

FIG. 2. Typical raw $A3$-$B2$ time difference. $\pi^+d\rightarrow pp$ events appear in center channel.

traction of these "target empty" events from the "target full" time distribution, there remained a flat distribution of background events from the deuterium in the region of the $\pi^+ d \to pp$ peak. These events were fitted to a straight line and then subtracted, leaving only pion absorption events.

Three systematic corrections were applied to the data. First, although the *B* arm was intended to be large enough to detect all protons in coincidence with the *A* arm, in practice there was some leakage when the *A* arm was at small angles. The problem was studied with a Monte Carlo routine designed to model the apparatus and incorporate multiple scattering, the beam phase space, and the effects of the spatial resolution of the wire chambers. The maximum correction for this effect was 7%, which occurred for the smallest angle at 417 MeV. At large angles no correction was necessary. Second, a mistake in the computer program responsible for recording data resulted in some duplication of events. Recovery from this error was possible by rejecting all duplicate events, which generally comprised less than 5% of the events in a run. This rejection did not bias the data or affect the normalization. Third, scintillation counter efficiency was found to be constant to within 0.25%, except for one point, which required a 2% correction. We estimate that all systematic corrections are known to better than 1% of the total event count.

Loss of protons due to nuclear scattering in the detector arms was calculated to be of the order of 1%. The variation over the angular and energy range of the experiment is negligible so that no correction was applied.

NORMALIZATION

Knowledge of detector solid angles and efficiencies, and beam and target size and composition could be combined into a single normalization factor dependent on incident beam energy. Since many of these quantities were known with only moderate precision, we measured the normalization factor by filling the target with liquid hydrogen and detecting both the scattered pion and recoil proton from $\pi^+ p$ elastic scattering. This measurement could then be compared to known $\pi^+ p$ cross sections to obtain the overall normalization. However, two effects complicate the comparison. First, the pions can decay within the spectrometer. Second, the kinematical relationship between the pion and proton is so different from that of the two protons for $\pi^+ d \to pp$, that both detector arms contributed significantly to the effective solid angle for $\pi^+ p$ scattering. These two effects were included in the Monte Carlo model and introduced corrections which grew very large at low energies. At 140 MeV the correction factor was approximately 2, and at 80 and 100 MeV, the uncertainties were too large to use the $\pi^+ p$ normalization at all. At higher energies, we compared our $\pi^+ p$ measurements to the phase shift analysis of Hohler *et al.*,[8] to calculate the normalizing factor. At 80 and 100 MeV, we normalized our $\pi^+ d \to pp$ data using Spuller and Measday's fit *C* (Ref. 9) to existing measurements.

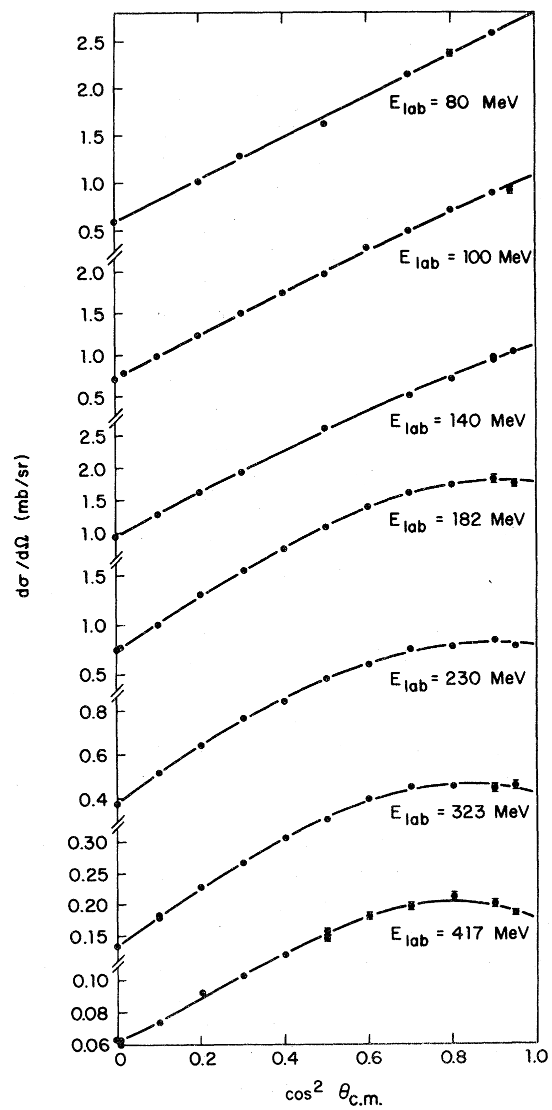

FIG. 3. Differential cross sections for $\pi^+ d \to pp$. The solid lines are Legendre polynomial fits, given in Table IV.

TABLE I. Differential cross sections for $\pi^+d\rightarrow pp$. Uncertainties are statistical only.

T_π (lab)	$\cos^2\theta^*$	$d\sigma/d\Omega$	Uncertainty
80 MeV	0.00	0.588 mb/sr	0.012 mb/sr
	0.20	1.014	0.030
	0.30	1.281	0.028
	0.50	1.625	0.034
	0.70	2.147	0.028
	0.80	2.370	0.033
	0.90	2.586	0.030
100	0.00	0.7003	0.0069
	0.02	0.7810	0.0084
	0.10	0.9706	0.0069
	0.20	1.232	0.012
	0.30	1.486	0.014
	0.40	1.745	0.017
	0.50	1.966	0.018
	0.60	2.281	0.020
	0.70	2.490	0.024
	0.80	2.736	0.022
	0.90	2.946	0.025
	0.94	2.974	0.046
140	0.00	0.933	0.0099
	0.10	1.285	0.015
	0.20	1.627	0.018
	0.30	1.947	0.019
	0.50	2.618	0.023
	0.70	3.123	0.029
	0.80	3.370	0.032
	0.90	3.694	0.025
	0.95	3.802	0.028
182	0.00	0.743	0.0073
	0.01	0.764	0.0053
	0.10	0.998	0.0098
	0.20	1.306	0.011
	0.30	1.547	0.015
	0.40	1.766	0.016
	0.50	1.989	0.018
	0.60	2.194	0.018
	0.70	2.324	0.020
	0.80	2.411	0.015
	0.90	2.482	0.018
	0.95	2.423	0.033
230	0.00	0.377	0.0042
	0.10	0.520	0.0070
	0.20	0.646	0.0063
	0.30	0.765	0.0077
	0.40	0.846	0.0086
	0.50	0.947	0.0098
	0.60	1.008	0.0099
	0.70	1.083	0.011
	0.80	1.092	0.011
	0.90	1.124	0.012
	0.95	1.097	0.012

TABLE I. (*Continued.*)

T_π (lab)	$\cos^2\theta^*$	$d\sigma/d\Omega$	Uncertainty
323 MeV	0.00	0.1336 mb/sr	0.0017 mb/sr
	0.10	0.1810	0.0016
	0.20	0.2271	0.0025
	0.30	0.2655	0.0027
	0.40	0.3053	0.0031
	0.50	0.3364	0.0034
	0.60	0.3673	0.0038
	0.70	0.3859	0.0038
	0.80	0.3873	0.0041
	0.90	0.3849	0.0038
	0.95	0.3889	0.0043
417	0.00	0.0639	0.0012
	0.01	0.0617	0.0009
	0.10	0.0736	0.0012
	0.20	0.0920	0.0014
	0.30	0.1025	0.0017
	0.40	0.1156	0.0018
	0.50	0.1282	0.0014
	0.60	0.1399	0.0021
	0.70	0.1459	0.0022
	0.80	0.1520	0.0026
	0.90	0.1475	0.0024
	0.95	0.1420	0.0020

RESULTS

The differential cross sections for $\pi^+d\rightarrow pp$ are shown in Fig. 3 and Table I. Some points in Fig. 3 are double or triple, indicating that data were taken at that angle more than once as a test of consistency. These multiple runs were combined for Table I. The uncertainty in systematic corrections is everywhere less than the statistical uncertainty. Hence, errors shown are statistical, only. The overall normalization is accurate to about 10%.

To compare experiments, it has been traditional to fit the data to the γ coefficients of Eq. (1):

$$\frac{d\sigma}{d\Omega}=\gamma_0+\gamma_2\cos^2\theta^*+\gamma_4\cos^4\theta^* \ . \qquad (1)$$

This parametrization is an unfortunate choice, because the values of the γ coefficients are unstable with respect to addition of higher order terms. We use it here only for a comparison to previous work. The results of fitting our data to Eq. (1) are shown in Table II. A comparison to other measurements is shown in Fig. 4, where the A and B coefficients are the ratios, γ_0/γ_2 and γ_4/γ_2. Solid lines connect

TABLE II. Coefficients of $\cos^2\theta^*$ polynomial. Uncertainties are statistical only.

T_π (lab)	γ_0 (mb/sr)	γ_2 (mb/sr)	γ_4 (mb/sr)
80 MeV	0.590 ± 0.018	2.151 ± 0.040	0.080 ± 0.052
100	0.706 ± 0.011	2.680 ± 0.038	-0.206 ± 0.055
140	0.932 ± 0.016	3.591 ± 0.039	-0.601 ± 0.049
182	0.731 ± 0.011	3.156 ± 0.039	-1.332 ± 0.045
230	0.376 ± 0.007	1.525 ± 0.017	-0.783 ± 0.022
323	0.130 ± 0.003	0.556 ± 0.006	-0.295 ± 0.009
417	0.060 ± 0.001	0.180 ± 0.004	-0.092 ± 0.005

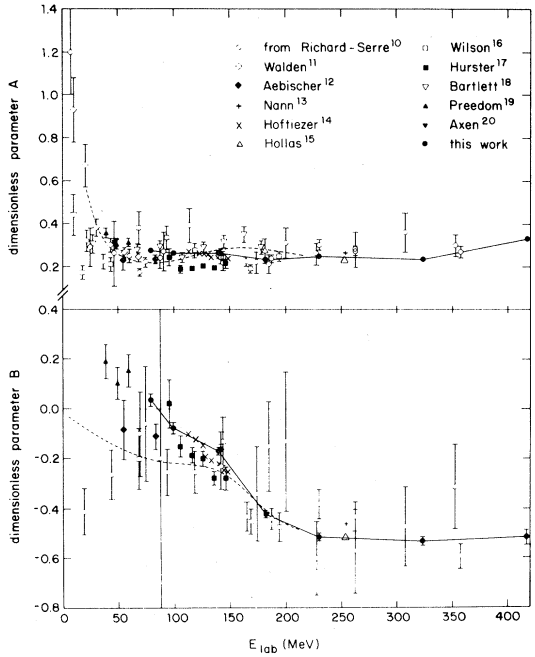

FIG. 4. Dimensionless parameter A, B. Parameters result from fits to the data of the form $d\sigma/d\Omega=c(A+\cos^2\theta^*+B\cos^4\theta^*)$. Solid lines connect the points from the present experiment. Dashed lines are the theoretical calculation of Niskanen (Ref. 21). Reactions represented are $\pi^+d\rightarrow pp$, $pp\rightarrow\pi^+d$, and $np\rightarrow\pi^0d$.

the data from this experiment. Also included in Fig. 4 are measurements of the $pp\rightarrow\pi^+d$ and $np\rightarrow\pi^0d$ reactions, related to $\pi^+d\rightarrow pp$ through time reversal symmetry and isospin rotation. The agreement with the average trend of the older data is good. For more quantitative comparisons, Table III shows the ratios of our measurements of A and B to those of other experiments. Since the energies selected by different workers seldom coincide, an interpolation procedure was used before forming the ratio. Assuming detailed balance and isospin invariance, all ratios should be unity to within experimental uncertainty. With the exception of the comparison to the $np\rightarrow\pi^0d$ data, the ratios for the A parameter show no statistically significant deviations from unity; the number of cases differing by more than one standard deviation is about what should be observed. However, for the B parameter, there are four cases involving comparisons to the reactions $np\rightarrow\pi^0d$ and $pp\rightarrow\pi^+d$ at low energies where the discrepancies are many standard deviations. Additional inconsistencies below 80 MeV, our lowest energy, do not appear in Table III. At the present time, these discrepancies should probably be interpreted as evidence for experimental systematic errors. It is clearly desirable to improve the measurements of the $np\rightarrow\pi^0d$ reaction as well as all measurements in the regions of 80 MeV.

The dashed lines in Fig. 4 are the theoretical predictions of Niskanen.[21] The theory of Niskanen generally follows the data quite well. The recent $pp\rightarrow\pi^+d$ data of Walden[11] follow the A parameter curve of Niskanen almost perfectly. It is worth noting that the Niskanen calculation predicts that at approximately 130 MeV there will be an abrupt change in the slope of the curve for B vs T_{π^+}. Although the magnitude of B obtained from calculation is apparently wrong, the change in slope appears in our data at essentially the same energy. This abrupt change in the slope of $B=\gamma_4/\gamma_2$ occurs at the peak in the coefficient γ_2, resulting from the π^+N 3-3 resonance. The coefficient γ_4 varies much more slowly through this region.

At energies above 180 MeV, the three parameter fit of Eq. (1) is inadequate and higher powers of $\cos\theta^*$ are needed. Rather than add an additional power of $\cos\theta^*$ to a nonorthogonal series, we fitted the data to Legendre polynomials whose coefficients are given in Table IV. The Legendre polynomial fits appear as solid lines in Fig. 3. Only even terms are included in the fit. We used the F test to determine how many terms in the polynomial were justified by our data. The probability of having added an insignificant term $P(F,1)$ is tabulated along with each polynomial coefficient in Table IV. Additional terms beyond those shown result in an abrupt rise in this probabilitity, indicating an unjustified term.

The total cross sections obtained from the data and listed in Table V have large errors due to the systematic uncertainties in the normalization to π^+p cross sections. They are graphed in Fig. 5. The dashed line is again the theoretical prediction of Niskanen. At the higher energies where the normalization is reliable, the agreement with previous data is good. The lower two energies have been normalized to literature values and do not represent new information.

CONCLUSIONS

As measurements of the $\pi^+d\rightarrow pp$ reaction have improved and higher energies have been reached, additional powers of cosine have been required to describe the shape of the cross section. This experiment shows that an additional $\cos^6\theta^*$ term is necessary to fit the $\pi^+d\rightarrow pp$ differential cross sections

TABLE III. A, B parameter comparison of selected data to present experiment.

		A parameter comparison			
Energy	$np\to\pi^0 d$	$pp\to\pi^+ d$	$\pi^+ d\to pp$	Author	Note
80	0.76±0.11			Wilson (Ref. 16)	a
	0.96±0.07			Bartlett (Ref. 18)	a
		1.17±0.10		Aebischer (Ref. 12)	a
100	1.18±0.14			Hurster (Ref. 17)	a
		1.11±0.21		Meshcheryakov (Ref. 22)	a
			0.83±0.17	Stadler (Ref. 23)	a
	0.82±0.11			Wilson (Ref. 16)	a
	1.00±0.07			Bartlett (Ref. 18)	a
		1.10±0.08		Aebischer (Ref. 12)	a
140	1.30±0.08			Hurster (Ref. 17)	a
		1.16±0.17		Meshcheryakov (Ref. 22)	a
	1.05±0.11			Wilson (Ref. 16)	a
	0.84±0.06			Bartlett (Ref. 18)	a
		1.02±0.09		Aebischer (Ref. 12)	a
		1.07±0.03		Hoftiezer (Ref. 14)	a
142			1.08±0.11	Richard-Serre (Ref. 10)	b
182		1.10±0.22		Meshcheryakov (Ref. 22)	a
			0.99±0.06	Richard-Serre (Ref. 10)	a
			0.93±0.15	Neganov (Ref. 24)	a
	1.05±0.07			Wilson (Ref. 16)	a
230			0.82±0.06	Richard-Serre (Ref. 10)	b
			0.94±0.17	Neganov (Ref. 24)	a
253	1.05±0.02			Hollas (Ref. 15)	b
254		0.93±0.02		Nann (Ref. 13)	b
262			0.88±0.07	Richard-Serre (Ref. 10)	b
262			0.86±0.25	Neganov (Ref. 24)	b
307			0.65±0.16	Neganov (Ref. 24)	b
352		0.88±0.15		Chapman (Ref. 25)	b
357		1.00±0.09		Heinz (Ref. 26)	b
		B parameter comparison			
Energy	$np\to\pi^0 d$	$pp\to\pi^+ d$	$\pi^+ d\to pp$	Author	Note
80	−0.17±0.12			Wilson (Ref. 16)	a
		−0.34±0.27		Aebischer (Ref. 12)	a
100	1.84±2.81			Hurster (Ref. 17)	a
	0.30±0.12			Wilson (Ref. 16)	a
		0.62±0.27		Aebischer (Ref. 12)	a
140	0.60±0.07			Hurster (Ref. 17)	a
	1.12±0.63			Wilson (Ref. 16)	a
		1.04±0.45		Aebischer (Ref. 12)	a
		0.77±0.11		Hoftiezer (Ref. 14)	a
142			0.77±0.31	Richard-Serre (Ref. 10)	b
182			0.96±0.09	Richard-Serre (Ref. 10)	a
			1.54±0.88	Neganov (Ref. 24)	a
	0.91±0.08			Wilson (Ref. 16)	a
230			1.35±0.22	Richard-Serre (Ref. 10)	b
			0.86±0.20	Neganov (Ref. 24)	a
253	1.02±0.03			Hollas (Ref. 15)	b
254		1.13±0.03		Hann (Ref. 13)	b
262			1.21±0.17	Richard-Serre (Ref. 10)	b
262			0.91±0.27	Neganov (Ref. 24)	b
307			1.12±0.38	Neganov (Ref. 24)	b
352		1.67±0.92		Chapman (Ref. 25)	b
357		0.88±0.08		Heinz (Ref. 26)	b

TABLE III. (*Continued.*)

[a]Ratio is result from present experiment divided by interpolated value using two nearby points from author listed. Energy is that of present experiment.
[b]Ratio is result of interpolation using two neighboring points from this experiment, divided by result from author listed. Energy is that of experiment by author listed.

TABLE IV. Legendre polynomial coefficients.

	a_0	a_2	a_4	a_6
T_π (lab): 80 MeV				
Polynomial coefficients:	1.325	1.477		
Errors on coefficients:	a	0.017		
Reduced chi-square[b]:	1184	1.17		
$P(F,1)$:	<0.001	<0.001		
T_π (lab): 100 MeV				
Polynomial coefficients:	1.558	1.669	−0.047	
Errors on coefficients:	a	0.011	0.014	
Reduced chi-square[b]:	2383	2.39	1.44	
$P(F,1)$:	<0.001	<0.001	0.025	
T_π (lab): 140 MeV				
Polynomial coefficients:	2.0	2.051	−0.137	
Errors on coefficients:	0.3	0.013	0.018	
Reduced chi-square[b]:	2847	8.07	1.14	
$P(F,1)$:	<0.001	<0.001	<0.001	
T_π (lab): 182 MeV				
Polynomial coefficients:	1.51	1.3470	−0.335	−0.080
Errors on coefficients:	0.51	0.0083	0.013	0.015
Reduced chi-square[b]:	2412	56.2	3.60	1.38
$P(F,1)$:	<0.001	<0.001	<0.001	0.001
T_π (lab): 230 MeV				
Polynomial coefficients:	0.727	0.5706	−0.1820	−0.0149
Errors on coefficients:	0.073	0.0053	0.0069	0.0084
Reduced chi-square[b]:	1218	71.6	1.35	1.13
$P(F,1)$:	<0.001	<0.001	<0.001	0.18
T_π (lab): 323 MeV				
Polynomial coefficients:	0.257	0.2033	−0.0695	−0.0147
Errors on coefficients:	0.026	0.0018	0.0025	0.0030
Reduced chi-square[b]:	1008	64.9	2.92	0.811
$P(F,1)$:	<0.001	<0.001	<0.001	0.001
T_π (lab): 417 MeV				
Polynomial coefficients:	0.101	0.0691	−0.0219	−0.0102
Errors on coefficients:	0.010	0.0010	0.0013	0.0017
Reduced chi-square[b]:	430	27.8	5.03	1.79
$P(F,1)$:	<0.001	<0.001	<0.001	0.001

[a]Normalized using Spuller and Measday's (Ref. 9) fit C to literature values.
[b]The reduced chi-square value for each coefficient is appropriate to a fit using only polynomials up to and including it.

TABLE V. Total $\pi^+d \to pp$ cross sections.

T_π (lab)	$\sigma_{c.m.}$	Note
80	8.28 mb	a
100	9.79	a
140	12.6 ± 1.9	
182	9.6 ± 0.95	
230	4.6 ± 0.46	
323	1.6 ± 0.16	
417	0.64 ± 0.06	

[a]Normalized using Spuller and Measday's (Ref. 9) fit C to literature values.

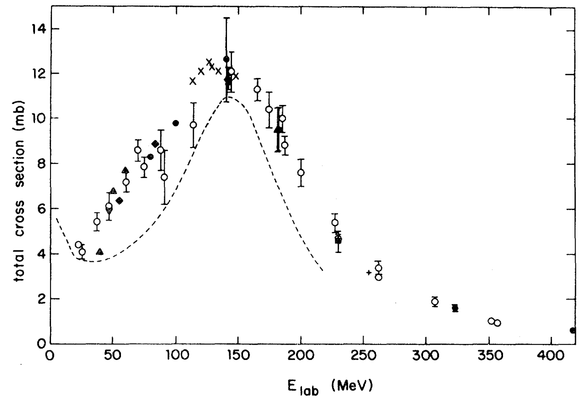

FIG. 5. Total $\pi^+d \to pp$ cross section. Dashed line represents the calculation of Niskanen (Ref. 21). The lowest two points from this experiment (solid circles at 80 and 100 MeV) were normalized using Spuller and Measday's fit C (Ref. 9) to existing measurements. Some data result from $pp \to \pi^+d$ experiments, transformed to equivalent $\pi^+d \to pp$ cross sections using detailed balance. Symbol definitions are the same as those of Fig. 4.

above a pion energy of 180 MeV. We present our results in terms of Legendre polynomials, as well as the traditional parametrization of Eq. (1). It must be stressed that Eq. (1) is a nonorthogonal power series. Thus, addition of a $\cos^6\theta^*$ term results in dramatic changes to lower order terms. Conversely, fits to data which exclude a $\cos^6\theta^*$ term will misrepresent lower order terms significantly. Because of this, recent theoretical work[27,28] compares favorably to the differential cross sections themselves, but not to the parametrization of Eq. (1). We recommend that use of Eq. (1) be discontinued, and Legendre polynomials used instead.

As a test of detailed balance and isospin invariance, this experiment provides excellent shape definition of the cross section. The shape, as shown in Fig. 4, compares well to the trends of previous experiments on the same reaction, as well as the inverse reaction, with few discrepancies larger than experimental error. The difficult $np \to \pi^0 d$ experiments generally show differences greater than reported uncertainties. We conclude that, to within a constant normalization factor, the principle of detailed balance is upheld at the 10% level of the best $pp \to \pi^+d$ shape measurements.

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