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Nuclear excitation by the inelastic photoelectric effect

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The possibility of a nucleus being excited during an interaction involving an incident photon, a bound orbital electron, and the nucleus has been investigated. In this inelastic photoelectric effect the angular distribution of the ejected photoelectrons has been calculated and the dependence of the total cross section on the photon energy and the nuclear excitation energy has been investigated.

NUCLEAR REACTIONS Calculated probability of nuclear excitation in interaction involving incident photon and bound atomic electron. Dependence of σ on Z, E_γ, and photoelectron emission angle θ studied.

I. INTRODUCTION

The possibility of an inelastic photoelectric effect has been considered recently.[1] Feynman diagrams for this process are shown in Fig. 1. In this third order process nuclear excitations occur in an interaction involving an incident photon, a bound orbital electron, and the nucleus. The nuclear excitation is produced by a dynamic interaction between the electron and the ground state of the nucleus. The process bears obvious resemblances to both the internal Compton effect and Coulomb excitation.

The electron-photon interaction can be treated in first order perturbation theory as the photoelectric effect, and the interaction between the electron and the nucleus can be treated by second order perturbation theory as the exchange of a virtual photon. In our approach nuclear recoil is neglected and nuclear degrees of freedom are described through the nuclear multipole transition operators between the ground state and excited nuclear levels. These operators enter in the respective γ-ray emission probabilities. Except for very high energies there is expected to be a close relation between the probabilities of the two processes. A similar situation was noted by Schiff[2] in an analysis of inelastic electron scattering, where the relationship persists for the leading term in the electric multipole transition even for large electron momentum transfers as compared to the photon momentum associated with the nuclear transition energy.

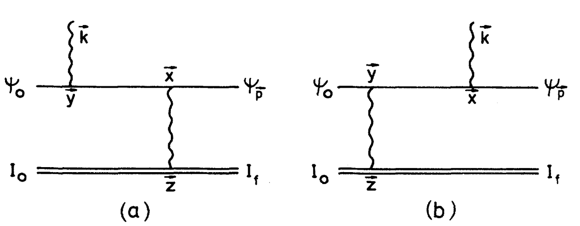

FIG. 1. The Feynman diagrams for the inelastic photoelectric effect.

In our calculations we have neglected the effects of the nuclear Coulomb field on the intermediate and final electron states, and the static electron nucleus interaction is only included through the zero momentum contribution of the bound electrons. These approximations are expected to be reasonable for incident photon momenta large compared to the momenta of the initial electrons and for ejected photoelectrons energies large compared with electron binding energy. We expect our approach to be accurate for rather large photon energies, low Z, and for relatively low nuclear levels.

An independent suggestion of the same nuclear excitation mechanism has been made by Batkin,[3] who discusses it in terms of a Compton excitation of nuclear levels. Two photon energy regions were studied, one near mc^2, where a rough estimate of the total cross section was made by using the Born approximation and neglecting process (*b*) in Fig. 1, and by considering only the zeroth component of the electric transition operator. An estimate was also made for photons with energies near the electron binding energy. Because of the dominant effect of the Coulomb field on the electron motion in this energy region, we do not expect our calculations to be applicable in this situation.

In this analysis we derive the differential cross section for the inelastic photoelectric effect (IPE), some aspects of the energy and Z dependence of the angular distribution of the ejected photoelectron, and the total cross section for various multipoles are calculated and discussed.

II. EVALUATION OF THE MATRIX ELEMENT AND CROSS SECTION

The orbital electron is described by the wave function Ψ_0 and the ejected electron with momentum $\vec{p}$ by the wave function $\Psi_{\vec{p}}$. The nuclear electromagnetic current between the spin I_0 ground state and the spin I_f excited state is J_μ. The matrix element corresponding to these Feynman diagrams can be written as:

$$M = 2\pi e^2 \delta(k+\epsilon_0-\omega-\epsilon) \times \int d\vec{x}\, d\vec{y}\, d\vec{z}\, \overline{\Psi}_{\vec{p}}(\vec{x})[\gamma_\mu D(\vec{z}-\vec{x}) J_\mu(\vec{z}) S^{(C)}(\vec{x},\vec{y})\hat{a}(\vec{y}) + \hat{a}(\vec{x}) S^{(C)}(\vec{x},\vec{y})\gamma_\mu D(\vec{z}-\vec{y}) J_\mu(\vec{z})]\Psi_0(\vec{y}) \,, \tag{1}$$

where k, ϵ_0, ω, and ϵ are the energies of the photon, orbital electron, excited nuclear level, and ejected electron, respectively. $S^{(C)}$ and D are, respectively, the electron Dirac-Coulomb and photon propagators. The incident photon four-potential a_μ is described by the momentum vector $\vec{k}$ and polarization vector $\vec{e}$. We have employed the units $m=c=\hbar=1$ (m denotes a mass of the electron), $e^2/4\pi=\alpha\simeq 1/137$. The Dirac matrices are $\vec{\gamma}=-i\beta\vec{\alpha}$, $\gamma_4=\beta$, and $\overline{\Psi}=\Psi^+\gamma_4$. The scalar product of two four-vectors $a_\mu=(\vec{a},ia_0)$ and $b_\mu=(\vec{b},ib_0)$ is $a\cdot b=a_\mu b_\mu=\vec{a}\cdot\vec{b}-a_0b_0$, and $\hat{a}=a_\mu\gamma_\mu$ is understood.

We have evaluated the matrix element M in the αZ expansion, retaining only the leading term in the expansion. In this evaluation we have made some mutually consistent approximations. We assume free motion of the electron in the intermediate states; this results in the electron propagator in the external Coulomb field $S^{(C)}$ being replaced by the free propagator. $\Psi_{\vec{p}}$ is assumed to be a plane wave of the momentum $\vec{p}$ and only the zero momentum component of Ψ_0 is considered. The binding energy of the electron is assumed to be negligible as compared with mc^2.

It is expected that these approximations will be valid for small Z, large k, and for $\epsilon \gtrsim mc^2$. The contribution of the zero momentum part of Ψ_0 is proportional to its value at $r=0$ and we only consider K-shell electrons. M is then given by:

$$M = e^2/k \left[\frac{\pi}{2k}\right]^{1/2} (\alpha Z)^{3/2} \frac{\delta(k+\epsilon_0-\omega-\epsilon)}{(q^2-\omega^2+i2\alpha Z\omega)} \bar{u}(\vec{p})[\hat{J}(\vec{q})(i\hat{j}-1)\vec{e}\cdot\vec{\gamma} + f\vec{e}\cdot\vec{\gamma}(i\hat{t}-1)\hat{J}(\vec{q})]u(0) \,, \tag{2}$$

where $u(0)$ and $u(\vec{p})$ are the electron spinor functions of momentum zero and $\vec{p}$. From energy-momentum conservation we have

$$\vec{q}=\vec{k}-\vec{p} \,,$$
$$j=[\vec{k},i(1+k)] \,, \tag{3}$$
$$t=[-\vec{q},i(1-\omega)] \,,$$

and the factor f is

$$f=(\,|\vec{p}\,|\cos\theta-\epsilon)^{-1} \,, \tag{4}$$

where θ is the angle between $\vec{k}$ and $\vec{p}$.

In Eq. (2) the Fourier-transform of the nuclear electromagnetic transition current $J_\mu(\vec{q})$ is

$$J_\mu(\vec{q}) = \int d\vec{z}\, e^{i\vec{q}\cdot\vec{z}} J_\mu(\vec{z}) \,. \tag{5}$$

For nuclear states with defined spin and parity, we can write the magnetic transition current defined by Eq. (5) in the form of the leading multipole term[4]

$$\vec{J}^{(m)} = Q_{LM}^{(m)}\,|\vec{q}\,|^L \vec{Y}_{LLM} \,,$$
$$J_4^{(m)} = 0 \,, \tag{6}$$

and for electric transitions in the form

$$\vec{J}^{(e)}=\left[\frac{2L+1}{L}\right]^{1/2} Q_{LM}^{(e)}\omega\,|\vec{q}\,|^{L-1}\vec{Y}_{LL-1M}\ ,$$

$$J_4^{(e)}=i\left[\frac{L}{L+1}\right]^{1/2} Q_{LM}^{(e)}\,|\vec{q}\,|^{L}Y_{LM}\ . \tag{7}$$

The quantities $Q_{LM}^{(e,m)}$ are the nuclear matrix elements of the 2^L-multipole transition operators between the initial and final nuclear states. Y_{LM} and $\vec{Y}_{Llm}$ are the usual spherical and spherical vector functions of the argument $\vec{q}^{\,0}=\vec{q}/|\vec{q}\,|$.

The cross section for the IPE for unpolarized photons and electrons, and unoriented nuclei, can be found by averaging over the initial, and summing over the final spin states using the squared matrix element M of Eq. (2). In the frame where $\vec{k}$ is directed in a $\vec{z}$ axis, and $\vec{p}$ is lying in the yz plane, the differential cross section for the IPE is given by:

$$\frac{d\sigma}{d\Omega}=\frac{\alpha^5}{2\pi}g\frac{\Gamma_0|\vec{p}\,|}{k^3}\frac{Z^3}{(q^2-\omega^2)^2+(2\omega\alpha Z)^2}$$
$$\times\Big\{[\omega+k(1+p\cdot j)+(j\cdot j)(1-\epsilon)]S_0^{e,m}-4kR^{e,m}(p,j)$$
$$+2(1+j\cdot j)I^e(p)-f^2[k+\omega(1+2\bar{p}\cdot t)-(t\cdot t)(1-\epsilon)]S_0^{e,m}$$
$$-4f^2(1+\bar{p}\cdot t)I^e(t)+2f^2(1+t\cdot t)I^e(\bar{p})+f\sum_{i=1}^{2}G_i^{e,m}\Big\}\ , \tag{8}$$

where $g=(2I_f+1)/(2I_0+1)$ and Γ_0 is the ground state transition width of the excited nuclear level in units of mc^2. The polarization dependent quantities $G_{1,2}$ are

$$G_i^{e,m}=(\epsilon-1)[R^{e,m}(j^{(i)},t)+R^{e,m}(t^{(i)},j)]$$
$$-k[R^{e,m}(p,t)-R^{e,m}(t^{(i)},p^{(i)})]$$
$$-\omega[R^{e,m}(p,j)+R^{e,m}(j^{(i)},p^{(i)})]$$
$$+(1-p^{(i)}\cdot j)[I^e(t)-I^e(t^{(i)})]$$
$$+(1+j\cdot t)[I^e(p^{(i)})-I^e(p)]$$
$$+[(j\cdot t)(1-\epsilon)-k(p^{(i)}\cdot t)+\omega(p^{(i)}\cdot j)]S_i^{e,m}\ . \tag{9}$$

In Eqs. (8) and (9) we introduced the four-vectors

$$\bar{p}=(0,0,p_3,i\epsilon)\ ,$$
$$j^{(1)}=j^{(2)}=(0,0,-k,-ij_0)\ ,$$
$$p^{(1)}=(0,-p_2,-p_3,-i\epsilon)\ ,$$
$$p^{(2)}=(0,p_2,-p_3,-i\epsilon)\ ,$$
$$t^{(1)}=(0,-t_2,-t_3,-it_0)\ ,$$
$$t^{(2)}=(0,t_2,-t_3,-it_0)\ ,$$

and the quantities S, R, G, and I are defined and calculated in the Appendix.

Figures 2–4 illustrate various aspects of our calculations for different multipoles assuming a nucleus with $Z=40$. In Fig. 2, the angular distribution of an ejected photoelectron is shown for an incident photon energy of 800 keV and a transition

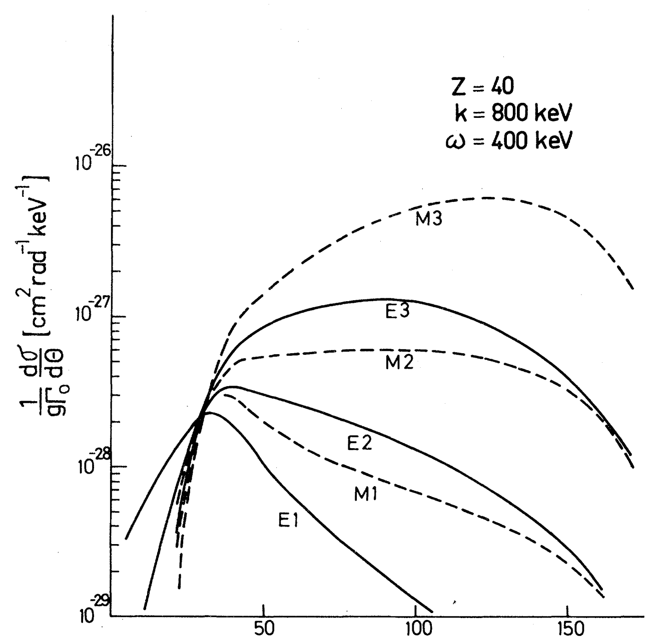

FIG. 2. The angular distribution of an ejected photoelectron for electric multipole (full curves) and magnetic multipole (dashed curves) transitions.

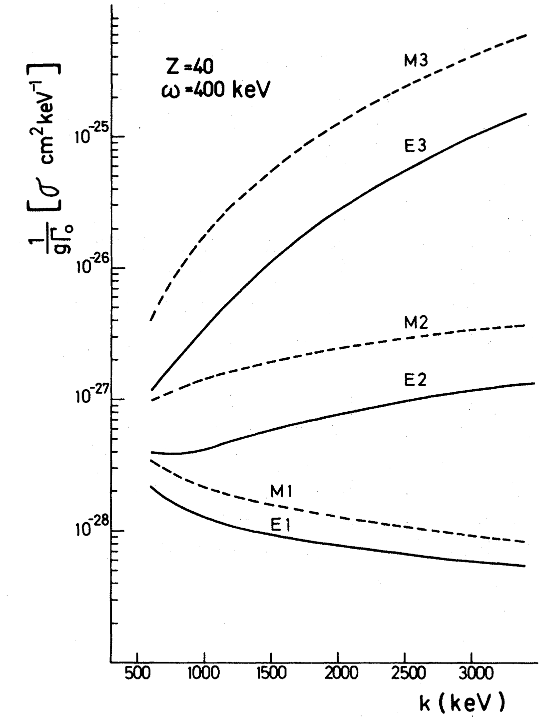

FIG. 3. The dependence of the total cross sections for IPE on the photon energy for a given transition energy.

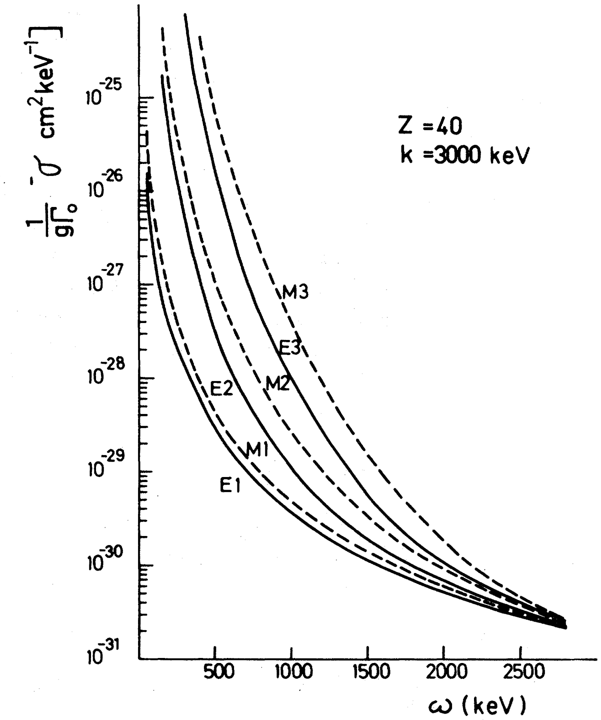

FIG. 4. The dependence of the total cross section for IPE on the transition energy for a given photon energy.

energy of 400 keV. In Fig. 3, the dependence of the total cross section for IPE on the photon energy is illustrated, and in Fig. 4, for a given photon energy, the dependence on the transition energy is shown.

III. CONCLUSIONS

Figure 2 bears some quantitative resemblance to the angular distributions obtained at these photon energies in the atomic photoelectric effect. However, in the case of the IPE, the differential cross section also depends on the nuclear transition operators and we should expect there to be differences from the pure atomic effect.

In the region where $q \neq \omega$ the cross section varies as Z^3. Near $q = \omega$ the term

$$[(q^2-\omega^2)^2+(2\omega\alpha Z)^2]^{-1}$$

gives a Z^{-2} dependence, while the width has a Z dependence. The integral then varies as Z^{-1} and the cross section has a Z^2 dependence. The same Z dependence is observed in the internal Compton effect.[5]

It is difficult to compare our results with those of Batkin,[3] as in the latter case the results are not expressed to show a Z dependence, and the analysis was not made for different multipoles. However, we can conclude that there is general qualitative agreement in the energy dependence of his estimate and our IPE cross section calculations. Although the ground state transition width Γ_0 decreases sharply with increase of the multipole order L, the IPE does not necessarily follow this trend. This is a common feature with both Coulomb excitation[6] and high-energy electron inelastic scattering.[2] At a given photon energy, the cross section for the IPE falls off rapidly as the nuclear excitation increases; a similar effect is observed in Coulomb excitation.[6]

The IPE will always make some contribution to nuclear level excitations in photon-nucleus investigations. The process is nonresonant and even in resonance fluorescence experiments contributions from off-resonance photons will occur. In practice, typical cross sections for the IPE are such that these contributions will be negligible. For example, in the case of the 1078 keV level of ^{115}In excited by ^{60}Co photons, the cross section was calculated to be $\sim 5\times10^{-37}$ cm^2 to 5×10^{-36} cm^2. As has been shown in investigations of the photoactivation of ^{115}In (Ref. 1) and ^{111}Cd (Ref. 7) other known nonresonant contributions will be of greater significance. Although the IPE is an interesting physical phenomenon, its experimental observation will be a formidable task.

ACKNOWLEDGMENTS

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APPENDIX

The transition current of Eqs. (6) and (7) give the 2^L-multipole transition rates for the emission of a photon of energy ω in the nuclear deexcitation of the I_f level to the ground state I_0 in the form

$$W_0=\frac{\omega^{2L+1}}{8\pi^2}\frac{1}{2I_f+1}\sum_{\substack{m_1m_2\\M}} |Q_{LM}^{(e,m)}|^2 , \qquad \text{(A1)}$$

where m_1,m_2 are the spin projections of the initial and final nuclear states. It is assumed that Eq. (A1) relates the nuclear matrix elements Q in the IPE process and the respective γ-transition probabilities W_0.

The averaging and summation over the nuclear spin states in the IPE cross section is accomplished by using the algebraic properties of the spherical and spherical vector functions. After some calculations the indicated quantities in Eqs. (8) and (9) are found for the 2^L multipole electric transition as:

$$\frac{1}{2I_0+1}\sum_{\substack{m_1m_2\\M}}(JI)\equiv S_0^e=C^e[(2L+1)\omega^2-Lq^2] , \qquad \text{(A2)}$$

$$\frac{1}{2I_0+1}\sum_{\substack{m_1m_2\\M}}(J^{(1)}I)\equiv S_1^e=C^eL(q^2-\omega^2) , \qquad \text{(A3)}$$

$$\frac{1}{2I_0+1}\sum_{\substack{m_1m_2\\M}}(J^{(2)}I)\equiv S_2^e=C^e\left[L(q^2-\omega^2)+(L-1)\frac{\omega^2}{q^2}p^2\sin^2\theta\right] , \qquad \text{(A4)}$$

$$\frac{1}{2I_0+1}\sum_{\substack{m_1m_2\\M}}\mathrm{Re}(aJ)(bI)\equiv R^e(a,b)$$

$$=\tfrac{1}{2}C^e[\omega^2(L+1)\vec{a}\vec{b}+\omega^2(L-1)(\vec{a}\vec{q}^0)(\vec{b}\vec{q}^0)-\omega qL(b_0\vec{a}\vec{q}^0-a_0\vec{b}\vec{q}^0)+Lq^2a_0b_0] , \qquad \text{(A5)}$$

$$\frac{1}{2I_0+1}\sum_{\substack{m_1m_2\\M}}\mathrm{Im}(aI)J_0\equiv I^e(a)=qLC^e[\omega\vec{a}\vec{q}^0-qa_0] , \qquad \text{(A6)}$$

where Re and Im mean the real and imaginary part, respectively. The four-vector I is defined as $\vec{I}=-\vec{J}^*$, $I_4=J_4^*$, and

$$J^{(1)}=(J_1,-J_2,-J_3,-J_4) ,$$

$$J^{(2)}=(-J_1,J_2,-J_3,-J_4) .$$

The C^e is given as

$$C^e=-\frac{2\pi}{L+1}\left[\frac{2I_f+1}{2I_0+1}\right]\Gamma_0\frac{q^{2(L-1)}}{\omega^{2L+1}} . \qquad \text{(A7)}$$

For the 2^L multipole magnetic transitions we found

$$S_0^m=C^m , \qquad \text{(A8)}$$

$$S_1^m=0 , \qquad \text{(A9)}$$

$$S_2^m = -C^m \frac{p^2}{q^2} \sin^2\theta \; , \tag{A10}$$

$$R^m(a,b) = \tfrac{1}{2} C^m [\vec{a}\vec{b} - (\vec{a}\vec{q}^0)(\vec{b}\vec{q}^0)] \; , \tag{A11}$$

which are defined in the same way as the electric quantities. The C^m is given as

$$C^m = -2\pi \left[\frac{2I_f+1}{2I_0+1} \right] \Gamma_0 \frac{q^{2L}}{\omega^{2L+1}} \; . \tag{A12}$$

In the above relations a,b are any two four-vectors, $q = |\vec{q}|$, $\vec{q}^0 = \vec{q}/q$, $p = |\vec{p}|$, and θ is the angle between $\vec{k}$ and $\vec{p}$.

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