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Multi-quark cluster effect on ^3He and ^4He form factors

Mikio Namiki, Keisuke Okano, and Noriyuki Oshimo
Department of Physics, Waseda University, Tokyo 160, Japan
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A multi-quark theoretical approach to ^3He and ^4He form factors based on the relativistic harmonic oscillator quark model is presented to reproduce those experimental data at large momentum transfers, which have not so far been explained in terms of ordinary nuclear physics.

[NUCLEAR REACTIONS Calculated elastic electromagnetic form factors of ^3He, ^4He, Q^2 up to 3.5 and 2.5 $(GeV/c)^2$, respectively. Multi-quark cluster effect.]

It is widely known that ordinary nuclear theoretical approaches to the structure functions of elastic electron-^3He and -^4He scatterings could not so far reproduce those experimental data[1] in the region of large momentum transfers. In Fig. 1 we can easily observe two characteristic discrepancies between the nuclear theoretical ^3He form factors and experimental data. The first is that the theoretical curve is about three times smaller than the experimental plot for $0.8 \lesssim Q^2 \lesssim 2$ $(GeV/c)^2$, Q^2 being momentum transfer squared, and the second is that the theoretical dip a little above $Q^2 \simeq 2.5$ $(GeV/c)^2$ does not clearly appear in the experimental plot. Here the theoretical form factors mean those which were obtained by the Faddeev method within the framework of ordinary nuclear physics. We can also observe quite similar situations in the ^4He case, as will be seen later. Such situations would suggest to us that the ordinary nuclear physics should be supplemented by possible quark configurations. In this paper, therefore, expecting that the discrepancies can be removed by taking the possible existence of multi-quark clusters in nuclei into account, we propose a semiphenomenological model of the ^3He and ^4He form factors based on the relativistic harmonic oscillator quark model (RHOM).[2]

In previous papers[3,4] we have reproduced fairly well the deuteron form factor assuming[5]

$$F_D(Q^2) = \cos^2\theta F_{NP}(Q^2) + \sin^2\theta F_{6q}(Q^2) \qquad (1)$$

for it, where $F_{NP}(Q^2)$ and $F_{6q}(Q^2)$ are, respectively, the ordinary proton-neutron bound state part and the six-quark bound state part, both being normalized by $F_{NP}(0)=F_{6q}(0)=1$. Equation (1) may be schematized as in Fig. 2(a). The parameter $\sin^2\theta$ represents the probability of finding the six-quark configuration in the deuteron state. The ordinary nuclear physics component $F_{NP}(Q^2)$ dominates in the region of lower Q^2's but rapidly decreases with increasing Q^2, while the six-quark component $F_{6q}(Q^2)$ becomes dominant at higher Q^2's. The RHOM gives us the formula[3]

$$F_{nq}(Q^2) = [1+(Q^2/2M_n{}^2)]^{-n+1} \times \exp\left[-\frac{n-1}{4\alpha_n}\frac{Q^2}{1+(Q^2/2M_n{}^2)}\right] \qquad (2)$$

for an n-quark bound system, where $\alpha_n = n^{3/2}\kappa$, κ being the universal coupling constant in the RHOM to be so determined as to give the Regge slope $2\alpha_3 = 1$, i.e., $\kappa = 0.096$ $(GeV/c)^2$. M_n is not necessarily equal to the physical mass of the n-quark bound state but the symmetric mass of the corresponding multiplet. However, $\sin^2\theta$ and M_n are to be adjusted here as free phenomenological parameters in order to fit our theoretical form factor to the experimental data. In the deuteron case we have actually obtained a nice fit with $\sin^2\theta = 0.07$ and $M_6 = 1.2$ (GeV/c^2) as is seen in Fig. 3, and with $\sin^2\theta = 0.05$ and $M_6 = 1.3$ (GeV/c^2) for $F_{6q}(Q^2)$ modified by taking the spin-isospin dependence into account.[4] It was also shown that the theoretical value of the deuteron magnetic moment[4] is much improved with $\sin^2\theta = 0.05 \sim 0.07$, and that the presence of dibaryon resonances[6] recently reported is consistent with the above six quark configuration in the deuteron.[3] Finally it should be remarked that the asymptotic behavior of (2), $F_{nq}(Q^2) \simeq (Q^2)^{-(n-1)}$, contains the formulas given by the dimensional scaling quark model,[7] and that the formula (2) itself can reproduce experimental pion[8] and nucleon[2] elastic form factors very well over all the Q^2 range including the preasymptotic region.

Let us analyze here the ^{3}He form factor using a natural extension of (1) in the deuteron case. Along the same line of thought as in (1),[5] we can put

$$F(Q^2) = \cos^2\theta_1 F_{NP}(Q^2) + \sin^2\theta_1 F_{MQ}(Q^2) , \tag{3a}$$

where $F_{MQ}(Q^2)$ stands for the contribution from multi-quark configurations which is decomposed as

$$F_{MQ}(Q^2) = \cos^2\theta_2 F_{6q-3q}(Q^2) + \sin^2\theta_2 F_{9q}(Q^2) . \tag{3b}$$

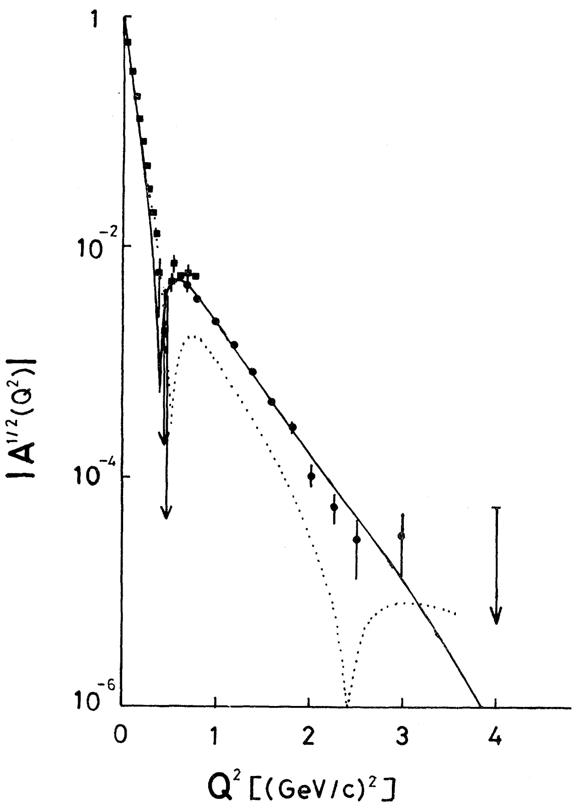

FIG. 1. Theoretical and experimental ^{3}He form factors. All curves represent theoretical form factors. The solid line stands for our final result and the dotted line for the ordinary nuclear theoretical form factor $F_{NP}(Q^2)$ given by Ref. 9.

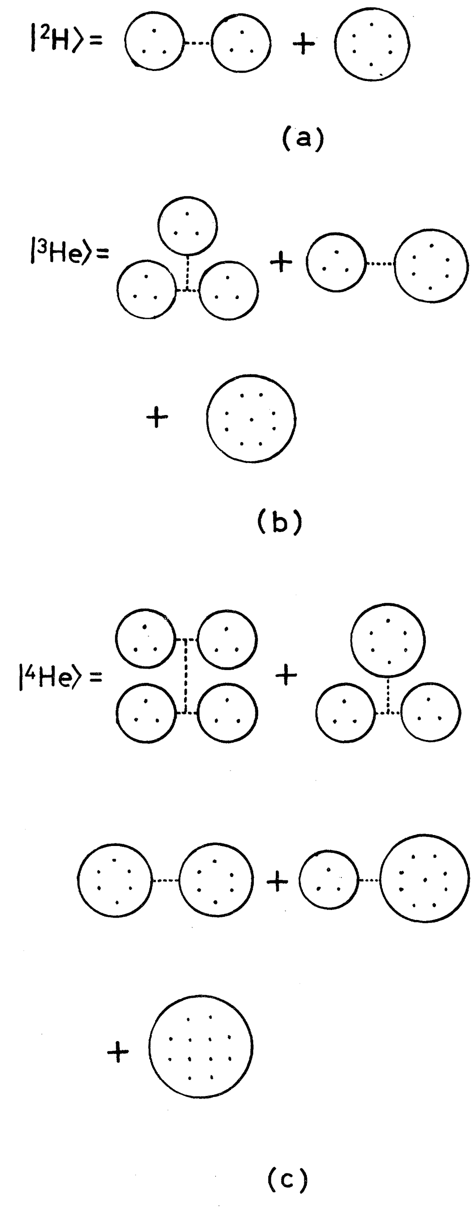

FIG. 2. Diagrams of (a) the deuteron state, (b) the ^{3}He state, and (c) the ^{4}He state.

Each term may be illustrated by the corresponding diagram in Fig. 2(b), and $F_{6q-3q}(Q^2)$ can be written as

$$F_{6q-3q}(Q^2) = \tfrac{1}{3}[\,2F_{6q}(Q^2)G_{3q}(Q^2) + F_{3q}(Q^2)G_{6q}(Q^2)]\,F_{2C}(Q^2) , \tag{3c}$$

in which the form factor $F_{nq}(Q^2)$ is given by (2) and the overlapping integral $G_{nq}(Q^2)$ by

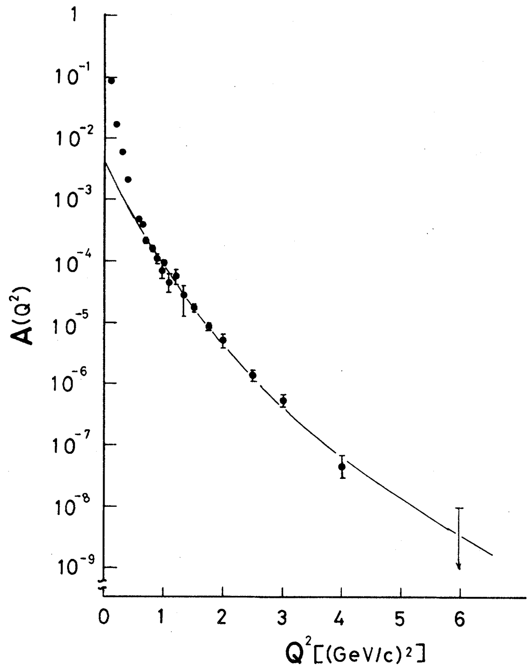

FIG. 3. The theoretical curve of the deuteron form factor is drawn here only for $\sin^2 F_{6q}(Q^2)$ to be dominant at higher momentum transfers. The form factor at lower momentum transfers is dominated by the nuclear physics component $\cos^2 F_{NP}(Q^2)$ which is not shown here.

$$G_{nq}(Q^2)=[1+(Q^2/2M_n{}^2)]^{-n+1} . \tag{4}$$

See the Appendix for derivation of (3c) with (2) and (4). The parameters $\cos^2\theta_1$, $\sin^2\theta_1\cos^2\theta_2$, and $\sin^2\theta_1\sin^2\theta_2$ in (3a) and (3b) are, respectively, probabilities of finding ordinary nuclear-theoretical, $(6q)-(3q)$, and $(9q)$ configurations in ^{3}He. $F_{NP}(Q^2)$ is to be identified with the ^{3}He form factor obtained by ordinary nuclear physics—practically, the Faddeev calculation given by Brandenberg and Sick[9] which is plotted by the dotted line in Fig. 1. As for $F_{2C}(Q^2)$ we can assume the one-boson exchange mechanism between $(6q)$ and $(3q)$ clusters, that is,

$$F_{2C}(Q^2)=(1+Q^2/m^2)^{-1} , \tag{5}$$

with m being a boson mass of the order of 1 (GeV/c^2). Instead of (5), we may put $F_{2C}(Q^2)=\exp(-Q^2/m^2)$ on the basis of the Reggeon exchange mechanism. Both are not so different from each other in the present Q^2 region, so that we exclusively used (5) with $m=1.0$ (GeV/c^2) as a natural choice in hadron physics. All the form factor functions and the overlapping integrals are normalized to unity at $Q^2=0$. We further assume that the relative sign of F_{NP} to F_{MQ} is negative for some

TABLE I. Mass and mixing parameters.

Mixing parameters				Mass parameters	
^{3}He		^{4}He			(GeV/c)
$\sin^2\theta_1$	0.04	$\sin^2\theta_1'$	0.12	M_3	1.097
$\sin^2\theta_2$	0.04	$\sin^2\theta_2'$	0.04	M_6	1.2
		$\sin^2\theta_3'$	0.03	M_9	1.5
		$\sin^2\phi$	0.5	M_{12}	2.0

theoretical reason—the same assumption has also been used in Ref. 10. Since parameters M_3 and M_6 have already been fixed in the analysis of nucleon[2] and deuteron form factors,[3,4] our free parameters are only M_9 and the mixing parameters ($\sin^2\theta_1$ and $\sin^2\theta_2$). We can easily guess that $\sin^2\theta_1 \cong \sin^2\theta_2 \cong \sin^2\theta$ hold in the sense of the order of magnitude, because $\sin^2\theta_1$ and $\sin^2\theta_2$ are considered to be probabilities of forming the multi-quark clusters to play the same role as $\sin^2\theta$ in the deuteron case mentioned above. Within the accuracy of this equality, these parameters can be so adjusted as to fit the theoretical form factor (3) to the experimental data. Final choice of the parameters listed in Table I gives us the theoretical ^{3}He form factor

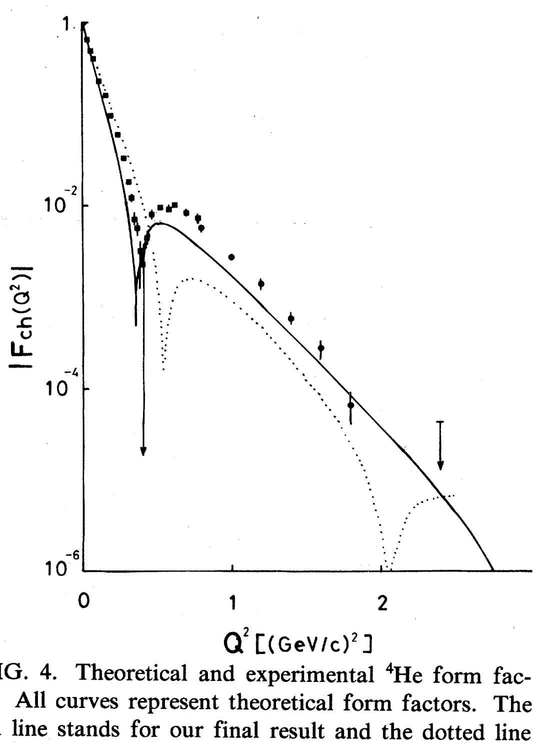

FIG. 4. Theoretical and experimental ^{4}He form factors. All curves represent theoretical form factors. The solid line stands for our final result and the dotted line for ordinary nuclear theoretical form factor $F_{NP}(Q^2)$ given by Ref. 11.

shown by the solid line in Fig. 1, which is really a very nice fit to the experimental plot. From this we can see the important role of our multi-quark part in the region of large Q^2. It is also worth mentioning that the position of diffraction dip shifts down to $Q^2 \cong 0.39$ (GeV/c)^2 from the original position $Q^2 \cong 0.54$ (GeV/c)^2 predicted by Brandenberg,[9] owing to the negative contribution of $F_{MQ}(Q^2)$ to $F_{NP}(Q^2)$. This tendency is consistent with the experiment. We must also remark that, in the Q^2 region below the first diffraction dip, our results have improved the theoretical fit given in Ref. 9, but are still slightly smaller than the experimental plot. The small gap can be removed by taking a contribution from the magnetic form factor or a possible rather hard two- or three-body nuclear force into account.

Finally we analyze the ^{4}He form factor on the same theoretical basis as in the ^{3}He and deuteron cases, by putting

$$F(Q^2)=\cos^2\theta_1' F_{NP}(Q^2)+\sin^2\theta_1' F_{MQ}(Q^2) , \tag{6a}$$

$$F_{MQ}(Q^2)= \cos^2\theta_2' F_{6q-3q-3q}(Q^2)+\sin^2\theta_2'\cos^2\theta_3' \times[\cos^2\phi F_{9q-3q}(Q^2)+\sin^2\phi F_{6q-6q}(Q^2)]+\sin^2\theta_2'\sin^2\theta_3' F_{12q}(Q^2) , \tag{6b}$$

$$F_{6q-3q-3q}(Q^2)=\tfrac{1}{2}\{F_{6q}(Q^2)[G_{3q}(Q^2)]^2+F_{3q}(Q^2)G_{6q}(Q^2)G_{3q}(Q^2)\}F_{3C}(Q^2) , \tag{6c}$$

$$F_{9q-3q}(Q^2)=\tfrac{1}{4}[3F_{9q}(Q^2)G_{3q}(Q^2)+F_{3q}(Q^2)G_{9q}(Q^2)]F_{2C}(Q^2) , \tag{6d}$$

$$F_{6q-6q}(Q^2)=F_{6q}(Q^2)G_{6q}(Q^2)F_{2C}(Q^2) , \tag{6e}$$

where we have used the similar notations as in (3). Each term in (6b) can be represented by the corresponding diagrams in Fig. 2(c). $F_{NP}(Q^2)$ should be identified with the ^{4}He form factor obtained by ordinary nuclear physics, for which we tentatively use the theoretical calculations given by Katayama and Tanaka[11] on the basis of the Hamada-Johnston potential and a special three-body interaction potential. Similarily as in the ^{3}He case, we use (2) for F_{nq} and (4) for G_{nq} and put

$$F_{2C}(Q^2)=[F_{3C}(Q^2)]^{1/2}=(1+Q^2/m^2)^{-1} \tag{7}$$

with $m=1$ (GeV/c^2). With the same choice of parameters listed in Table I, we can get our final result as shown by the solid line in Fig. 4, where we have also assumed that the relative sign of $F_{MQ}(Q^2)$ to $F_{NP}(Q^2)$ is negative as in the ^{3}He case. Figure 4 shows us that we have obtained a much better theoretical fit to the experimental plot than in the ordinary nuclear theory.

From the above analyses, it is concluded that the small mixture of multi-quark configurations in nuclear structure enables us to very much improve the ordinary nuclear physics results of ^{2}H, ^{3}He, and ^{4}He form factors.

We would like to express our sincere gratitude to Professor I. Sick and Dr. T. Katayama for kindly sending us the numerical tables of their nuclear theoretical ^{3}He and ^{4}He form factors. We are also indebted to Professor I. Ohba, Mr. T. Fukai, and Mr. M. Kato for valuable discussions.

APPENDIX: DERIVATION OF THE FORM FACTOR OF THE TWO CLUSTER COMPONENT

First of all, we have to remark that the inner orbital wave function of a relativistic bound system, governed by the Bethe-Salpeter equation or the RHOM equation, should depend on the center-of-mass momentum—see Ref. 2. Owing to the dependence, the final inner wave function is different from the initial one. Therefore, we can define the form factor by

$$F(Q^2)=\int \phi^*(P_F;r)e^{-q\cdot r}\phi(P_I;r)d^4r \tag{A1}$$

and the overlapping integral by

$$G(Q^2)=\int \phi^*(P_F;r)\phi(P_I;r)d^4r , \tag{A2}$$

where r stands for the four-dimensional relative coordinate, and P_F and P_I for the final and initial center-of-mass four momenta, respectively. $Q^2=-q^2$ represents the invariant momentum transfer squared. The RHOM wave function gives us the formulas (2) for $F(Q^2)$ and (4) for $G(Q^2)$. Note that $G(Q^2)$ is never equal to unity unless $P_F=P_I$.

Here we derive the form factor of a two cluster component $F_{2q-2q}(Q^2)$, each cluster being composed of two particles with unit charge, for simplicity, instead of $F_{6q-3q}(Q^2)$, etc.

The $(2q-2q)$ system can be represented by the wave function

$$\Psi_{2q-2q}(P;X,R,r_1,r_2)=N\,\Phi(P;R)\phi_1(P;r_1)\times\phi_2(P;r_2)e^{-iP\cdot X}\,, \tag{A3}$$

where P and X stand for, respectively, the center-of-mass momentum and coordinate, R for the relative coordinate between the two clusters, and r_1 and r_2 for the relative coordinates inside each cluster. N is the normalization constant. Using (A3) we can define the form factor of the two cluster component by

$$\begin{aligned}\delta^4(P_F-P_I-q)F_{2q-2q}(Q^2)\\ =\tfrac{1}{4}\int d^4X\int d^4R\int d^4r_1\int d^4r_2\Psi^*_{2q-2q}(P_F;X,R,r_1,r_2)\\ \times[e^{-iq\cdot(X-R+r_1)}+e^{-iq\cdot(X-R-r_1)}+e^{-iq\cdot(X+R+r_2)}+e^{-iq\cdot(X+R-r_2)}]\\ \times\Psi_{2q-2q}(P_I;X,R,r_1,r_2)\,,\end{aligned} \tag{A4}$$

which yields

$$F_{2q-2q}(Q^2)=F_{2q}(Q^2)G_{2q}(Q^2)F_{2C}(Q^2)\,, \tag{A5}$$

where $F_{2q}(Q^2)$ and $G_{2q}(Q^2)$ are given by (2) and (4), respectively, and

$$F_{2C}(Q^2)=\int d^4R\,\Phi^*(P_F;R)e^{-iq\cdot R}\Phi(P_I;R)\,. \tag{A6}$$

Note that $\Phi(P;R)$ is not a tightly bound state as described by the RHOM but a loosely bound state as governed by the ordinary hadron dynamics, then we have put (7) for $F_{2C}(Q^2)$.

Generalizing the above procedure, we can easily derive (3c) for $F_{6q-3q}(Q^2)$ corresponding to (A5) in the case of the $(2q-2q)$ system. It is noted that $G_{nq}(Q^2)$ has the same Q^2 dependence as $F_{nq}(Q^2)$ does in the region of higher momentum transfers.

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