Shape isomeric band in ^{32}S

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An unconstrained variation after angular momentum projection yields a rotational shape isomeric band in ^{32}S. The 8.507, 9.065, 10.276 MeV levels of ^{32}S are identified as the 0^+, 2^+, 4^+ members of the band. Any other set of three known levels (with at least approximate spin assignments) can be ruled out.

[NUCLEAR STRUCTURE ^{32}S; calculated shape isomeric band, moment of inertia. Variation after projection. Finite-range forces. Experimental levels assigned.]

Nuclear fission from shape isomeric states that are localized in a second prolate minimum of the deformation energy surface is a typical phenomenon of the actinide region. To date, more than 30 shape isomers have been observed. However, the region of elements with experimentally known shape isomers is rather narrow ($Z=92-97$); see, e.g., Ref. 1. It has been speculated[2,3] that shape isomers may also exist in light nuclei such as ^{40}Ca or ^{32}S, but there is no experimental evidence for shape isomeric levels in a nucleus lighter than $A=235$.

For ^{32}S we present in this paper both a calculated shape isomeric band and the identification of the corresponding lowest three levels in the experimental spectrum. Earlier conjectures about a hypothetical shape isomer in ^{32}S are based on a second minimum in the potential energy surface that has been obtained by different methods including the constrained Hartree-Fock,[4,5] Brueckner Hartree-Fock,[6] cluster model,[7,8] liquid drop with Strutinsky correction,[3,4,9,10] and a combination of cranking with the Strutinsky approach.[11]

The small particle number of ^{32}S causes problems in the conventional methods for heavy nuclei, such as the liquid-drop, Strutinsky, and cranking approach. Hartree-Fock results have led to contradictory conclusions (there are calculations with and without the second minimum[5,12]). In addition, most existing calculations are for the intrinsic state only; none have good angular momentum for both the ground and shape isomeric states. Therefore, the (possibly large) relative effect of angular momentum projection on the states in the first and the second minimum is not known. It is therefore difficult to deduce from the existing calculations information about the shape isomeric spectrum that is accessible to experimental verification, such as the excitation energy of the band head and the rotational constant (if any).

In order to avoid these difficulties, we have performed a variational search for isomeric states after projection onto good parity and angular momentum. The spectrum has been calculated for the standard Brink-Boeker B_1 soft-core Hamiltonian,[13] including the exact Coulomb and center-of-mass term. The intrinsic states are taken to be alpha-cluster model states,[14] i.e., Slater determinants Φ of $1s$ quartet structures centered around given positions $\vec{\mathbf{R}}$

$$\Phi(\vec{\mathbf{x}}_1,\ldots,\vec{\mathbf{x}}_A)\sim A\phi_1(\vec{\mathbf{x}}_1)\cdots\phi_A(\vec{\mathbf{x}}_A)\,, \tag{1}$$

where

$$\phi_i(\vec{\mathbf{x}}_i)\sim\exp[-(\vec{\mathbf{x}}_i-\vec{\mathbf{R}}_j)^2/2b^2]\chi_i\,,\quad (i=1,\ldots,A;j=1,\ldots,A/4)\,. \tag{2}$$

Here i labels the nucleons, j the quartet centers, and χ_i denotes the spin and isospin state of the nucleon. This type of variational many-body state has the technical advantage that no Lagrange constraint is needed to prevent the isomer search from going over into the ground state minimum. Thus the ambiguities associated with the choice of a shape constraint can be avoided.

The calculation is similar to Refs. 8 and 15 except that the variation has now been performed after projection onto 0^+, 2^+, 4^+, 6^+, 8^+, and 10^+ for the isomeric states and after 0^+ projection for the ground state of ^{32}S. The variational space has

been restricted to the D_{3d} and C_{2v} point symmetries of the intrinsic states that result from the variation[8,15] before angular momentum projection. This reduces the number of variational parameters to 7 and 4 in the ground state and isomer search, respectively.

The results are given in Table I. The shape isomer of ^{32}S turns out to be a very good rotor: The calculated level spacing follows the $J(J+1)$ pattern to within 1%:

$$E(J^+)-E(0^+)=(77.7\ \text{keV}\pm 1\%)J(J+1)\ ,$$
$$(J^\pi=0^+,2^+\ldots 10^+)\ . \qquad (3)$$

Moreover, the intrinsic state that minimizes the energy remains exactly the same for all angular momentum projections $0^+,2^+\ldots 10^+$. As considerable computer time is required for the present calculation, no attempt has been made to calculate levels beyond the 10^+, although the spacing and structure of the higher states would be important for the experimental population and observation of the isomeric band.

The deformation of the intrinsic state is in the vicinity of an axis ratio $c/a=2:1$, as one would expect for the secondary shells of a harmonic oscillator potential. The quadrupole moment of the intrinsic state $Q_0=208$ fm^2 would correspond to a classical uniformly charged spheriod with an axis ratio $c/a\sim 2.3$. The actual single-nucleon density, however, is necked-in and deviates substantially from a spheriodal distribution (cf. Fig. 1 and Ref. 8). It has an axis ratio of 1.7 for the 10% density contour and 2.1 for the 50% contour.

We note that the effect of angular momentum projection accounts for more than 50% of the calculated spacing between the 0^+ ground state and the 0^+ isomer level. It turns out that variation after angular momentum projection lowers the ground state of ^{32}S by about 12 MeV but the deformed isomeric state by only 8 MeV. This may explain why the calculated value of 7.51 MeV excitation for the 0^+ isomeric band head (over the -190.674 MeV variation-after-projection ground state) is much larger than previous estimates from unprojected calculations.[16]

Alpha-cluster model variations after projection for the B_1 interaction are known to yield spectra in light nuclei that are not too far from reality (e.g., the 16.75 MeV band[17] in ^{16}O, and other spectra; see Ref. 18). This is particularly true if one allows for a readjustment of the calculated energies by an appropriate overall scale factor. As a simple estimate, we have therefore adjusted the calculated band such that its 0^+ band head fits the nearest 0^+ level in the experimental spectrum.[19] This requires a 13% increase in the energy scale. The resulting band is given in column 3 of Table I. It turns out that there are two more states that fit the adjusted band within 0.025 MeV. The energies, spin, parity, and isospin assignments of these levels are listed in Table I.

In addition, we have also searched for alternative experimental 0^+, 2^+, 4^+ candidates (6^+, 8^+, and 10^+ are not known) by performing a numerical search among all experimental levels of Ref. 19 that have at least approximate spin assignments (e.g., $J\in 0,2$ or $J\in 2,3,4$). We find that the three experimental levels of Table I are unique if one rules out any three levels that

(i) deviate from rotational $J(J+1)$ spacing by more than 5%,

(ii) have no definite $J=0$ assignment,

TABLE I. Shape isomeric spectrum of ^{32}S. The entry "calc. E" gives the band resulting from variation after projection onto $0^+,\ldots,10^+$ for the B_1 interaction. The adjusted energies ("adj. E") are the calculated energies ("calc. E") multiplied by a factor of 1.133 to make the calculated 0^+ level fit the nearest experimentally known 0^x level. The experimental levels in columns 4 and 5 have been determined by a numerical search among all known levels (Ref. 19) with the requirements described in the text.

Calc. $J^\pi;T$	Calc. E (keV)	Adj. E (keV)	Expt. E (keV)	Expt. $J^\pi;T$
$0^+;0$	7510	8507	8507 ± 8	0^+
$2^+;0$	7981	9040	9065 ± 2	$(2^+,3^-,4^+);0$
$4^+;0$	9076	10281	10276 ± 8	4^+
$6^+;0$	10783	12214		
$8^+;0$	13085	14822		
$10^+;0$	15958	18076		

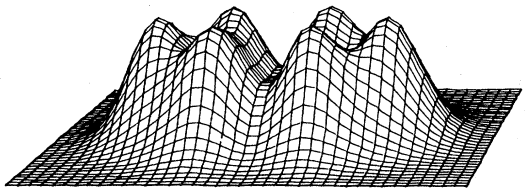

FIG. 1. Density of the intrinsic state of the shape isomeric band in ^{32}S. The figure indicates that the shape isomer has a high degree of ^{16}O+^{16}O clustering. The plot is for a cut along the axis connecting the centers of the ^{16}O substructures.

(iii) do not have positive (or unknown) parity,

(iv) differ by more than 50% from the calculated ratio of rotational constant $\Delta E/J(J+1)$ over excitation energy $E(0^+)$ (i.e., they fail to fit an adjusted band no matter how large the scale factor).

The three experimental levels of Table I match these requirements rather well: They have rotational spacing within 2.5%,

$$E_{\text{expt}}(J^+)=E_{\text{expt}}(0^+)+(90.7\text{ keV}\pm 2.5\%)J(J+1)\ , \tag{4}$$

two definite spin assignments are known, and their relative slope $\Delta E/J(J+1):E(0^+)$ is within 3% of the calculated value. We conclude from this that the determination of both the experimental band and the adjusted band of Table I should be quite safe within the known spectrum[19] of ^{32}S.

In conclusion, we summarize the properties of the shape isomeric band in ^{32}S that may facilitate the experimental discrimination among many other states: The isomeric band is a particularly long rotational band (at least up to the 10^+) in a vibrational nucleus and should have a correspondingly long γ cascade. It starts at a high excitation (8.507 MeV), and has a large moment of inertia [$\Delta E/J(J+1)=90.7$ keV] and a large quadrupole moment ($Q_0=208$ fm^2). The overlap with the ground state is small ($4\cdot 10^{-9}$) and should guarantee a relatively long lifetime of the isomer. This may lead to γ-transitions that are sufficiently delayed to make their observation feasible. In addition, the shape isomer is 1.56 MeV above the (^{28}Si+α) threshold, and α-decay can possibly serve as a trigger similar to fission events in the spectroscopy of actinide shape isomers. We also note that the density distribution of the shape isomer is very similar to the density distribution of the ^{16}O-^{16}O resonance in Ref. 20. One may therefore speculate that the γ-decay of the ^{16}O-^{16}O resonance may serve to populate the shape isomeric band.

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[1]V. Metag, in *Proceedings of the Fourth International Atomic Energy Agency Symposium on Physics and Chemistry of Fission, Jülich, 1979* (IAEA, Vienna, 1980), Vol. I, p. 153.

[2]V. Metag, R. Repnow, P. von Brentano, F. Dickmann, and K. Dietrich, Phys. Lett. B34, 257 (1971).

[3]R. K. Sheline, I. Ragnarsson, and S. G. Nilsson, Phys. Lett. B41, 115 (1972).

[4]S. J. Krieger and C. Y. Wong, Phys. Rev. Lett. 28, 690 (1972).

[5]P. G. Zint and U. Mosel, Phys. Lett. B58, 269 (1975); Phys. Rev. C 14, 1488 (1976).

[6]R. Y. Cusson, R. Hilko, and D. Kolb, Nucl. Phys. A270, 437 (1976).

[7]H. Schultheis, R. Schultheis, K. Wildermuth, A. Faessler, and F. Grümmer, Z. Phys. A 286, 65 (1978).

[8]W. Bauhoff, H. Schultheis, and R. Schultheis, Phys. Lett. B94, 285 (1980).

[9]G. Leander and S. E. Larsson, Nucl. Phys. A239, 93 (1975).

[10]I. Ragnarsson, S. G. Nilsson, and R. K. Sheline, Phys. Rep. 45, 1 (1978).

[11]I. Ragnarsson, S. Åberg, and R. K. Sheline, Lund Institute of Technology, Report No. Lund-MPh-80/19, 1980.

[12]H. Flocard, Phys. Lett. B49, 129 (1974).

[13]D. M. Brink and E. Boeker, Nucl. Phys. A91, 1 (1967).

[14]D. M. Brink, in *Many-Body Description of Nuclear Structure and Reactions,* Proceedings of the International School of Physics, "Enrico Fermi," Course XXXVI, Varenna, edited by C. Bloch (Academic, New York, 1966), p. 247.

[15]W. Bauhoff, H. Schultheis, and R. Schultheis, Phys. Rev. C 22, 861 (1980).

[16]We do not count here the projection-after-variation result of Ref. 8 for the V_1 force, which is not fitted to nuclei beyond the p shell.

[17]D. M. Brink, H. Friedrich, A. Weiguny, and C. W. Wong, Phys. Lett. 33B, 143 (1970).

[18]A. Arima, H. Horiuchi, K. Kubodera, and N. Takigawa, *Advances in Nuclear Physics,* edited by M. Baranger and E. Vogt (Plenum, New York, 1972),Vol. V, p. 345.

[19]P. M. Endt and C. van der Leun, Nucl. Phys. A310, 1 (1978).

[20]D. Baye and G. Reidemeister, Nucl. Phys. A258, 157 (1976).