Role of configuration mixing on absolute alpha-decay width in Po isotopes

F. A. Janouch and R. J. Liotta
Research Institute of Physics, S-104 05 Stockholm 50, Sweden
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Absolute alpha-decay widths for the alpha decays of ^{210,212,214,216}Po are calculated within the framework of the nuclear field theory. A tremendous enhancement factor produced by configuration mixing is found.

[RADIOACTIVITY α decay, absolute width in $^{210-216}$Po ground to ground state calculated by nuclear field theory. Configuration mixing.]

The evaluation of absolute alpha-decay rates requires a good description of alpha-cluster formation inside the mother nucleus. From a pure mathematical viewpoint the shell model is able to provide such a description including even the effects induced by the Pauli principle. However, to achieve this the continuum part of the spectrum must be included and then even the most simple calculation may become a difficult undertaking. To avoid this problem it was proposed by Fliessbach and Mang[1] to use the resonating group method to account for both the continuum and the Pauli principle in the exit channel. However, as was pointed out by Tonozuka and Arima,[2] this method has a number of uncertainties, e.g., the choice of the effective potential between the alpha particle and the daughter nucleus. At the same time Tonozuka and Arima found that highly excited shell-model configurations can play an important role in alpha decay, as one would expect.[3,4]

An appropriate tool to include a large number of configurations is the nuclear field theory (NFT).[5] In this paper, we apply the formalism developed in Ref. 5 for the calculation of absolute alpha-decay rates in the Po isotopes.

A proper shell-model treatment of alpha-decay (i.e., including the continuum) may allow us to write the alpha-decay width in the classical form[4,11]

$$\Gamma_L = 2\gamma_L{}^2 P_L/\hbar^2 \,, \tag{1}$$

where P_L is the Coulomb penetration factor and γ_L is the alpha-particle formation amplitude. For the formula (1) to be valid one must be able to define a region in space where the following two conditions are fulfilled: (i) The "alpha-particle" wave function inside the mother nucleus, represented by γ_L, can be smoothly connected with the corresponding wave function outside the mother nucleus G_L.[2] (ii) The width Γ_L must not depend strongly upon the nuclear radius in that region.[4] In all cases, presented in this paper, we checked that both conditions were fulfilled.

The amplitude γ_L is graphically displayed in Fig. 1. In fact, this diagram is the only one which contributes to γ_L within the Tamm-Dancoff approximation[5] and therefore our results coincide exactly with those obtained by the shell model.

To evaluate the diagram in Fig. 1, one has first to decide how to include the continuum part of the spectrum. It does not seem to be appropriate to utilize harmonic oscillator wave functions, since no continuum is present in harmonic oscillator potentials. Yet, this assumption is sometimes made in order to be able to include many configurations as well as to facilitate the description of the internal structure of the alpha particle. That is, if one assumes the neutrons and protons to be bound separately in a relative s-wave state,[2,3,6] one has to introduce the Moshinski transformation and consequently also the harmonic oscillator wave functions.[2] Another way to include the continuum was first introduced by Bayman[7] in two-particle transfer reactions. Bayman realized that the inclusion of the continuum is necessary in order to get a proper account of the tail of the cluster wave function. This effect can be imitated by the artifice of using for each single particle state a Woods-Saxon potential which binds the particle with $\frac{1}{2}$ of the separation energy of the two-particle cluster.[8] A similar procedure was recently shown to be adequate also for

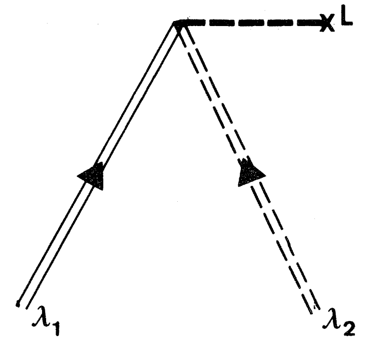

FIG. 1. Lowest order NFT diagram representing the alpha decay from a two-boson system (as ^{212}Po) to the core ground state. The other cases calculated in Table I can easily be derived from this diagram. Full (dashed) lines represent proton (neutron) pairing boson.

alpha-particle transfer reactions.[6] We use this procedure with the assumption that the alpha particle is formed when the four nucleons are on a sphere of radius R centered in the mother nucleus.[5,9] This assumption does not require further approximations[2] regarding the relative motion of the neutrons and protons that eventually constitute the alpha particle. We thus obtain, from Fig. 1

$$\gamma_L = \frac{(-1)^{\lambda_\pi+\lambda_\nu}}{(2L+1)}(4S_\alpha/3)^{1/2}\sqrt{\hbar^2 R/8\pi\mu} \times \langle \lambda_\pi 0 \lambda_\nu 0 \,|\, L\,0\rangle A \;, \tag{2a}$$

$$A = B(\lambda_\pi)B(\lambda_\nu) \;, \tag{2b}$$

$$B(\lambda) = \Sigma X(ab;\lambda)Q(ab;\lambda) \;, \tag{2c}$$

$$Q(ab;\;) = (-1)^{-j_a+1/2-l_b} \times \langle j_a \tfrac{1}{2} j_b - \tfrac{1}{2} \,|\, \lambda 0\rangle \times \hat{J}_a \hat{J}_b R_a(R) R_b(R) \;, \tag{2d}$$

where the index $\pi(\nu)$ refers to a proton (neutron) pair, and R_a is the radial part of the single-particle (Woods-Saxon) wave function a. S_α is the alpha-particle radius. The two-particle wave function amplitude is $X(ab;\lambda)$. If one uses a delta force to describe the two particle system[10] (as is the case in this paper), one finds that X is proportional to Q. Therefore all configuration mixing components contribute with the same phase to (2c), and one may expect to obtain a large, unlimited value of B (and thus of γ_L) if enough configurations are included.

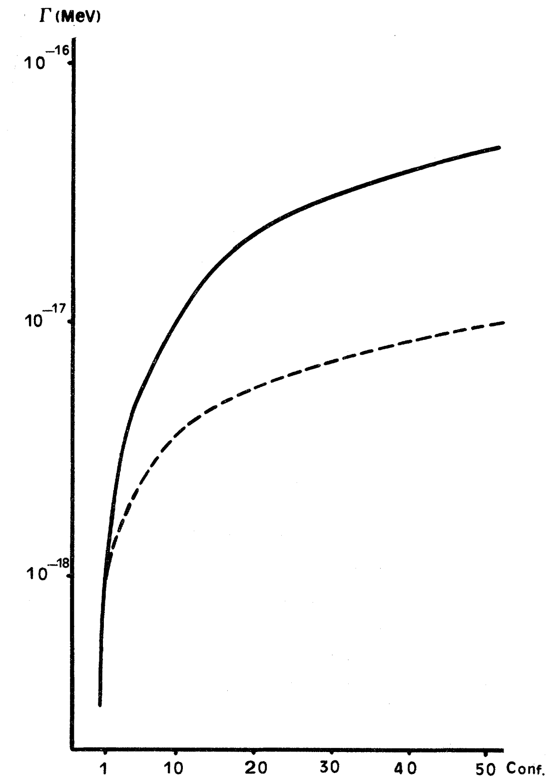

FIG. 2. Absolute alpha-decay width as a function of the number of shell-model configuration for $^{212}\text{Po}\rightarrow\alpha+^{208}\text{Pb}$ at $R=8$ fm. The solid (dashed) line, which represents proton (neutron) contribution, was calculated taking for neutrons (protons) only the leading shell-model configuration.

However, this is not the case because, as can be seen in Fig. 2., the amplitude γ_L is saturated rather rapidly with the number of configurations. Using this procedure, we have calculated the absolute alpha-decay width for the ground-state to ground-state transitions of the ^{210,212,214,216}Po isotopes.

In Fig. 2. we show the dependence of Γ_L as a function of the number of configurations. The tremendous enhancement factor (around 2500) produced by the configuration mixing is mostly due to proton correlation, as can be seen from this figure. This rather surprising result may explain why the calculated absolute cross sections for two-proton reactions are too small in the lead region compared with experimental data.

Furthermore, it is worthwhile to point out that Γ_L depends only very slightly upon the radius R for values between 7.5 and 9.5 fm, i.e., around the expected region where formula (1) must be valid.

As one would expect, our simple treatment of the relative motion of the alpha particle with respect to

TABLE I. Experimental and theoretical absolute alpha-decay width for the pure and full configurations. The single-particle wave functions were obtained using a Wood-Saxon potential that binds the particle with $\frac{1}{4}$ of the alpha-particle binding energy to the mother nucleus. The single-particle states were taken from Ref. 12.

Decay	$(\Gamma_{exp}/\Gamma_{NFT})_{pure}$	$(\Gamma_{exp}/\Gamma_{NFT})_{full}$
$^{210}Po \rightarrow \alpha + {}^{206}Pb$	8.61×10^{4}	2.37×10^{1}
$^{212}Po \rightarrow \alpha + {}^{208}Pb$	7.82×10^{4}	9.62×10^{1}
$^{214}Po \rightarrow \alpha + {}^{210}Pb$	1.45×10^{5}	1.74×10^{2}
$^{216}Po \rightarrow \alpha + {}^{212}Pb$	1.75×10^{5}	2.02×10^{1}

the daughter nucleus gives results which still differ from the corresponding experimental data. We have properly taken into account the Pauli principle among the active particles but we considered the core (i.e., ^{208}Pb) to be an inert system, as it is usually done in shell-model calculations. Although in Ref. 1 it was reported that the core may play an important role to enhance alpha-decay width, recent calculations show that, actually, this is not the case.[14]

The very important effect produced by configuration mixing on alpha-decay width (seen in Fig. 2 and Table I) would not depend upon the alpha-particle relative motion. Therefore, one can say that calculations which consider the leading configuration only to describe the formation of the alpha-particle[1,13] are not very reliable.

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