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Nuclear form factors in the timelike region and photoproduction of pions in nuclei

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The requirement of gauge invariance is applied to the matrix element for pion photoproduction near threshold for spin $1/2^{\pm}\rightarrow 1/2^{\pm}$ and $0^{\pm}\leftrightarrow 1^{\pm}$ transitions and leads to relations between $F_A(\mu^2)$ and $F_A(-\mu^2)$, and to partially conserved axial-vector current relations. These relations lead to theoretical predictions for cross sections consistent with experiment within the level of approximations made. Straightforward analytic continuation of the axial-vector form factor to the timelike region is also examined.

[NUCLEAR REACTIONS Gauge invariance, PCAC, weak nuclear form factors for ^{6}Li$(\gamma,\pi^+)^6$He and ^{12}C$(\gamma,\pi^+)^{12}$B.]

I. INTRODUCTION

Matrix elements of the electromagnetic and weak currents between any two single-particle states with four-momenta p and p', respectively, can be expressed in terms of form factors $F_i(t)$ with $t=(p'-p)^2$ which contain information about the structure of the initial and final states. In principle, form factors can be calculated if the dynamics of the constituent particles is known. In practice, however, form factors are usually obtained from experiment, and information about the structure is extracted from analysis of the data.

From the elastic and inelastic scattering of ν, e^-, and μ^-, one can obtain the form factors for spacelike t ($t<0$). In muon-capture processes, the form factors for fixed values of t ($t\simeq -m_\mu{}^2$) are probed, whereas β-decay processes involve timelike t ($t>0$). In the latter case, however, $|t|$ is usually small and only the F_i ($t=0$) are obtained. The behavior of $F_i(t)$ for the timelike region is experimentally less well known since the directly accessible timelike region is $t\geq(m+m')^2$ and annihilation experiments such as $i+\bar{f}\leftrightarrow\gamma\leftrightarrow e^+ + e^-$ and $i+\bar{f}\leftrightarrow e+\bar{\nu}_e$ are exceedingly difficult. Although some data exist for the nucleon case, no information is available at present for any of the nuclear cases.

In spite of this difficulty, in the nucleon case, in order to obtain, e.g., F_i ($t=\mu^2$), one can safely analytically continue $F_i(t)$ from the spacelike t region to $t=\mu^2$. This procedure is valid because the pole is far away from $t=\mu^2$. (μ is the pion mass.)

In the nuclear case, because of the nuclear size, the form factors vary much more rapidly as functions of t than the nucleon form factors do. The apparent pole lies roughly at $t\simeq R^{-2}\simeq\mu^2 A^{-2/3}$. This makes the corresponding analytic continuation from $t=-\mu^2$ to $t=\mu^2$ very uncertain.

In this paper we attempt to find the behavior of the nuclear form factors in the small t timelike region (i.e., $t\simeq\mu^2$). Our investigation is based on the observation that pion photoproduction in nuclei near threshold is described, when calculated using the elementary particle treatment in th first Born approximation, by nuclear form factors evaluated both at $t=-\mu^2$ and μ^2. The gauge invariance of the transition amplitude then yields a relationship between F_A ($t=\mu^2$) and F_A ($t=-\mu^2$). This prediction is used to calculate the cross sections for $\gamma+{}^6\text{Li}\rightarrow{}^6\text{He}+\pi^+$ and $\gamma+{}^{12}\text{C}\rightarrow{}^{12}\text{B}+\pi^+$ near threshold. The results are compared with calculations in which F_A ($t=\mu^2$) is obtained from F_A ($t=-\mu^2$) by a straightforward analytic continuation.

II. $\frac{1}{2}^{\pm} \to \frac{1}{2}^{\pm}$ TRANSITIONS

The partially conserved axial vector current relation of Gell-Mann and Lévy[1] in the first order in the electromagnetic interaction is, in standard notation,

$$(\partial_\mu \pm ie\mathscr{A}_\mu)A_\mu^{(\pm)}(x)=f_\pi \mu^2 \phi_\pi^{(\pm)}(x) . \qquad (1)$$

With use of $(\Box^2+\mu^2)\phi_\pi^{(\pm)}(x)=j_\pi^{(\pm)}(x)$, one finds[2]

$$\langle f\,|\,j_\pi^{(+)}(0)\,|\,i,\gamma\rangle=\frac{i}{f_\pi}[e\epsilon_\mu\langle f\,|\,A_\mu^{(+)}(0)\,|\,i\rangle - q_\mu\langle f\,|\,A_\mu^{(+)}(0)\,|\,i,\gamma\rangle'] , \qquad (2)$$

where the prime on the last term denotes the removal of the pion pole. In the limit of $q\to 0$, only the first term (with $q^2=0$) survives, which is the Kroll-Ruderman theorem.[3] Instead of taking the limit $q\to 0$, one can actually calculate the right-hand side of Eq. (2) in the Born approximation. The first term is described by form factors evaluated at $t=(p'-p)^2\simeq -\mu^2$ near the threshold for pion production. On the other hand, the second term, when calculated in the first order Born approximation with the standard three diagrams,[4,5] contains form factors evaluated at $t=0$ and μ^2 owing to the fact that γ and π^+ are on the mass shell.

First we consider the case of $\gamma+p\to n+\pi^+$ near threshold. The calculation of Eq. (2) is straightforward.[4] Gauge invariance is then imposed on the final amplitude. This requirement leads to the following relation:

$$ie\bar{u}(p')\left\{\gamma\cdot q[F_A^n(-\mu^2)-F_A^n(\mu^2)]+(m_m+m_p)F_A^n(-\mu^2)-\frac{q\cdot k}{\mu^2}F_P^n(-\mu^2)-\tfrac{1}{2}f_\pi g_{\pi np}(-\mu^2)\right\}\gamma_5 u(p)=0 , \qquad (3)$$

where $F_A^n(\pm\mu^2)$ and $F_p^n(\pm\mu^2)$ are the nucleon axial-vector and pseudoscalar form factors, respectively, and q and k are the momenta of the pion and the photon, respectively. In order to eliminate the kinematical factors we multiply Eq. (3) by $\bar{u}(p)\gamma_\mu\gamma_5 u(p')$ from the right and sum over all the spins.[6] After some straightforward algebra, we obtain

$$[F_A^n(-\mu^2)-F_A^n(\mu^2)](P\cdot qp_\mu - p'\cdot pq_\mu + p\cdot qp'_\mu - m_n m_p q_\mu) + [(m_n+m_p)F_A^n(-\mu^2)-\frac{q\cdot k}{\mu^2}F_P^n(-\mu^2)-\tfrac{1}{2}f_\pi g_{\pi np}(-\mu^2)](m_n p_\mu - m_p p'_\mu)=0 . \qquad (4)$$

Since p_μ, p'_μ, and q_μ are independent, their coefficients must vanish. This leads to the following two independent relations:

$$F_A^n(\mu^2)=F_A^n(-\mu^2) , \qquad (5a)$$

$$(m_n+m_p)F_A^n(-\mu^2)+\mu F_p^n(-\mu^2) = \tfrac{1}{2}f_\pi g_{\pi np}(-\mu^2) . \qquad (5b)$$

That is, if the above relations are satisfied, then the final amplitude calculated in the first order Born approximation is automatically gauge invariant. The second relation is nothing but the PCAC relation obtained by taking $t=-\mu^2$ in the matrix element of Eq. (1), in the absence of the electromagnetic interaction, between the neutron and proton states,

$$(m_n+m_p)F_A^n(t)-\frac{t}{\mu}F_P^n(t)=\frac{\mu^2 f_\pi g_{\pi np}(t)}{\mu^2-t} . \qquad (6)$$

Equation (5a) is not exactly true, since we have, experimentally,

$$F_A^n(t)=\frac{F_A^n(0)}{(1-t/m_A{}^2)^2},\quad m_A\simeq 1\text{ GeV} . \qquad (7)$$

However, our results are expected to be valid in the first order of the Born approximation (up to the order of μ/m_p). Clearly, up to this order, Eq. (7) implies Eq. (5a). Thus, the gauge condition Eq. (5a) is consistent with Eq. (7), which is expected to hold in the region $t\le\mu^2$.

The above formulas and discussions may, in fact, be applied to any nuclear transitions of the type $\frac{1}{2}^{\pm}\to\frac{1}{2}^{\pm}$. However, the only example where sufficient data are available is the transition $^3He\to{}^3H$. In this case, as was true in the nucleon case, the condition for gauge invariance [Eq. (5a)] is essentially satisfied to the order μ/m_p because of the relatively large size of the mass $M_A{}^2=8\mu^2$. Thus, no new information is obtained from this case.

The situation for the transition $0^\pm\leftrightarrow 1^\pm$ is more interesting. The transitions $^6Li(1^+)\to{}^6He(0^+)$ and

$^{12}C(0^+)\rightarrow{}^{12}B(1^+)$ are examples for which the data necessary for our discussion are available.

III. $0^{\pm}\leftrightarrow 1^+$ TRANSITIONS

In the following, we discuss, for definiteness, the transition $^{12}C(0^+)\rightarrow{}^{12}B(1^+)$. [The result for $^6Li(1^+)\rightarrow{}^6He(0^+)$ is trivially obtained from that for the ^{12}C case.] The form factors that appear in the transition $\gamma(k,\epsilon)+{}^{12}C(p)\rightarrow{}^{12}B(p',\xi)+\pi^+(q)$, where k, p, p', and q are the four-momenta, and ϵ and ξ are the polarization four vectors, respectively, are defined by[5]

$$\langle {}^{12}C(p')\,|\,j_\mu^{\text{e.m.}}(0)\,|\,{}^{12}C(p)\rangle = F(t)(p'+p)_\mu,\quad F(0)=6\ ,$$

$$\langle {}^{12}B(p')\,|\,j_\mu^{\text{e.m.}}(0)\,|\,{}^{12}B(p)\rangle = -(p'+p)_\mu\{\,\xi'\cdot\xi F_1(t)+(\xi'\cdot p)(\xi\cdot p')F_2(t)-F_3(t)(\xi'_\mu\xi\cdot p'+\xi_\mu\xi'\cdot p)\,\}\ ,$$

$$F_1(0)=5,\quad F_2(0)=\frac{5\tau}{2m^2},\quad F_3(0)=5(1+\kappa)\ ,$$

$$\langle {}^{12}B(p')\,|\,A_\mu^{(+)}(0)\,|\,{}^{12}C(p)\rangle = i\left[\xi'_\mu F_A(t)+(p'-p)_\mu\frac{\xi'\cdot q}{\mu^2}F_p(t)\right]\ , \tag{8}$$

$$\langle {}^{12}B(p')\,|\,j_\pi^{(+)}(0)\,|\,{}^{12}C(p)\rangle = -\xi'\cdot(p'-p)g_{\pi if}(t)\ ,$$

where $\mu=5(1+\kappa)$ and $Q=5(\tau-\kappa)/m^2$ are, respectively, the total magnetic dipole and electric quadrupole moments of ^{12}B and m is the nuclear mass. Noting that $t=(p'-p)^2\simeq\mu^2$ near threshold production and calculating the second term in Eq. (2) in the standard first Born approximation, we obtain

$$\begin{aligned}\langle {}^{12}B\,|\,j_\pi^{(+)}(0)\,|\,{}^{12}C,\gamma\rangle = {}&-e\left[\xi'\cdot\epsilon F_A(-\mu^2)-\xi'\cdot(k-q)\epsilon\cdot q\frac{F_p(-\mu^2)}{\mu^2}\right]\\ &+e\left[\frac{p'\cdot\epsilon}{p'\cdot k}\right]F_A(\mu^2)\left\{-\xi'\cdot qF_1(0)+\xi'\cdot k\frac{m'^2-m^2+q^2}{2m'^2}[F_1(0)-\tfrac{1}{2}F_3(0)]\right.\\ &\left.\qquad-\frac{\xi'\cdot kp'\cdot k}{m'^2}F_1(0)+\xi'\cdot k\left[q\cdot k+\frac{p'\cdot kq\cdot(p'-k)}{m'^2}\right]F_2(0)\right\}\\ &-\frac{e}{2p'\cdot k}F_A(\mu^2)F_3(0)\left[-\xi'\cdot\epsilon q\cdot k+\xi'\cdot k\epsilon\cdot q+p'\cdot k\frac{m'^2-m^2+q^2-2p'\cdot k}{2m'^2}\xi'\cdot\epsilon\right]\\ &+e\left[\frac{p\cdot\epsilon}{p\cdot k}\right]\xi'\cdot qF_A(\mu^2)F(0)+\frac{e}{2q\cdot k}f_\pi g_{\pi if}(-\mu^2)\epsilon\cdot q(\xi'\cdot k-\xi'\cdot q)\ .\end{aligned} \tag{9}$$

We remark that the form factors F_i and F_A have arguments zero and μ^2, respectively, because of the fact that γ and π^+ are on the mass shell.

We now impose the gauge invariance condition on the amplitude. We obtain the following two relations which are the coefficients of the $(\xi'\cdot q)$ and $(\xi'\cdot k)$ terms, respectively, set equal to zero:

$$F_A(\mu^2)-F_P(-\mu^2)-\tfrac{1}{2}f_\pi g_{\pi if}(-\mu^2)=0\ , \tag{10a}$$

$$[F_A(-\mu^2)-F_P(-\mu^2)-\tfrac{1}{2}f_\pi g_{\pi if}(-\mu^2)]-F_A(\mu^2)\left[\frac{\mu}{m}\right][F_3(0)-F_1(0)]=0\ . \tag{10b}$$

In deriving Eq. (10), we have neglected the terms of order $(\mu/m)^2$.

The PCAC relation in the absence of the electromagnetic interaction obtained from Eqs. (1) and (8) is

$$F_A(t)+\frac{t}{\mu^2}F_p(t)-\frac{\mu^2 f_\pi g_{\pi if}(t)}{\mu^2-t}=0\ . \tag{11}$$

The quantity in the first set of square brackets in Eq. (10b) is identical with Eq. (11) at threshold, i.e., $t=-\mu^2$, and thus, Eq. (10b) is consistent with PCAC only if terms of the order (μ/m) are neglected. This is in contrast to the spin $\frac{1}{2}$ case, where at threshold Eq. (5b) was consistent with the PCAC condition to this order.

In order to recover gauge invariance of the amplitude to the order (μ/m) while not affecting Eq. (10a), we add to the amplitude terms of the type

$$-e\left[A\xi'\cdot\epsilon+\xi'\cdot k\epsilon\cdot q\frac{B}{\mu^2}+\xi'\cdot k\epsilon\cdot p'\frac{C}{m\mu}\right], \tag{12}$$

where A, B, C are to be determined. Equation (10b), thus modified, yields the relation

$$A+B+C=F_A(\mu^2)\left[\frac{\mu}{m}\right][F_3(0)-F_1(0)] \tag{13}$$

under the assumption of PCAC [Eq. (11)].

Although the addition of these terms appear to be somewhat arbitrary, the standard three Born terms used here, not representing a true lowest order perturbation expansion, are not necessarily gauge invariant to the order (μ/m), and other Born terms must also be considered. Since the single nuclear excited state (without pions and others) make the dominant contribution at the energies involved here, we consider only this possibility. The intermediate states themselves can be of spin one or spin zero. However, because we wish to correct Eq. (10b) without affecting Eq. (10a), which allows us to recover PCAC as well as the gauge condition in the spin $\frac{1}{2}$ case, we want only those terms which on application of the gauge condition will be proportional to $\xi\cdot k$, but contain no $\xi\cdot q$ term. These contributions will all come from the Born diagram in which the photon is coupled to the spin one states.

From the ^{12}B-photon vertex we see that the ratio of A, B, and C must be -2, 1, and -1, respectively. This, with Eq. (13), yields the result

$$\begin{aligned}A&=F_A(\mu^2)\left[\frac{\mu}{m}\right][F_3(0)-F_1(0)]\,,\\ B&=-\tfrac{1}{2}A\,,\\ C&=\tfrac{1}{2}A\,.\end{aligned} \tag{14}$$

With the additional terms described by Eqs. (12)–(14) near threshold, we satisfy PCAC and obtain the same gauge conditions for the $0^{\pm}\leftrightarrow 1^{\pm}$ transitions as for the spin $\frac{1}{2}$ case, i.e., to the order (μ/m) and to this order:

$$F_A(\mu^2)=F_A(-\mu^2)\,. \tag{15}$$

Near the threshold, the $\xi'\cdot\epsilon$ terms in Eq. (9) dominate in the differential cross section, and are given by

$$\langle {}^{12}\mathrm{B}\,|\,j_\pi^{(+)}(0)\,|\,{}^{12}\mathrm{C},\gamma\rangle \simeq -\frac{e}{f_\pi}(\xi'\cdot\epsilon)\left[F_A(-\mu^2)-\left[\frac{\mu}{m}\right]F_1(0)F_A(\mu^2)\right]. \tag{16}$$

The slope of the center-of-mass-system differential cross section is defined by

$$\left[\frac{a}{4\pi}\right]=\left[\frac{d\sigma}{d\Omega}\right]\frac{|\vec{\mathrm{k}}|}{|\vec{\mathrm{q}}|}\frac{1}{\rho(z,|\vec{\mathrm{q}}|)}\,, \tag{17}$$

where $\rho(Z,|\vec{\mathrm{q}}|)$ is the final Coulomb correction factor. From Eqs. (16) and (17) we find

$$\left[\frac{a}{4\pi}\right]_{\mathrm{theor}}=\frac{1}{(2\pi)^2}\frac{1}{16(m+\mu)^2}\left[\frac{e}{f_\pi}\right]^2 \times\left[F_A(-\mu^2)-\left[\frac{\mu}{m}\right]F_1(0)F_A(\mu^2)\right]^2. \tag{18}$$

So far our discussion has been based on the calculation of the amplitude in the first Born approximation with no reference to Low's theorem.[7] Suppose we keep in Eq. (9), in accordance with Low's theorem, only the terms of order k^{-1} and k^0 in the Born approximation of the second term in Eq. (2). Then it can be seen that the resulting amplitude with the gauge invariance requirement gives rise to the constraints identical to those of the spin $\frac{1}{2}$ case. The slope of the cross section in this case is given by

$$\left[\frac{q}{4\pi}\right]_{\mathrm{theor}}=\frac{1}{(2\pi)^2}\frac{1}{16(m+\mu)^2}\left[\frac{e}{f_\pi}\right]^2\left[F_A(-\mu^2)-\left[\frac{\mu}{2m}\right]F_3(0)F_A(\mu^2)\right]^2. \tag{19}$$

Although the second terms in Eqs. (18) and (19) are different in nature, the two approaches lead to the same numerical result, since $[F_3(0)/2F_1(0)] = \frac{1}{2}(1+\kappa) \sim 1$ for both ^{6}Li and ^{12}B.

Employing the standard dipole fit we may write

$$F_A(t) = \frac{F_A(0)}{(1-t/m_A{}^2)}, \quad \text{for } t \le 0 \tag{20}$$

where experiments give[8]

$$F_A(0) = 1.03, \quad m_A{}^2 = 2.74\,\mu^2 . \tag{21}$$

Using Eqs. (15), (18), (19), and (21) we obtain

$$\left[\frac{a}{4\pi}\right]_{\text{theor}} = 1.6\ \mu\text{b/sr} , \tag{22}$$

which may be compared with an experimental value of[9]

$$\left[\frac{a}{4\pi}\right]_{\text{expt}} = 1.31\ \mu\text{b/sr} . \tag{23}$$

The matrix elements for the $^6\text{Li} \leftrightarrow {}^6\text{He}$ system are identical to those for the $^{12}\text{C} \leftrightarrow {}^{12}\text{B}$ system so that it is only necessary to replace $F_A(-\mu^2)$ and $F_1(0)$ by the appropriate value[10]

$$F_A(0) = 2.26, \quad m_A{}^2 = 2.12\,\mu^2 . \tag{24}$$

These values lead to a result

$$\left[\frac{a}{4\pi}\right]_{\text{theor}} = 1.6\ \mu\text{b/sr} \tag{25}$$

versus an experimental value[11] of

$$\left[\frac{a}{4\pi}\right]_{\text{expt}} = 1.12\ \mu\text{b/sr} . \tag{26}$$

Next, instead of using the relation (15), we calculate $F_A(\mu^2)$ directly from Eq. (20) by analytically continuing to the timelike value $t=\mu^2$. The results are

$$\frac{F_A(\mu^2)}{F_A(-\mu^2)} = \begin{cases} 4.62, & \text{for } ^{12}\text{C} \\ 7.76, & \text{for } ^{6}\text{Li} \end{cases} \tag{27}$$

which yield, with use of Eq. (18) or (19),

$$\left[\frac{a}{4\pi}\right]_{\text{theor}} = \begin{cases} 0.90\ \mu\text{b/sr}, & \text{for } ^{12}\text{C} \\ 0.51\ \mu\text{b/sr}, & \text{for } ^{6}\text{Li} . \end{cases} \tag{28}$$

We therefore see that the experimental values lie exactly in the middle of the two predictions. This implies that since the correction term in Eq. (18) or (19) is negative, the prediction of $F_A(\mu^2)$ based on the gauge invariance is an underestimate, whereas the value obtained from a straightforward analytic continuation is an overestimate of the true value of $F_A(\mu^2)$.

IV. SUMMARY AND CONCLUSIONS

In deriving the above results we have made a number of assumptions. We have treated the Gell-Mann-Levy version of PCAC as exact. Electric quadrupole contributions to the pion photoproduction amplitude are ignored. This was done for consistency because these contributions are all of order $(\mu/m)^2$ or higher. We have also ignored the pseudotensor term in the axial-vector current matrix element. From experience[12] with muon-capture and neutrino reaction calculations, we estimate that its contribution to the pion photoproduction amplitude should be no more than 5% to ~10% of the axial-vector form factor contributions to the Born terms. Finally, we have ignored pion distortion effects.

Within these assumptions we have found that the imposition of gauge invariance on the amplitude for threshold pion photoproduction consisting of the catastrophic term (Kroll-Ruderman term) and the standard three Born diagrams leads, for the spin $\frac{1}{2}$ case, to two requirements. First, it implies that $F_A(\mu^2)=F_A(-\mu^2)$, and second, that the PCAC relation is satisfied at $t=-\mu^2$. The former requirement for the nucleon and light nuclei cases is trivially satisfied by the axial-vector form factor which may be analytically continued from the physical region to $t=\mu^2$. This is due to the large value for $M_A{}^2$, which reflects the fact that the pole in $F_A(t)$ is far from $t=\mu^2$.

On the other hand, the same is not true for the transition $0^{\pm} \leftrightarrow 1^{\pm}$. In this case, one of the gauge invariance constraints is inconsistent with the PCAC relation. Since we expect the PCAC relation to hold at $t=-\mu^2$, it is necessary to modify the amplitude in such a way that the PCAC relation is recovered. The modification means that we have to go beyond the standard Born diagrams. With some reasonable assumptions this modification can be made unambiguously. The modified amplitude is then gauge invariant up to the order of (μ/m) and the situation becomes identical to that of the transition $\frac{1}{2}^{\pm} \rightarrow \frac{1}{2}^{\pm}$. However, if Low's theorem is applied to the amplitude by ignoring the terms linear in k in the Born approximation, PCAC is automatically satisfied and $F_A(\mu^2) = F_A(-\mu^2)$ follows.

We have found that in both the $\frac{1}{2}^\pm \to \frac{1}{2}^\pm$ and $0^\pm \leftrightarrow 1^\pm$ transitions, the gauge invariance requirement implies the relation $F_A(\mu^2)=F_A(-\mu^2)$. This may serve as a prescription for obtaining the form factor at the timelike region of t from its spacelike behavior. Using the two examples $\gamma + {}^{12}C \to {}^{12}B + \pi^+$ and $\gamma + {}^6Li \to {}^6He + \pi^+$, we have demonstrated that this prescription yields results not inconsistent with experiment up to the level expected in the approximation used here. However, the predictions are consistently larger than the experimental values implying, as noted already, that the reflection $F_A(\mu^2)=F_A(-\mu^2)$ seems to underestimate the value of $F_A(\mu^2)$.

On the other hand, a straightforward analytic continuation of the $F_A(t)$ to $t=\mu^2$ gives overestimated values of $F_A(\mu^2)$. This is not unexpected, because the presence of the anomalous mass threshold associated with the nucleon breakup as well as the nuclear size make the pole in $F_A(t)$ closer to the physical region ($t \le 0$) than in the nucleon and light nuclei cases. Furthermore, since near threshold the final state interaction between the outgoing pion and the final nucleus, which we have neglected so far, has a tendency to decrease[13] the theoretical predictions of $(a/4\pi)$, the analytic continuation is actually less favorable than the comparison between theory [Eq. (28)] and experiment [Eqs. (23) and (26)] indicates. In view of this correction, it seems that the relation $F_A(\mu^2) = F_A(-\mu^2)$ is not too unreasonable after all.

Finally, we note that we obtain, from our analysis, no information about $g_{\pi if}(\mu^2)$ in contrast to the previous analysis.[5] However, for the physically attainable region for $F_A(t)$ [but not for $g_{\pi if}(t)$] we obtain the result $F_A(t)=f_\pi g_{\pi if}(t)$, where $-\mu^2 \le t \le 0$.

Finally, we remark that for the $0^\pm \leftrightarrow 1^\pm$ transitions considered here, $F_A(0)$ is substantially larger than $F_A(-\mu^2)$ and cannot be substitued for it, unlike the case for the nucleon and light nucleus. This accounts for most of the disagreement in Ref. 5 between the theoretical value and the experimental value for $(a/4\pi)$.

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