

Cluster representation of ${}^6\text{Li}$ and the finite-range distorted-wave Born approximation analysis of ${}^6\text{Li}$ -induced alpha transfer reactions

D. R. Chakrabarty and M. A. Eswaran

Nuclear Physics Division, Bhabha Atomic Research Centre, Bombay 400085, India

(Received 16 June 1981)

With a view to examine the suitability of the various different forms of α - d cluster representation of ${}^6\text{Li}$ used in the literature based on different criteria for the α -transfer reaction analysis, finite-range distorted-wave Born approximation calculations have been made in the present work for $({}^6\text{Li}, d)$ reactions with various forms of cluster representation of ${}^6\text{Li}$ and they are compared with the angular distribution data on the targets ${}^{26}\text{Mg}$, ${}^{27}\text{Al}$, ${}^{29}\text{Si}$, and ${}^{31}\text{P}$ at a bombarding energy of 36 MeV. It is found that (a) the single node wave function ψ_{ad} introduced by Kubo and Hirata with the corresponding Woods-Saxon form of V_{ad} is not found to fit the angular distribution data at all, (b) the zero node Eckart function form of ψ_{ad} with the corresponding V_{ad} with attractive and repulsive parts is also not successful, and (c) the zero node wave function generated by a Woods-Saxon form of a potential with a hard repulsive core of $R_{\text{core}} = 1.25$ fm introduced by Watson *et al.* is also unsatisfactory. However, the Watson form modified by reducing the hard core radius to $R_{\text{core}} = 0.6$ fm predicts angular distributions satisfactorily. These results are not affected by the different choices of optical model parameters.

[NUCLEAR REACTIONS (${}^6\text{Li}, d$), FRDWBA calculations with different α - d cluster representations of ${}^6\text{Li}$. Compared $\sigma(\theta)$ data.]

INTRODUCTION

The $({}^6\text{Li}, d)$ reactions are being extensively employed for the study of the aspects of alpha clustering in nuclei¹ with the use of exact finite range DWBA analysis.^{2,3} However, the alpha-deuteron cluster representation of ${}^6\text{Li}$ to be used in these analyses is not free from ambiguities. Specifically for the relative motion wave function of the alpha particle and deuteron in ${}^6\text{Li}$ and the effective interaction between them, various different forms are in use. The ${}^6\text{Li}$ induced alpha transfer reaction cross sections and angular distributions calculated by the finite range DWBA analysis are sensitively dependent on the α - d intercluster relative wave function at small separations. Furthermore, absolute cross sections calculated from these analyses assume significance when the cluster strengths obtained from transfer reactions are to be used for comparison with the widths for the inverse process of α decay. The forms of the α - d intercluster relative motion wave function giving satisfactory predictions of knockout reactions on ${}^6\text{Li}$ are also in use

in the ${}^6\text{Li}$ induced alpha transfer reactions, although knockout reactions on ${}^6\text{Li}$ analysis are sensitive only to the intercluster wave function at larger separations, contrary to the case of the transfer reactions, where the wave functions at small separations are important.

With a view to examine the suitability of the different forms of α - d cluster representation of ${}^6\text{Li}$ for the transfer reaction analysis, finite-range DWBA calculations^{4,5} have been made in the present work for $({}^6\text{Li}, d)$ reactions with different forms of cluster representation of ${}^6\text{Li}$ and they are compared with the data on the targets ${}^{26}\text{Mg}$, ${}^{27}\text{Al}$, ${}^{29}\text{Si}$, and ${}^{31}\text{P}$. Some preliminary results of this study were reported earlier.⁶

EXPERIMENTAL DATA

The $({}^6\text{Li}, d)$ reaction angular distribution data at a ${}^6\text{Li}$ energy of 36 MeV on the odd mass targets ${}^{27}\text{Al}$, ${}^{29}\text{Si}$, and ${}^{31}\text{P}$ are taken from the work of Eswaran *et al.*,³ and on ${}^{26}\text{Mg}$ from the work of

Sikora *et al.*⁷ The ground state to ground state transitions in the case of the above reactions on odd mass targets have the pure L transfer of 2 and these are the ones included for comparisons with the present calculations. In the case of the even mass target ^{26}Mg , the ground state to ground state transition of L transfer zero and the transition to the first excited state $^{26}\text{Mg} \rightarrow ^{30}\text{Si}^*$ (2.24 MeV) with an L transfer of 2 are included. In the case of the $(^6\text{Li},d)$ reaction on ^{26}Mg included in the present work, the optical model potential parameters needed for ^6Li on ^{26}Mg at 36 MeV are available in the literature from the work of Schumacher *et al.*,⁸ who obtained these parameter sets from the analysis of ^6Li elastic scattering data on ^{26}Mg at the same energy of 36 MeV.

CLUSTER REPRESENTATION OF ^6Li

The treatment of finite-range DWBA analysis of the $(^6\text{Li},d)$ transfer reactions requires a knowledge of the relative motion wave function ψ_{ad} of the α and deuteron in ^6Li , and their interaction potential V_{ad} . Specifically, the FRDWBA integrand⁵ is proportional to the product $V_{ad}\psi_{ad}$, and hence the reaction is sensitive to this product and not to the individual wave function and potential.

Various different forms of relative d - α cluster wave functions have been available in literature. A single radial node $2S$ wave function is predicted by the shell model and this type of wave function is adopted by Kubo and Hirata⁹ in their treatment of some α transfer reactions. They approximated the nuclear interaction V_{ad} by a Woods-Saxon potential

$$V_{ad} = V_0 \left[1 + \exp \left(\frac{r - R_0}{a} \right) \right]^{-1}, \quad (1)$$

with parameters $V_0 = -77.8$ MeV, $R_0 = 1.9$ fm, and $a = 0.65$ fm, where the potential depth was determined to give an α - d binding energy of 1.474 MeV for the single node $2S$ state wave function. This potential is adopted by Kubo *et al.*⁹ to express the shape of the potential obtained based on nucleon-nucleon interactions in Refs. 10 and 11. The parameters for the potential determine the root mean square radius of the relative motion wave function which is appropriate to reproduce the ^6Li density distribution.⁹ This potential V_{ad} and the relative motion wave function ψ_{ad} calculated as a solution with the potential V_{ad} including the Coulomb interaction are marked by K in Figs.

1(a) and (b), respectively, and this is one of the forms we used in the present investigation. The potential parameters are denoted by set K in Table I. It may be noted here that for this cluster representation the product $V_{ad}\psi_{ad}$ will have one node away from the origin. This wave function with the correct radial quantum number (i.e., a $2S$ state) is required to account phenomenologically for antisymmetrization effects at small α - d separation distances.

The second form that was tried in our analysis was the Eckart function used by Noble.¹² In this approach the relative wave function ψ_{ad} is of the form $(1 - e^{-\alpha r})^3 (e^{-Kr}/r)$, which is an eigenstate of the real potential with a repulsive core

$$V_{ad} = -44.4[e^{\alpha r} - 1]^{-1} + 47.5[e^{\alpha r} - 1]^{-2}, \quad (2)$$

with $\alpha = 0.714 \text{ fm}^{-1}$ and $K = [2\mu E_B / \hbar^2]^{1/2} = 0.307 \text{ fm}^{-1}$.

This wave function has no radial node outside the origin; however, the potential with which this is generated as an eigenstate has an infinite repulsive core (necessary to produce the node at the origin) which may be thought of as a manifestation of the Pauli principle. Noble¹² has used this wave

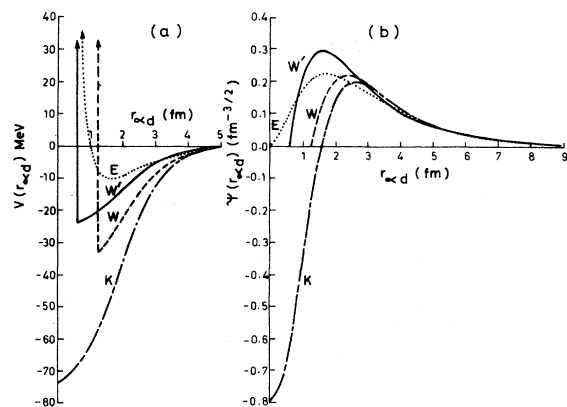


FIG. 1.(a) The nuclear part of the interaction potential V_{ad} between α cluster and deuteron as a function of separation distance r_{ad} . The curves marked K , E , W , and W' denote different potentials, as described in the text. (See Table I.) (b) The ψ_{ad} relative motion wave function as a function of the α - d separation distance r_{ad} calculated as solutions with the nuclear interaction potentials shown in (a). The curves marked K , E , W , and W' denote the solutions with the corresponding potentials in (a).

TABLE I. Interaction potentials $V_{ad}(r_{ad})$ used to construct the $\alpha+d$ relative motion wave functions ψ_{ad} .

Set	Form of $V_{ad}(r_{ad})$	Ref.
K	$-V_0[1+\exp(r-R_0)/a]^{-1}+V_{\text{Coul}}(r)$ $V_0=77.8$ MeV, $R_0=1.9$ fm, $a=0.65$ fm, $R_c=1.9$ fm, $(N,L)=1,0$	9
E	$-V_0(e^{ar}-1)^{-1}+V_1(e^{ar}-1)^{-2}$ $V_0=44.4$ MeV, $V_1=47.5$ MeV, $\alpha=0.714$ fm $^{-1}$, $(N,L)=0,0$	12,13
W	$-V_0[1+\exp(r-R_0)/a]^{-1}+V_{\text{Coul}}(r)$ for $r > R_{\text{core}}$ ∞ for $r \leq R_{\text{core}}$ $V_0=45.25$ MeV, $R_0=2.0$ fm, $a=0.7$ fm, $R_{\text{core}}=1.25$ fm, $R_c=2.0$ fm, $(N,L)=(0,0)$	15
W'	Same as W $V_0=27.26$ MeV, $R_{\text{core}}=0.6$ fm	

function to describe the static $E2$ and $M1$ moments and elastic charge form factors of ${}^6\text{Li}$ as well as cluster knockout data. The Eckart function is also employed by Schwandt *et al.*¹³ to construct a folded ${}^6\text{Li}$ optical potential. This form is denoted by E in Table I and the potential with a repulsive core and the wave function are shown in Figs. 1(a) and (b), respectively. In this form also the product $V_{ad}\psi_{ad}$ has a node away from the origin. Jain *et al.*¹⁴ have successfully used a similar function, which is analytically defined but is not the solution of the one body Hamiltonian, to describe cluster knockout reactions such as $(\alpha,2\alpha),(p,pd)$ on ${}^6\text{Li}$.

The third form we examined in our analysis was the one introduced by Watson *et al.*¹⁵ In describing ${}^6\text{Li}(\alpha,2\alpha)d$ knockout data, Watson *et al.*¹⁵ generated the $\alpha+d$ cluster wave function by a Woods-Saxon potential with a hard repulsive core. The parameters are shown in Table I (listed under W) and the potential and wave function are plotted in Figs. 1(a) and (b). The form of the nuclear potential is

$$V_{ad}(r) = V_0 \left[1 + \exp \left[\frac{r-R_0}{a} \right] \right]^{-1} \quad \text{for } r > R_{\text{core}}$$

$$= \infty \quad \text{for } r \leq R_{\text{core}} . \quad (3)$$

This potential with a hard core radius chosen as $R_{\text{core}} = 1.25$ fm, a depth $V_0 = -45.25$ MeV and $R_0 = 2.0$ fm, and a diffuseness parameter $a = 0.7$ fm, correctly yields the α - d binding energy of ${}^6\text{Li}$ ($=1.474$ MeV), the low energy 3S_1 , α - d scattering phase shifts, and the rms charge radius of the ${}^6\text{Li}$ ground state close to the experimental value.¹⁵ It should, however, be noted that for the description of knockout reaction data on ${}^6\text{Li}$, the cluster wave

function at larger separations than about 5 fm is what is important.¹⁵

All three forms (K , E , and W) of the wave functions are similar at large separations [Fig. 1(b)], but very different at small separation distances. The transfer reaction data are found to be sensitive to the product $V_{ad}\psi_{ad}$ at small separations, as described in the next section.

The above forms of the wave functions ψ_{ad} were generated by a separate computer program as solutions of a one body Hamiltonian using the corresponding potentials V_{ad} (see Table I). The Coulomb part of V_{ad} is then removed retaining only the nuclear part of V_{ad} , and these V_{ad} and ψ_{ad} were then introduced in the LOLA⁵ program for the finite range DWBA analyses.

FINITE RANGE DWBA ANALYSIS

The exact finite-range DWBA code LOLA⁵ was used to calculate the theoretical angular distributions. For the bound state in the final nucleus, a target plus α -particle cluster wave function was generated in a Woods-Saxon well of radius $R = 1.30 A_i^{1/3}$ fm and diffuseness $a = 0.65$ fm. The well depth was adjusted to reproduce the known binding energy of the α particle in the final nuclear state. The number of radial nodes was fixed by the oscillator conservation relation

$$2(N-1)+L = \sum_{i=1}^4 [2(n_i-1)+l_i], \quad (4)$$

where n_i and l_i are the individual nucleon shell model quantum numbers. For the transfer of four nucleons in the $2s-1d$ shell the above equation yields $2(N-1)+L=8$, and this is the value used in the calculation in this work. The α - d interclus-

ter relative motion wave function in ${}^6\text{Li}$ as well as the interaction potential $V_{ad}(r_{ad})$ are to be known for the calculation of the form factor in the analysis. The various forms, K , E , and W discussed in the previous section were introduced in the program for the FRDWBA analysis. The optical model potentials used to generate the distorted waves consisted of the ${}^6\text{Li}$ parameters of Strobusch *et al.*¹⁶ and Schumacher *et al.*,⁸ and the average deuteron parameters of Newman *et al.*¹⁷ The α -cluster spectroscopic strengths S for the residual nuclear states are related by the equation

$$\left(\frac{d\sigma}{d\Omega} \right)_{\text{exp}} = S \frac{(2J_f + 1)}{(2J_i + 1)} \left(\frac{d\sigma}{d\Omega} \right)_{\text{LOLA}}, \quad (5)$$

where J_i and J_f are the spins of the target and final nuclear states. In the above equation the spectroscopic factor describing the overlap of ${}^6\text{Li}$ with $d + \alpha$, is taken as unity. All the computations were done on a DEC-10 computer. The program LOLA was adapted for use in this computer with the provision that it read in the external wave function and interaction potentials. The number of partial waves up to 25 were included in the calculation and integrations were extended up to a maximum radius of 16 fm.

RESULTS AND DISCUSSION

The calculated finite-range DWBA angular distributions for the reaction ${}^{29}\text{Si}({}^6\text{Li}, d){}^{33}\text{S}_{\text{g.s.}}$ at a ${}^6\text{Li}$ energy of $E = 36$ MeV are shown in Fig. 2. The L transfer is of a single value of 2 in this case

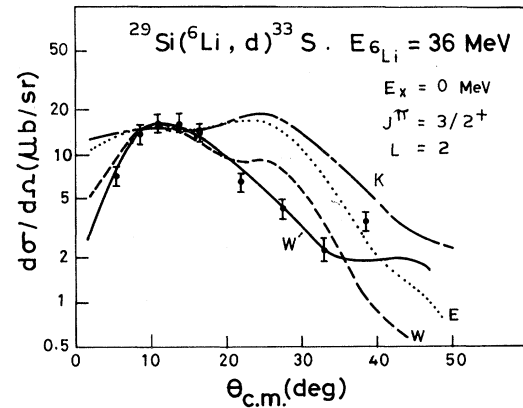


FIG. 2. Angular distributions of the reaction ${}^{29}\text{Si}({}^6\text{Li}, d){}^{33}\text{S}_{\text{g.s.}}$. The curves K , E , W , and W' are the present FRDWBA calculations for L transfer 2 with different choices of V_{ad} and ψ_{ad} corresponding to K , E , W , and W' for the α - d cluster representation of ${}^6\text{Li}$ (see Table I). The data are from Ref. 3.

since the spins of ${}^{29}\text{Si}$ and ${}^{33}\text{S}$ are $\frac{1}{2}^+$ and $\frac{3}{2}^+$. The optical model parameters for ${}^6\text{Li}$ and the deuteron are the ones used for $2s - 1d$ shell targets earlier in the literature,^{3,18} and are listed in Table II. The data of Eswaran *et al.*³ are also plotted in Fig. 2. The different angular distribution curves marked K , E , and W are the ones calculated with V_{ad} and ψ_{ad} of Kubo *et al.*,⁹ the Eckart function¹² and those of Watson *et al.*,¹⁵ respectively, discussed in the previous section. The calculated angular distribution curves are shown normalized at the maximum in the data at $\theta_{\text{c.m.}} = 11^\circ$. Obviously both curves K and E do not fit the angular distri-

TABLE II. Optical model Woods-Saxon potential parameters used in the FRDWBA analysis.

Channel	Set	V (MeV)	R_R (fm)	a_R (fm)	W (MeV)	$W' = 4W_D$ (MeV)	R_I (fm)	a_I (fm)	R_{Coul} (fm)
${}^6\text{Li} + {}^{26}\text{Mg}$		161.9	3.58	0.80	17.3		5.48	0.89	3.85
$d + {}^{30}\text{Si}$		94.2	3.29	0.81		44.7	4.19	0.74	3.88
${}^6\text{Li} + {}^{29}\text{Si}$		72.6	4.21	0.87	8.0		7.07	0.81	7.68
$d + {}^{33}\text{S}$		94.6	3.40	0.81		45.6	4.29	0.74	4.01
${}^6\text{Li} + {}^{27}\text{Al}$	A	72.6	4.11	0.87	8.0		6.90	0.81	7.50
	B	161.9	3.63	0.80	17.3		5.55	0.89	3.90
$d + {}^{31}\text{P}$		94.4	3.33	0.81		45.0	4.21	0.74	3.93
${}^6\text{Li} + {}^{31}\text{P}$	A	72.6	4.30	0.87	8.0		7.23	0.81	7.85
	B	161.9	3.80	0.80	17.3		5.81	0.89	4.08
$d + {}^{35}\text{Cl}$		94.8	3.48	0.81		46.1	4.37	0.75	4.09

bution data at all. As discussed in the previous section, in both these forms the product $V_{ad}\psi_{ad}$ has a node away from the origin. The FRDWBA calculation with the form of Watson *et al.* for the V_{ad} and ψ_{ad} is shown by curve W in Fig. 2. The fit is not found to be satisfactory. Although the first maximum in the curve fits the data well, there is a second hump in the curve which is not observed in the data.

The W form of the interaction potential V_{ad} has an infinite repulsive core with a core radius $R_{\text{core}} = 1.25$ fm. We chose to modify this form by reducing the R_{core} radius, keeping the radius R_0 and the diffuseness at the same values of 2.0 fm and 0.7 fm, respectively, and adjusting the depth V_0 so that the α - d binding energy of 1.474 MeV is reproduced for the ground state of ${}^6\text{Li}$. The value of $R_{\text{core}} = 0.6$ fm was adopted, which needed a depth of $V_0 = -27.26$ MeV. This modified form is shown as W' in Figs. 1(a) and (b). With this form FRDWBA calculation for the angular distribution is shown by curve W' in Fig. 2.

It is found that this modified form W' reproduces the angular distribution satisfactorily. This form of wave function W' is essentially the same as W at longer separations and differs from it only slightly at smaller distances. To compare the modification in the radius the mean square radius of ${}^6\text{Li}$ is computed from the expression¹⁹

$$\langle r_A^2 \rangle = \frac{Z_1}{Z} \langle r_{A_1}^2 \rangle + \frac{Z_2}{Z} \langle r_{A_2}^2 \rangle + \frac{Z_1 A_2^2 + Z_2 A_1^2}{Z A^2} \langle r_{\text{Rel}}^2 \rangle,$$

where Z_1 , Z_2 and A_1 , A_2 are the charges and masses of the two clusters α and deuteron, respectively, and $Z = Z_1 + Z_2$, $A = A_1 + A_2$. The value of the root mean square radius is $\langle r_A^2 \rangle^{1/2} = 2.72$ fm according to the original form W of Watson *et al.*,¹⁵ while the modified form W' yields $\langle r_A^2 \rangle^{1/2} = 2.55$ fm, the experimental value¹⁵ being 2.56 ± 0.05 fm. Hence the modified form, with the reduced value of the repulsive core radius $R_{\text{core}} = 0.6$ fm and the relative motion wave function modified essentially only at smaller separation distances, reproduces the $({}^6\text{Li}, d)$ transfer reaction data well, while retaining the rms radius of ${}^6\text{Li}$ close to the experimental value and with the relative motion wave function at larger separations, the same as the original form of Watson *et al.*¹⁵ [denoted by W in Fig. 1(b)] which is necessary for explaining the cluster knockout reaction data on ${}^6\text{Li}$.¹⁵

Figure 3 shows a similar comparison of the calculations and the data for the reactions ${}^{26}\text{Mg}({}^6\text{Li}, d){}^{30}\text{Si}_{\text{g.s.}}$ and ${}^{26}\text{Mg}({}^6\text{Li}, d){}^{30}\text{Si}^*$ (2.24 MeV). The α -transfer transition ${}^{26}\text{Mg} \rightarrow {}^{30}\text{Si}_{\text{g.s.}}$ has the L transfer value of 0 and the ${}^{26}\text{Mg} \rightarrow {}^{30}\text{Si}^*$ (2.24 MeV) transition has L transfer = 2. For both these cases (Fig. 3) experimental data of Sikora *et al.*⁷ and our FRDWBA calculations with V_{ad} and ψ_{ad} forms of K , W , and W' are shown. As found in the case of the previous reaction, the form K does not fit the angular distribution at all and the form W also deviates significantly from the data. The modified form W' fits the data of the first excited state well and for the ground state $L=0$ transition, the slope of the steep fall in the cross section in the forward angles is also well predicted.

The optical model parameters for ${}^6\text{Li}$ on ${}^{26}\text{Mg}$ at 36 MeV required in this analysis were taken from Schumacher *et al.*⁸ who determined the optical model parameters by fitting their elastic scattering data on the same target at the same energy as

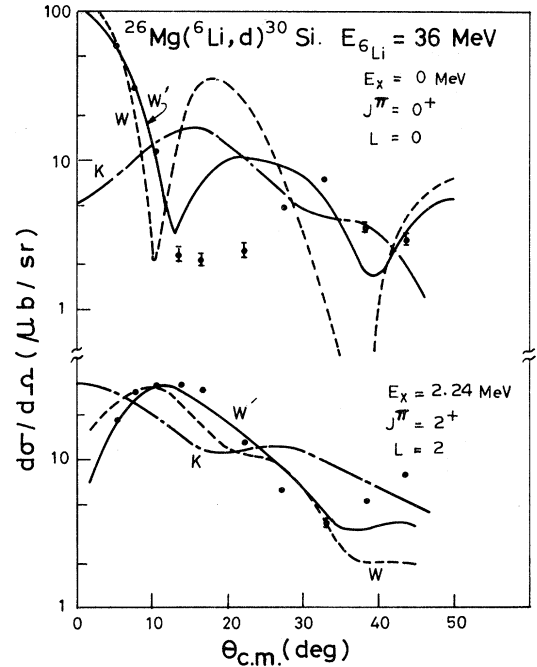


FIG. 3. Angular distributions of the reaction ${}^{26}\text{Mg}({}^6\text{Li}, d){}^{30}\text{Si}$ leading to the g.s. and the 2.24 MeV first excited state in ${}^{30}\text{Si}$. The curves K , W , and W' are the FRDWBA calculations of the present work for an L transfer of 0 for the ground state and of 2 for the first excited state with the choices of V_{ad} and ψ_{ad} corresponding to K , W , and W' for the α - d cluster representation of ${}^6\text{Li}$ (see Table I). The data are of Ref. 7.

above. This parameter set is listed in Table II.

In the cases of odd mass targets in the $2s - 1d$ shell there are no elastic scattering data for ${}^6\text{Li}$ to obtain the optical model parameters required in the present analysis. In the case of the ${}^{29}\text{Si}({}^6\text{Li}, d){}^{33}\text{S}$ reaction discussed above, the ${}^6\text{Li}$ optical model parameters used were those used for several other $2s - 1d$ shell nuclei earlier in the literature^{3,18} obtained from Strohmusch *et al.*¹⁶ and listed in Table II. However, it is found that the conclusion regarding the dependence of the FRDWBA calculation on the cluster representation of ${}^6\text{Li}$ is not affected by the optical model parameter set used in the analysis. In Fig. 4 the reaction data ${}^{27}\text{Al}({}^6\text{Li}, d){}^{31}\text{P}_{\text{g.s.}}$ and ${}^{31}\text{P}({}^6\text{Li}, d){}^{35}\text{Cl}_{\text{g.s.}}$ from Eswaran *et al.*³ are shown along with the comparison with the present FRDWBA calculations. The L transfer is pure 2 in both cases. The FRDWBA calculations with the form W for the α - d cluster representation of ${}^6\text{Li}$ with the optical model set of Strohmusch *et al.*¹⁶ are shown, and it is clear that the form W does not fit the angular distribution satisfactorily. The calculations with the modified form W' are shown with the ${}^6\text{Li}$ optical model parameter set A based on Strohmusch *et al.*¹⁶ and set B based on the values for the ${}^{26}\text{Mg}$

target obtained by Schumacher *et al.*⁸ (see Table II).

From the comparisons with data in Figs. 2–4 it is seen that the form of V_{ad} and ψ_{ad} of Kubo *et al.*⁹ (K) and the Eckart function form of the wave function (E) with the corresponding V_{ad} with attractive and repulsive parts [Eq. (2)] do not predict the angular distributions for $({}^6\text{Li}, d)$ reactions, and the form of Watson *et al.*¹⁵ with zero node wave function with V_{ad} having an infinite repulsive core with $R_{\text{core}} = 1.25$ fm is not satisfactory. The modified form W' with an infinite repulsive core radius reduced to 0.6 fm fits the $({}^6\text{Li}, d)$ angular distributions satisfactorily.

It is emphasized that in the present FRDWBA analysis it is stipulated that the ψ_{ad} that is used should be obtained as a solution of the one body Hamiltonian with V_{ad} as the potential. Bang and Gignoux derived²⁰ ψ_{ad} from three body model calculations for ${}^6\text{Li}$ and this ψ_{ad} is a one node wave function, essentially similar to the Kubo and Hirata form. As pointed out by the authors,²⁰ this form is close to the form that can be derived as a solution of the one body Hamiltonian with the attractive Woods-Saxon form of V_{ad} with parameters $R_0 = 2.22$ fm and $a = 0.7$ fm, and $V_0 = -60.87$ MeV as used by Aurdal, Bang, and Hans- teen.²² We find that this combination of ψ_{ad} and V_{ad} is also not successful in predicting the $({}^6\text{Li}, d)$ angular distributions.

If it is assumed arbitrarily that a combination of V_{ad} and ψ_{ad} need not be restricted by the stipulation that ψ_{ad} should be a solution of the one body Hamiltonian with V_{ad} as the potential, then it appears feasible to obtain good fits to the $({}^6\text{Li}, d)$ angular distributions. For example, the Eckart form (E) of the wave function ψ_{ad} is obtained as a solution with the potential V_{ad} which has an attractive part and a repulsive core [Eq. (2)]. If this ψ_{ad} is used in conjunction with only the attractive part of V_{ad} (neglecting the repulsive part) in the FRDWBA calculations, we find that the angular distributions are well predicted. Furthermore, such an arbitrary procedure with no justifiable basis has also been very much adopted in the literature, obtaining good fits in the case of FRDWBA analyses of the $(\alpha, {}^6\text{Li})$ reaction²³ and $({}^6\text{Li}, d)$.^{3,18} In these analyses the form of the repulsive core was chosen somewhat arbitrarily to be equivalent to the centrifugal potential for $L = 2$ and the parameters of the Woods-Saxon well were chosen to produce a radial shape similar to that of the Eckart function and nearly like that used by Jain *et al.*,¹⁴ and then

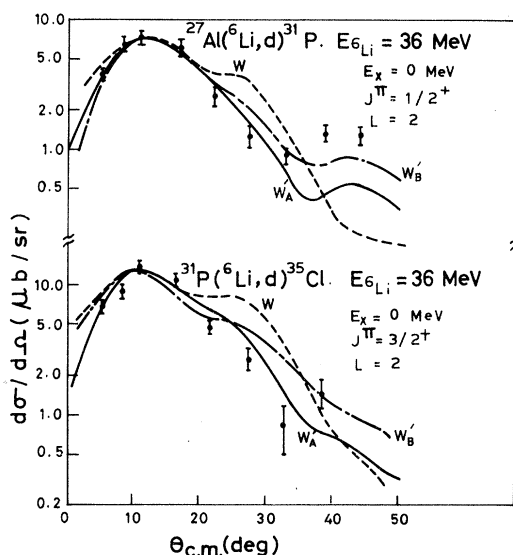


FIG. 4. Angular distributions of the reactions ${}^{27}\text{Al}({}^6\text{Li}, d){}^{31}\text{P}$ (g.s.) and ${}^{31}\text{P}({}^6\text{Li}, d){}^{35}\text{Cl}$ (g.s.). The curves W , W'_A , and W'_B are the FRDWBA calculations of the present work for an L transfer of 2 with choices of V_{ad} and ψ_{ad} corresponding to W and W' for the α - d cluster representation of ${}^6\text{Li}$ (see Table I), while the subscripts A and B on W' refer to the calculations with optical model sets A and B (see Table II), respectively.

the repulsive core part of the interaction was dropped for V in the finite range calculations.

So the basic question arising as a result of these observations is whether the $\psi_{\alpha d}$ and $V_{\alpha d}$ that are used in the FRDWBA analysis should be such that $\psi_{\alpha d}$ is derived as a solution of the one body Hamiltonian with $V_{\alpha d}$ as the potential. If this condition is to be relaxed, then, at present it is not clear as to what should be the basis for such a procedure.

Regarding the absolute cross section as calculated from different choices of $\psi_{\alpha d}$ and $V_{\alpha d}$, it is found that the form K gives much larger cross sections than W and the Eckart form gives much lower cross sections. As an example, in the case of ${}^{29}\text{Si}({}^6\text{Li},d){}^{33}\text{S}_{g.s.}$ with an L transfer of 2, at the maximum of the observed cross section at 11.7° c.m. the calculated cross sections $(d\sigma/d\Omega)_{\text{LOLA}}$ with the forms K, E, W , and W' are 48.50, 3.84, 19.74, and 12.36 $\mu\text{b}/\text{sr}$, respectively, calculated with the Strobusch *et al.*¹⁶ optical model parameter set for ${}^6\text{Li}$.

In the study of the systematics of ground state α cluster spectroscopic strengths in an earlier work^{2,3} with a different choice of cluster representation of ${}^6\text{Li}$ in the FRDWBA analysis, it was deduced that the ground state α cluster spectroscopic strengths of ${}^{31}\text{P}$, ${}^{33}\text{S}$, and ${}^{35}\text{Cl}$ as deduced from the $({}^6\text{Li},d)$ reactions on ${}^{27}\text{Al}$, ${}^{29}\text{Si}$, and ${}^{31}\text{P}$ are 0.65, 0.30, and 0.29 relative to unity for ${}^{16}\text{O}({}^6\text{Li},d){}^{20}\text{Ne}_{g.s.}$. From the present analysis discussed above with the form W' the corresponding values are 0.81, 0.39, and 0.40, showing that although the values are slightly higher, the conclusion regarding relative variations of these strengths with mass number remains the

same.

Janecke *et al.*²¹ have studied the dependence of the finite range DWBA cross sections in the analysis of the $(d, {}^6\text{Li})$ reaction on heavier even-even targets such as ${}^{144}\text{Nd}$ on the cluster representation of ${}^6\text{Li}$. Comparing our results with theirs, it is noted that while we find that the angular distributions are sensitive to the hard core radius used in the form W , they have not reported a similar result. Their comparisons are for $L=0$ transfers in even nuclei, while in the present work on odd and even mass targets of the $2s-1d$ shell, both $L=0$ and $L=2$ transfers are included in the analysis.

In summary, we find the α - d cluster representation of ${}^6\text{Li}$ introduced by Kubo *et al.*⁹ with one node wave function and a Woods-Saxon form of an attractive potential, or the Eckart function form of the wave function with an interaction potential including a repulsive core do not predict the angular distributions in the $({}^6\text{Li},d)$ α -transfer reactions. The form introduced by Watson *et al.*¹⁵ with an infinite repulsive core with a hard core radius $R_{\text{core}} = 1.25$ fm and no node wave function is not satisfactory for predicting the $({}^6\text{Li},d)$ angular distributions. This form modified by reducing the hard core radius to $R_{\text{core}} = 0.6$ fm satisfactorily fits the angular distribution data in $({}^6\text{Li},d)$ reactions.

We thank B. Sikora *et al.* (see Ref. 7) for use of their data on ${}^{26}\text{Mg}({}^6\text{Li},d){}^{30}\text{Si}$ in our study. Our thanks are also due to R. M. DeVries for providing us the LOLA program which we adapted for use in the DEC-10 computer.

¹F. D. Becchetti, in *Clustering Aspects of Nuclear Structure and Nuclear Reactions (Winnipeg, 1978)* Proceedings of the Third International Conference on Clustering Aspects of Nuclear Structure and Nuclear Reaction, edited by W.T.H. Van Oers *et al.* (AIP, New York, 1978), p. 308, and references cited therein.
²M. A. Eswaran and H. E. Gove, *Phys. Rev. C* **18**, 1528 (1978).
³M. A. Eswaran, H. E. Gove, R. Cook, and B. Sikora, *Nucl. Phys.* **A325**, 269 (1979).
⁴N. Austern, R. M. Drisko, E. C. Helbert, and G. R. Satchler, *Phys. Rev.* **133**, B3 (1968).
⁵R. M. DeVries, *Phys. Rev. C* **8**, 951 (1973).
⁶D. R. Chakrabarty and M. A. Eswaran, Proceedings of the International Conference on Nuclear Physics, Berkeley, 1980, Lawrence Berkeley Laboratory Report

No. LBL-11118, Vol. I., p. 522.

⁷B. Sikora, R. E. Cook, and B. R. Fulton, *Bull. Am. Phys. Soc.* **23**, 540 (1978).

⁸P. Schumacher, N. Ueta, H. H. Duhm, K. I. Kubo, and W. J. Klages, *Nucl. Phys.* **A212**, 573 (1973).

⁹K. I. Kubo and M. Hirata, *Nucl. Phys.* **A187**, 186 (1972).

¹⁰J. L. Gammel, B. J. Hill, and R. M. Thaler, *Phys. Rev.* **119**, 267 (1960).

¹¹H. Hasegawa and S. Nagata, *Prog. Theor. Phys.* **38**, 1188 (1967).

¹²J. V. Noble, *Phys. Rev. C* **9**, 1209 (1974).

¹³P. Schwandt, W. Jacobs, W. Ploughe, M. Kaitchuk, J. Meek, P. P. Singh, F. D. Becchetti, and J. Janecke, Indiana University Cyclotron Facility Technical Scientific Report, 1978, p. 85.

- ¹⁴A. K. Jain, J. Y. Crossiord, M. Chevallier, P. Gaillard, A. Guichand, M. Gusakow, and J. R. Pizzi, Nucl. Phys. A216, 519 (1973); A. K. Jain, N. Sarma, and B. Banerjee, *ibid.* A142, 330 (1970).
- ¹⁵J. W. Watson, H. G. Pugh, P. G. Roos, D. A. Goldberg, R. A. J. Riddle, and D. I. Bonbright, Nucl. Phys. A172, 513 (1971).
- ¹⁶U. Strohmusch, C. L. Fink, B. Zeidman, R. G. Markham, H. W. Fulbright, and R. N. Horoshko, Phys. Rev. C 9, 965 (1974).
- ¹⁷E. Newman, L. C. Becker, B. M. Freedom, and J. C. Heibert, Nucl. Phys. A100, 225 (1967).
- ¹⁸N. Anantaraman, H. E. Gove, J. P. Trentleman, J. P. Draayer, and F. C. Jundt, Nucl. Phys. A276, 119 (1977); and A279, 474 (1977).
- ¹⁹B. Buck and A. A. Pilt, Nucl. Phys. A280, 133 (1977).
- ²⁰J. Bang and C. Gignoux, Nucl. Phys. A313, 119 (1979).
- ²¹J. Janecke, F. D. Becchetti, and D. Overway, Nucl. Phys. 343A 161 (1980).
- ²²A. Aurdal, J. Bang, and J. M. Hansteen, Nucl. Phys. A135, 632 (1969).
- ²³R. G. Markham and M. A. M. Shahabuddin, Phys. Rev. C 14, 2037 (1976).