Back-angle scattering of ^{12}C on ^{28}Si

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A simple model is constructed for the $^{12}\mathrm{C}+^{28}\mathrm{Si}$ elastic angular distribution and excitation function at back angles. A "structure" concentrated at a critical value of angular momentum *smaller* than the grazing value in l space is suggested as a viable mechanism. Possible connection to α-particle transfer contribution is pointed out. A short comparison with other models is made.

[NUCLEAR REACTIONS Back angle and 90° scattering of $^{28}\mathrm{Si}+^{12}\mathrm{C}$ is discussed.]

I. INTRODUCTION

The recent measurements[1] of light heavy ion back-angle angular distributions and excitation functions has created intensive discussion concerning the mechanism responsible for the anomalous phenomenon. Several seemingly different interpretations of the phenomenon have been given that try to account partially for the data. So far no unifying description has been suggested. The "state of the art" of this topic has been nicely described in a recent review by Barrette and Kahana.[2]

Recently, Frahn, Hussein, Canto, and Donangelo[3] (this paper will be referred to as FHCD) have put forward a simple model that emphasizes the abnormal nature of the back-angle scattering of light heavy ions. By the explicit introduction of a complex l window, which contains a small, albeit important, parity dependent component, they were able to account rather well for most of the salient features of the back-angle scattering of $^{16}\mathrm{O}+^{28}\mathrm{Si}$. The complex l window introduced by FHCD was shown to peak at a value of angular momentum significantly lower than the grazing one, thus diverging, in an important way, from the conventional philosophy.[2] Through the intricate interference effects that resulted from this anomalous window, FHCD were able to pin down the main characteristics of the 180°-excitation function, namely its being dominated by oscillations (called E oscillations) due to the interference between the parity independent and parity dependent parts of the anomalous window. The period of these oscillations was found to be related exclusively to the position of the peak of the l window. The crucial point that underlies the FHCD approach is that these E oscillations are almost completely accounted for by the anomalous l window. Only at smaller energies ($E_{\mathrm{c.m.}}\sim 20$ MeV for $^{16}\mathrm{O}+^{28}\mathrm{Si}$) does the normal, "$E$-18" part of the elastic S function come into play.

In this paper we extend the FHCD analysis to the $^{12}\mathrm{C}+^{28}\mathrm{Si}$ system and, in particular, consider the 90°-excitation function[4] as another constraint that would help in fixing the model. The $^{12}\mathrm{C}+^{28}\mathrm{Si}$ system has several common features with the $^{16}\mathrm{O}+^{28}\mathrm{Si}$ system. Firstly, it is also an n-α system. As such, the origin of the anomalous window may also be traced to an α-transfer process.[3] Secondly, both systems have been extensively studied experimentally. Therefore, in order to understand the phenomenon of anomalous back-angle scattering, it is very important to consistently use the same model, such as the FHCD, to account for *all* the features of the data of at least these two systems.

This paper is organized as follows. In Sec. II, we present a brief account of the FHCD model, with the relevant specifications for the $^{12}\mathrm{C}+^{28}\mathrm{Si}$ system. In Sec. III we present the results of our analysis. These results are then discussed and compared with those of other models in Sec. IV. Finally, in Sec. V we make several concluding remarks.

II. THE FHCD MODEL AND THE $^{12}\mathrm{C}+^{28}\mathrm{Si}$ SYSTEM

The FCHD model is based on the premise that heavy ion elastic scattering may be described by an elastic S function composed of a normal, strong-absorption part and an anomalous, windowlike part. The strong-absorption part of S is responsible for forward-angle scattering, whereas the anomalous part mostly accounts for back-angle scattering. The intermediate angle regime is determined by the interference of the contributions to the elastic amplitude arising from both the normal and abnormal parts of $S(l)$. Furthermore, to account for the 180°-excitation function, a small parity-dependent part is allowed as a second component of the complex anomalous l window. Through the interference between the parity-dependent and parity-independent parts of the anomalous window (which dominates the scattering in the back-angle region), the "unnatural" period of the E oscillation of the 180°-excitation function is obtained.[3]

As will become clear later, the parity-dependent part of the anomalous window *does not* have any major role in the 90°-excitation function. This point is very important in so far as fixing the relative phase between the parity-dependent and parity-independent parts of the anomalous l window is concerned.

In detail, the FHCD elastic S function is given by

$$S_l=\bar{S}_l+d(E)\left[1+(-1)^l\gamma(E)\right]\omega(l) , \tag{1}$$

where $\bar{S}_l$ is the strong absorption part, responsible for forward-angle scattering and generated, in e.g., $^{16}O+^{28}Si$, from the E-18 optical potential.[4] The anomalous window, given by the second term in Eq. (1), was specified by its position $\tilde{l}(E)$ and width $\tilde{\Delta}(E)$. The strength functions, d and γ, depend on energy. It was found in Eq. (3) that $\tilde{\Delta}$ is quite different from $\bar{\Delta}$, the "width" of $\bar{S}_l$. Furthermore, it was found that $\tilde{\Lambda}\equiv\tilde{l}+\frac{1}{2}$ was several units below the grazing $\bar{\Lambda}$ which characterizes $\bar{S}_l$. The same, as above, is assumed for $^{12}C+^{28}Si$. It is suggested that a particularly strongly coupled channel to the elastic one, e.g., $^{12}C+^{28}Si\rightarrow{}^{16}O+^{24}Mg$, is responsible for the anomalous part of S_l. Since the form factor for α transfer drops quickly to insignificance within a few fermis, the resulting window in the elastic channel should peak at an l value smaller than the grazing l.[5] Although this assumption would have to await a detailed confirmation through coupled channels calculation, it is certainly plausible. The normal part of S was generated from an optical potential used previously by De Vries *et al.*[5] to fit the elastic scattering data of $^{12}C+^{28}Si$ at $E_{c.m.}=20.3$ MeV. We assumed that this potential (which as a typical strong absorption potential as the E-18) accounts for the forward part of the angular distribution at other energies as well. Notice that the normal part of S enters in our calculation only in the description of the 90°-excitation function to be discussed later. This is so since at 180° the contribution of the normal S is quite small, giving rise to a small σ_{el}/σ_R (typically $\sim10^{-4}$) which decreases with increasing center-of mass energy, and therefore may be neglected.[2]

Finally, the parametrized normal part of S, $\bar{S}$, is chosen to have an Ericson form

$$\bar{S}(\lambda)=\left\{1+\exp\left[\left[\frac{\bar{\Lambda}-\lambda}{\bar{\Delta}}\right]-i\bar{\alpha}\right]\right\}^{-1} ,$$
$$\lambda\equiv l+\tfrac{1}{2} . \tag{2}$$

III. RESULTS

The window function $\omega(l)$ has been parametrized as in FHCD, namely in the form of a derivative of an Ericson function

$$\omega(\lambda)=\frac{1}{2\tilde{\Delta}}\left[1+\cosh(\tilde{\mu}+i\tilde{\alpha})\right]^{-1} ,$$
$$\tilde{\mu}=\frac{\lambda-\tilde{\Lambda}}{\tilde{\Delta}} . \tag{3}$$

Equation (3) represents a complex window centered about $\lambda=\tilde{\Lambda}$ and having a width $\tilde{\Delta}$. The energy dependence of $\tilde{\Lambda}$ is found from the period of the oscillations in the 180°-excitation function (see above) to be

$$\tilde{\Lambda}(E_{c.m.})=\tilde{A}(E_{c.m.}-\tilde{E})^{1/2} ,$$
$$\tilde{A}=4.44\ \mathrm{MeV}^{-1/2} , \tag{4}$$
$$\tilde{E}=11.0\ \mathrm{MeV} .$$

The other parameters were found to be

$$\tilde{\Delta}=0.35 ,$$
$$\alpha=0.2 . \tag{5}$$

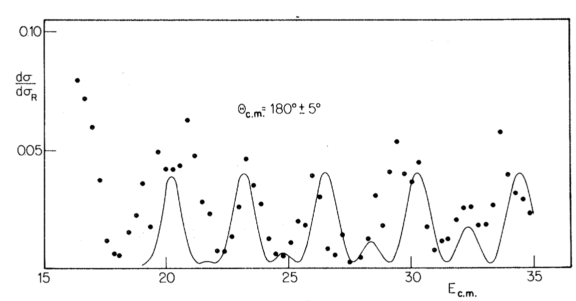
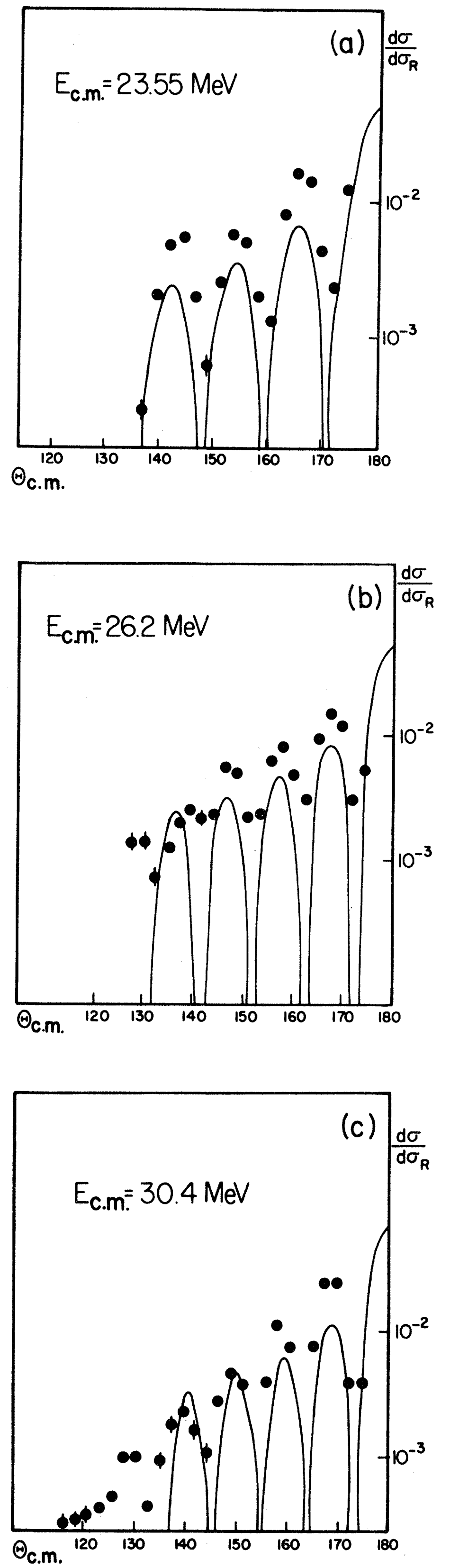


FIG. 1. The 180° excitation of $^{12}C+^{28}Si$. Data points are from Ref. 4. The full curve is a result of our calculation using the abnormal part of S of Eq. (1). See text for the values of the parameters.



The strength function γ and d were allowed the following values

$$\gamma = -4.0\exp(-0.1149E_{c.m.}) \, , \quad d = 0.08\exp(-0.08E_{c.m.}) \, . \tag{6}$$

The result of the calculation of the excitation function is shown in Fig. 1. The overall agreement with the data is reasonable, considering the lack of parameter optimization.

The results for the angular distributions at three energies ($E_{c.m.} = 23.55$, 26.2, and 30.4 MeV), are shown in Fig. 2. Again the overall agreement with the data is reasonable. In particular, the angular period of the θ oscillation does come out right and is roughly equal to $\pi/\tilde{\Lambda}$. Notice that the grazing $\overline{\Lambda}(E)$ is larger than $\tilde{\Lambda}$ by one unit at $E_{c.m.} = 23.55$ MeV, with the difference increasing to about 2 to 3 units at $E_{c.m.} = 30$ MeV. This is in accord with similar behavior seen in $^{16}O + ^{28}Si$.[2]

Finally, in Fig. 3 we present the result of our calculation of the 90°-excitation function. The overall trend of the oscillations as well as the envelope, as compared to the 180°-excitation function, follows the trend of the data of Kubono *et al.*[4] Two important points should be stressed at this stage. The first is that the relative phase between the parity-dependent and parity-independent parts of the anomalous window is immaterial in the 90° excitation. It is, of course, important in the 180° excitation, as the oscillations here reflect the interference [i.e., linear dependence on γ, see Eq. (3)] between the two parts of the anomalous window. Thus we could unambiguously fix the relative phase. Secondly, the interference phenomenon observed in the 90°-excitation function is more complex than that seen in the 180°-excitation function. The reason is already referred to earlier, namely, the more important role of the normal part of S in the former.

To see this point clearly, we use the closed expressions for $d\sigma/d\sigma_R$ (180°) and $d\sigma/d\sigma_R$ (90°) worked out by Frahn,[7]

$$\frac{d\sigma}{d\sigma_R}(180°) \cong \left[2d \cot\frac{\tilde{\theta}_R}{2} \right]^2 [A + B \sin\pi\tilde{\Lambda}(E)] \, ,$$

FIG. 2. Back-angle angular distribution of $^{12}C + ^{28}Si$ at (a) $E_{c.m.} = 23.55$ MeV, (b) $E_{c.m.} = 26.2$ MeV, and (c) $E_{c.m.} = 30.4$ MeV. The data points are from Ref. 6. The full curves were obtained from the same abnormal S as the one used in Fig. 1.

where

$$A \equiv \left\{ H[\tilde{\Delta}(\tilde{\theta}_R - \pi)] \right\}^2 + \gamma^2 \left[H(\tilde{\Delta}\tilde{\theta}_R) \right]^2 ,$$

$$B \equiv 2\gamma H(\tilde{\Delta}\tilde{\theta}_R) H\left[\tilde{\Delta}(\tilde{\theta}_R - \pi) \right] \quad (7)$$

with

$$\tilde{\theta}_R \cong \arctan \frac{\eta}{\tilde{\Lambda}}$$

and

$$\frac{d\sigma}{d\sigma_R}(90^\circ) \simeq \left[[A' + B' \sin\pi\tilde{\Lambda}(E)] + C_1' \sin\left\{ \frac{\pi}{2}(\bar{\Lambda} - \tilde{\Lambda}) + 2[\sigma(\tilde{\Lambda}) - \sigma(\bar{\Lambda})] \right\} + C_2^1 \cos\left\{ \frac{\pi}{2}(\bar{\Lambda} + \tilde{\Lambda}) + 2[\sigma(\tilde{\Lambda}) - \sigma(\bar{\Lambda})] \right\} \right]$$

with

$$A' = \frac{\bar{\Lambda}}{2\pi} \frac{1}{\eta} \frac{1}{\left[\bar{\theta}_R - \frac{\pi}{2} \right]^2} \left\{ H\left[\bar{\Delta} \left[\bar{\theta}_R - \frac{\pi}{2} \right] \right] \right\}^2 + \left[\frac{d_0}{\eta} \right]^2 \frac{\tilde{\Lambda}}{2\pi}(1+\gamma)^2 \left[\left\{ H\left[\tilde{\Delta} \left[\tilde{\theta}_R + \frac{\pi}{2} \right] \right] \right\}^2 + \left\{ H\left[\tilde{\Delta} \left[\tilde{\theta}_R - \frac{\pi}{2} \right] \right] \right\}^2 \right] ,$$

$$B' = 2\left[\frac{d_0}{\eta} \right]^2 \frac{\tilde{\Lambda}}{2\pi}(1+\gamma)^2 H\left[\tilde{\Delta} \left[\tilde{\theta}_R + \frac{\pi}{2} \right] \right] H\left[\tilde{\Delta}(\tilde{\theta}_R - \frac{\pi}{2}) \right] ,$$

$$C_1' = 2\left[\frac{\bar{\Lambda}\tilde{\Lambda}}{4\pi^2} \right]^{1/2} \frac{1}{\eta^2}(1+\gamma) \frac{1}{\bar{\theta}_R - \frac{\pi}{2}} H\left[\bar{\Delta} \left[\bar{\theta}_R - \frac{\pi}{2} \right] \right] H\left[\tilde{\Delta} \left[\tilde{\theta}_R - \frac{\pi}{2} \right] \right] , \quad (8)$$

$$C_2' = 2\left[\frac{\bar{\Lambda}\tilde{\Lambda}}{4\pi^2} \right]^{1/2} \frac{1}{\eta^2}(1+\gamma) \frac{1}{\bar{\theta}_R - \frac{\pi}{2}} H\left[\bar{\Delta} \left[\bar{\theta}_R - \frac{\pi}{2} \right] \right] H\left[\tilde{\Delta} \left[\tilde{\theta}_R + \frac{\pi}{2} \right] \right] ,$$

$\sigma(\Lambda)$ is the Coulomb phase shift and

$$\bar{\theta} = \arctan \frac{\eta}{\bar{\Lambda}} .$$

Where the function $H(\Delta x)$ is given by the Fourier transform of a derivative of an Ericson function and is given by

$$H(\Delta x) = \frac{\pi \Delta x}{\sinh \pi \Delta x} e^{\alpha \Delta x} . \quad (9)$$

The first important qualitative difference between σ/σ_R (180°) and σ/σ_R, (90°) is that the parity-dependent part of the anomalous window decides upon the over-all sign of the oscillatory term in σ/σ_R (180°), whereas it has no similar role in the 90°-excitation function. Further, the interference pattern in the 90°-excitation function is more complex as can be seen from Eq. (8). Ignoring the difference $\sigma(\tilde{\Lambda})$-$\sigma(\bar{\Lambda})$ for the moment, one may recognize three different periods:

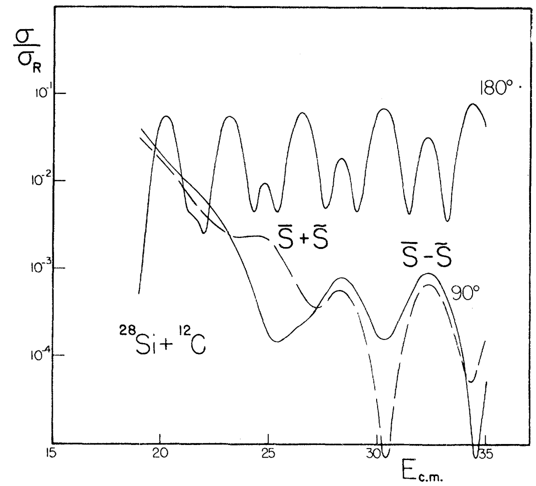


FIG. 3. A comparison of the 180° and 90° excitation function for $^{12}C+^{28}Si$. The full $S=\bar{S}+\tilde{S}$ was used for the 90° excitation function with the relative phase between $\bar{S}$ and $\tilde{S}$ considered to be 0° (dashed curve) and 180° (full curve). See text for the parameters of the normal $\bar{S}$.

$$P_1=2\left[\frac{d\tilde{\Lambda}}{dE}\right]^{-1},$$

$$P_2=4\left[\frac{d}{dE}(\bar{\Lambda}-\tilde{\Lambda})\right]^{-1},$$

and

$$P_3=4\left[\frac{d}{dE}(\bar{\Lambda}+\tilde{\Lambda})\right]^{-1}.$$

Our calculation, shown in Fig. 3, presents only those oscillations with periods P_1 and P_3, with P_2 completely absent as it corresponds to widely spaced oscillations. It would seem that at lower c.m. energies the interference term with the period P_3 becomes dominant. This term represents the interference between the near-side normal contribution and the far-side anomalous term. As Fig. 3 shows, a change in relative phase between these two terms by π brings about a 180° change in phase in the oscillations of σ/σ_R (90°). At higher energies this change in relative phase between the normal and abnormal contributions does very little insofar as the positions of maxima and minima are concerned. This conclusion is in line with our contention that at higher energies the anomalous part of S dominates.

It should be remarked that the expression for σ/σ_R (180°) of Eq. (7) is an approximate one obtained by dropping the interference term between the near-side and far-side contributions of the parity-independent window. Including this term brings in a term proportional to $\cos 2\pi\tilde{\Lambda}$. The smaller-magnitude oscillations that appear between the dominant $\sin\pi\tilde{\Lambda}$ oscillation in Fig. 1 are due to the above interference. The data also seem to exhibit this behavior.

IV. COMPARISON WITH OTHER INTERPRETATIONS

Several attempts have been made in the quest to understanding the phenomenon of anomalous back-angle heavy-ion scattering.

In one class of these approaches,[1] the structure in the excitation function is associated with isolated partial-wave resonances superimposed on a normal, strong-absorption scattering. Owing to the strong absorption present in the system, these resonances are possible only for those l values near the grazing l. We shall call those approaches invoking anomalies concentrated around l_g, "grazing anomaly approaches" (GAA). Another approach that is based on GAA is that of Lee.[8] He employs a semiclassical approximation on the scattering amplitudes of several optical potentials and shows that the structure seen in the 180°-excitation function arises from interference between the internal, S_I (reflected from the inner barrier), and the external, S_E (reflected from the outer barrier) parts of the amplitude. Since the interference term becomes appreciable only when the magnitude of S_I and S_E are comparable, which happens for l close to l_g, one reaches the conclusion that the effective anomalous "window" (which is parity-dependent) in Lee's analysis is also of the grazing type. Though Lee succeeds in qualitatively accounting for the E oscillations in the $^{16}O+^{28}Si$ 180°-excitation function at low energies, his calculated excitation function becomes almost smooth (with very slight modulations) at $E_{c.m.}\gtrsim 30$ MeV. Furthermore, his model is not expected to account for the 90°-excitation function, since the same mechanism, i.e., interference between S_I and S_E, must be invoked, and we have seen that the parity-dependent window has no role in this case.

Finally, we comment on attempts at using explicit parity dependence in the ion-ion potential in the form $[1+C(-1)^l](V+iW)$.[4,9] Since $|C|<<1$, the effect on the partial-wave S function arising from the parity-dependent term may be calculated using first-order perturbation theory. We find for the total $S(l)$, the following approximate expression

$$S(l)=\bar{S}(l)-i\frac{4\mu}{\hbar^2 k}C(-)^l \times \int_0^\infty \left[\psi_l^0(r)\right]^2\left[V(r)+iW(r)\right]dr \ . \quad (10)$$

The integral appearing in the second term in Eq. (10) resembles very much the DWBA radial integral for zero-angular momentum transfer amplitude, with $V+iW$ acting as a complex form factor. For strongly absorptive optical potential (W large) the above integral represents an l window sharply peaked around l_g. This is so since the only region in r which gives an appreciable contribution to the integral is concentrated around the barrier. In this region, the large values of k (i.e., the radial kinetic energy) present in $\psi^0_{l<l_g}(r)$ and the small value of both V and W at the outer turning points for $l>l_g$, force the l profile of the integral to peak at about l_g.

Thus, although the approaches of Refs. 4 and 9 discussed above are closest in spirit to our approach, they differ we believe, in an important detail; namely, the position of the anomalous l window. We believe that by fixing the general structure of the interference pattern in both the 180°- and 90°-excitation functions one is able to distinguish between the grazing $l,\bar{l}$ and the smaller position of the anomalous window, $\tilde{l}$, as we have shown for the system $^{12}C+^{28}Si$.

V. CONCLUSIONS

In this paper we have extended the analysis of Ref. 3 (FHCD) to the system $^{12}C+^{28}Si$. We were able to account rather well for the 180°-excitation function, the back-angle angular distributions for three energies ($E_{c.m.}=23.55$, 26.2, and 30.4 MeV) and the 90°-excitation function. Since the 90°-excitation function is insensitive to the sign of the parity-dependent part (relative to the parity-independent part) of the anomalous window, we used the 180°-excitation function to fix this sign (negative) unambiguously. We have also verified the insensitivity of the 90°-excitation function at higher energies ($E_{c.m.}\geq 28$ MeV, Fig. 3) to the relative phase between the normal $\bar{S}$ and the abnormal $\tilde{S}$, due to the smaller contribution of $\bar{S}$ at these energies. At lower energies, the relative phase becomes relevant as can be seen in Fig. 3. This observation helps in fixing this phase which came out to be π. We may therefore conclude that the FHCD model is capable of describing both the back-angle and the 90°-excitation function. This, we believe, demonstrates the viability of this model in actually pinning down the structure of the elastic S function that underlies the phenomenon of anomalous back-angle heavy-ion scattering.

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