

Neutrino reactions in ¹²C

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Total and differential cross sections for the reaction $\nu_\mu + ^{12}\text{C} \rightarrow \mu^- + ^{12}\text{N}$ and total cross sections for the reactions $\nu_e + ^{12}\text{C} \rightarrow e^- + ^{12}\text{N}$, $\nu + ^{12}\text{C} \rightarrow ^{12}\text{C}^* + \nu$ and the corresponding antineutrino reactions are calculated for neutrino energy from threshold to the 250 MeV range. The effect of the pseudotensor form factor on these results is examined.

[NUCLEAR REACTIONS Total and differential cross sections for ¹²C(ν_μ, μ^-)¹²N and total cross sections for ¹²C(ν_e, e^-)¹²N, ¹²C($\bar{\nu}_e, e^+$)¹²B, ¹²C(ν, ν)¹²C*, ¹²C($\bar{\nu}, \bar{\nu}$)¹²C* are obtained.

Neutrino reactions in ¹²C are interesting for a number of reasons. Recently, they have been used¹ and proposed² for tests of neutrino oscillations. In addition they offer the possibility for observing the pseudotensor or weak charge form factor.

In this paper we study the reactions

$$\nu_\mu + ^{12}\text{C} \rightarrow ^{12}\text{N}(\text{g.s.}) + \mu^-, \tag{1a}$$

$$\bar{\nu}_\mu + ^{12}\text{C} \rightarrow ^{12}\text{B}(\text{g.s.}) + \mu^+, \tag{1b}$$

$$\nu_e + ^{12}\text{C} \rightarrow ^{12}\text{N}(\text{g.s.}) + e^-, \tag{1c}$$

$$\bar{\nu}_e + ^{12}\text{C} \rightarrow ^{12}\text{B}(\text{g.s.}) + e^+, \tag{1d}$$

$$\begin{pmatrix} \nu \\ \bar{\nu} \end{pmatrix} + ^{12}\text{C} \rightarrow ^{12}\text{C}^* + \begin{pmatrix} \nu \\ \bar{\nu} \end{pmatrix}. \tag{1e}$$

These reactions are important in their own right and also as major components of inclusive neutrino reactions in ¹²C.

The method of treatment used here is the elementary particle model.³ This method of treatment is particularly suitable because the form factors are either known or can be approximated from other measurements and thus the use of nuclear wave functions can be avoided.

The initial state ¹²C(g.s.) is a 0⁺ state of isospin 0. The three final states we are considering, ¹²N(g.s.), ¹²B(g.s.), and ¹²C*(15.11), are all 1⁺ states with I = 1 and I_z = 1, -1, and 0, respectively. Thus, the current matrix elements for the processes described in Eq. (1) have the same Lorentz struc-

ture and we give only one as an example:

$$\langle ^{12}\text{N} | V_\mu^+(0) | ^{12}\text{C} \rangle = -i\sqrt{2}m \xi_{\mu\alpha\beta\gamma} q^{\alpha\xi\beta} \frac{Q_\gamma}{2m} \frac{F^M(q^2)}{2m_p}, \tag{2a}$$

$$\langle ^{12}\text{N} | A_\mu^+(0) | ^{12}\text{C} \rangle = \sqrt{2}m \left[\xi_\mu F_A(q^2) + q_\mu \xi \cdot q \frac{F_p(q^2)}{m_\pi^2} - \frac{Q_\mu}{2m} \frac{q\xi}{2m_p} F_E(q^2) \right], \tag{2b}$$

where $P_{i\mu}$ and $P_{f\mu}$ are the initial and final momentum of the nuclei, ξ_μ is the polarization vector of the final state nucleus, m and m_p are the mass of the nucleus and proton, respectively, $q_\mu = P_{f\mu} - P_{i\mu}$, and $Q_\mu = P_{i\mu} + P_{i\mu}$.

The matrix element for the process

$$\begin{pmatrix} \nu \\ \bar{\nu} \end{pmatrix} + ^{12}\text{C} \rightarrow f + \begin{pmatrix} l \\ \bar{l} \end{pmatrix}, \tag{3}$$

where f and l represent the final state nucleus and lepton, respectively, is given to the first order in G by

$$M = \frac{G}{\sqrt{2}} \bar{u}_l \gamma^\mu (1 - \gamma_5) u_\nu \langle f | J_\mu^{(i)} | ^{12}\text{C} \rangle, \tag{4}$$

where $i = V_\mu - A_\mu, V_\mu^+ - A_\mu^+, V_\mu^{(3)} - A_\mu^{(3)} - 2\sin^2\theta_w J_\mu(\text{em})$, as is appropriate. Equation (4) together with Eqs. (2a) and (2b) leads to the result

$$\begin{aligned}
|M|^2 = \frac{G^2}{2} & \left[\frac{F_M^{i2}}{(2m)^2(2mp)^2} \{2Q^2[(v \cdot l)^2 - m_l^2 v \cdot l] + 2(Q \cdot v)^2(v \cdot l - m_l^2)\} \right. \\
& + F_A^{i2}(l \cdot v + 2P_f \cdot l P_f \cdot v / m^2) \\
& + \frac{F_P^{i2}}{m_\pi^4} \left[m_l^2 v \cdot l \left\{ 2v \cdot l - m_l^2 + \frac{1}{m^2} [(v \cdot P_f)^2 + (l \cdot P_f)^2 - 2v \cdot P_f l \cdot P_f] \right\} \right] \\
& \pm \frac{F_m^i F_A^i}{mm_P} [(v \cdot l - m_l^2) Q \cdot v + v \cdot l Q \cdot l] \\
& \left. + \frac{F_A^i F_P^i}{m_\pi^2} (2m_l^2) \left\{ -v \cdot l + \frac{1}{m_f^2} [-(P_f \cdot v)^2 + (P_f \cdot v)(P_f \cdot l)] \right\} + |M_E|^2 \right], \quad (5)
\end{aligned}$$

where the \pm sign corresponds to a neutrino or an antineutrino reaction, respectively, and the i distinguishes among the form factors belonging to the charge lowering, charge raising, and neutral current matrix elements, respectively. Finally,

$$\begin{aligned}
|M_E|^2 = \frac{F_E^i F_A^i}{2mm_P} & \left[m_l^2 Q \cdot v + \frac{(v \cdot P_f - l \cdot P_f)}{m^2} (p_f \cdot l Q \cdot v + P_f \cdot v Q \cdot l - l \cdot v Q \cdot P_f) \right] \\
& + \frac{F_E^i F_P^i \vec{q}^2 Q \cdot v m_l^2}{2mm_P m_\pi^2} + \frac{2F_E^2 \vec{q}^2}{(2m)^2(2m_P)^2} (2Q \cdot l Q \cdot v - v \cdot l Q^2). \quad (6)
\end{aligned}$$

The form factor $F_M^i(q^2)$ is obtained from electron scattering data and from photon absorption data.⁴ By the use of the conserved vector current (CVC) hypothesis

$$\begin{aligned}
F_M^i(q^2) &= \sqrt{2}\mu(q^2) \\
&= 3.66/(1 - q^2/M_V^2)^2, \quad (7a)
\end{aligned}$$

where $M_V^2 = 2.7m_\pi^2$ for i corresponding to the charge raising or charge lowering current. For the neutral current case

$$F_M^{(3)}(q^2) = \frac{F_M(q^2)}{\sqrt{2}} (1 - 2\sin^2\theta_w), \quad (7b)$$

where $F_M(q^2)$ is given by Eq. (7a) and we take $\sin^2\theta_w = 0.25$.

The values for $F_A^i(q^2)$ are obtained from beta decay data⁴ and a result based on the impulse approximation relating $F_A(q^2)/F_A(0)$ to $F_M(q^2)/F_M(0)$. The results are given by⁵

$$F_A^i(q^2) = 1.03/(1 - q^2/M_A^2)^2, \quad (8a)$$

where $M_A^2 = M_V^2$ and i corresponds to the charge raising and lowering current cases. For the neutral current case

$$F_A^{(3)}(q^2) = F_A(q^2)/\sqrt{2}. \quad (8b)$$

The pseudotensor form factor $F_P^i(q^2)$ is obtained

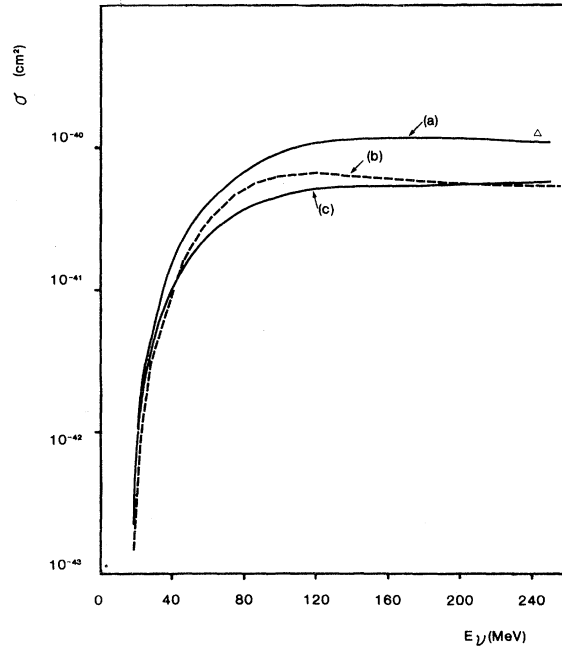


FIG. 1. Plot of the cross section for the reactions $\nu_e + {}^{12}\text{C} \rightarrow {}^{12}\text{N} + e^-$ and $\bar{\nu}_e + {}^{12}\text{C} \rightarrow {}^{12}\text{B} + e^+$ as a function of neutrino energy. Curve (a) represents the result obtained here for the electron neutrino reaction. Curve (b) is the result of Uberall *et al.* for the same reaction. Curve (c) is the result obtained here for the antielectron neutrino. The point indicated by a triangle is due to O'Connell *et al.* for the neutrino reaction.

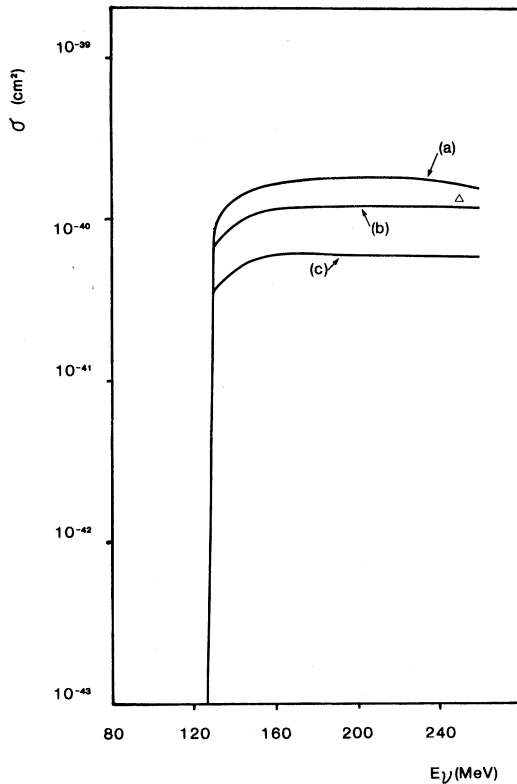


FIG. 2. Plot of the cross section for the reactions $\nu_\mu + {}^{12}\text{C} \rightarrow {}^{12}\text{N} + \mu^-$ and $\bar{\nu}_\mu + {}^{12}\text{C} \rightarrow {}^{12}\text{B} + \mu^+$ as a function of neutrino energy. Curve (a) represents the result obtained here for the neutrino reaction. Curve (b) represents the result obtained here for the antineutrino reaction. Curve (c) is the result of Uberall *et al.* for the neutrino reaction. The point indicated by the triangle is due to O'Connell *et al.* for the neutrino reaction.

from a modification⁶ of the Nambu⁷ result

$$F_P(q^2) = m_\pi^2 F_A(q^2) / (m_\pi^2 - q^2).$$

The form factor $F_E(q^2)$ has been generally assumed small. However, estimates from muon capture⁸ (including recoil polarization measurements⁹) have indicated a value⁸ of $F_E(0)/F_A(0) = 3.64 \pm 0.08$. Furthermore, Hwang and Primakoff⁸ have argued that F_E and F_M have the same q^2 dependence, i.e., $M_E = M_V$. We use this result in what follows. Using the values given by Eqs. (7) and (8) but keeping $F_E = 0$, we obtain results for the total cross section as a function of neutrino energy for reactions (1a)–(1d). Reactions (1a) and (1c) have been calculated previously by Uberall *et al.*¹⁰ and O'Connell *et al.*¹¹ The two previous results are not in agreement.¹² Our result is in reasonable agreement with the O'Connell point as can be seen in Figs. 1 and 2. We note that in Fig. 2 the

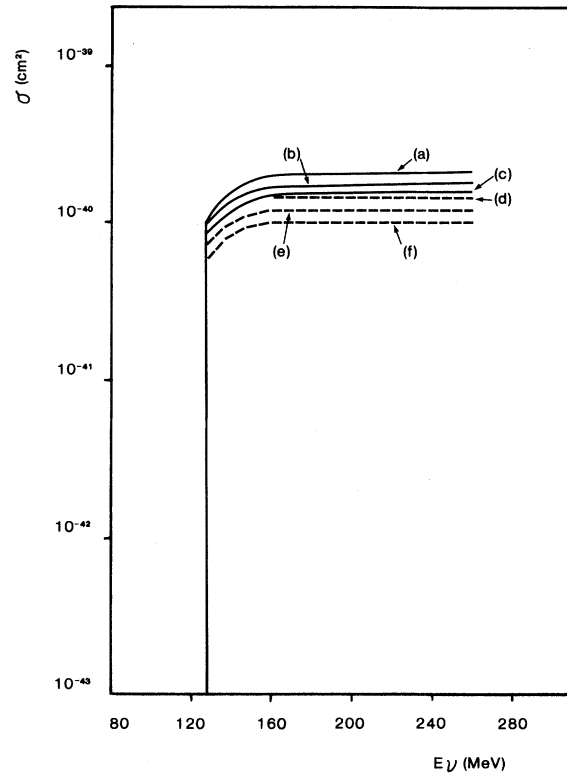


FIG. 3. Plot of the cross section for the reactions $\nu_\mu + {}^{12}\text{C} \rightarrow {}^{12}\text{N} + \mu^-$ and $\bar{\nu}_\mu + {}^{12}\text{C} \rightarrow {}^{12}\text{B} + \mu^+$ as a function of neutrino energy for various values of $F_E(0)$. Curves (a)–(c) represent results for the reaction $\nu_\mu + {}^{12}\text{C} \rightarrow {}^{12}\text{N} + \mu^-$ with $F_E(0)/F_A(0) = -3.64, 0, \text{ and } 3.64$, respectively. Curves (d)–(f) represent results for the reaction $\bar{\nu}_\mu + {}^{12}\text{C} \rightarrow {}^{12}\text{B} + \mu^+$ with $F_E(0)/F_A(0) = 0.364, 0, \text{ and } 3.64$, respectively.

O'Connell point represents an extreme relativistic limit, whereas the calculation presented here is not a limit.

When F_E is not set equal to zero but is allowed to have a magnitude in the range suggested by Hwang and Primakoff, the effect on the electron neutrino and neutral current neutrino cross sections are small due to the lepton mass size. However, in the case of the muon-neutrino reactions the effect is more noticeable and is around the 10–15% level. This can be seen both from the total cross section, Fig. 3, and from the differential cross section, Fig. 4.

Finally, in Fig. 5 we have plotted the total cross section for the neutral current neutrino reaction. Two calculations of this exist: one, an impulse approximation based calculation by Donnelly *et al.*,¹³ and the other, an elementary particle model calculation by Bernabeu and Pascual.¹⁴ Our results are

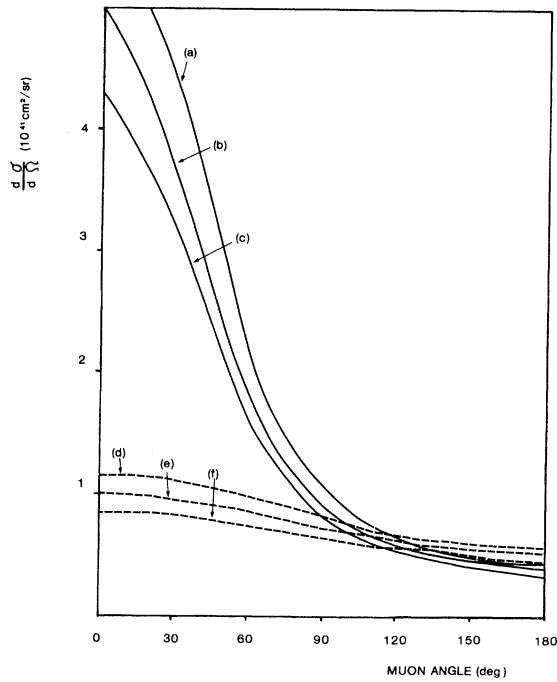


FIG. 4. Plot of the differential cross section for the reaction $\nu_\mu + {}^{12}\text{C} \rightarrow {}^{12}\text{N} + \mu^+$ for various values of $F_E(0)$. Curves (a)–(c) represent results for an incident neutrino energy of 200 MeV with $F_E(0)/F_A(0) = -3.64, 0,$ and 3.64 , respectively. Curves (d)–(f) represent results for an incident neutrino energy of 130 MeV with $F_E(0)/F_A(0) = -3.64, 0,$ and 3.64 , respectively.

closer to those of Donnelly *et al.* and would be even more so if we had used his value for θ_w .

In conclusion we remark that as the accuracy of neutrino experiments improve, it may be possible to determine the pseudotensor form factor F_E from

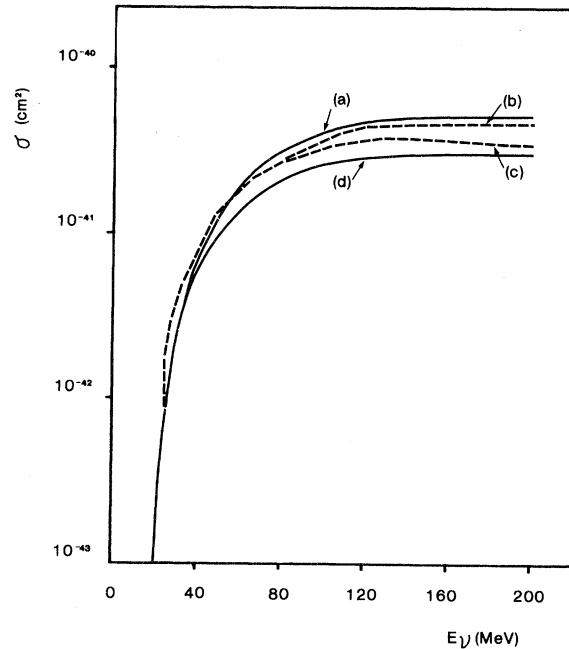


FIG. 5. Plot of the neutral current cross section as a function of neutrino energy. Curves (a)–(c) represent the results obtained here, by Donnelly *et al.*, and by Bernabeu and Pascual, respectively, for the reaction $\nu + {}^{12}\text{C} \rightarrow \nu + {}^{12}\text{C}^*$. Curve (d) represents the results obtained here for the reaction $\bar{\nu} + {}^{12}\text{C} \rightarrow \bar{\nu} + {}^{12}\text{C}^*$.

muon-neutrino reactions in ${}^{12}\text{C}$. Even with fixed neutrino spectra, if the muon angle can be measured so that a differential cross section can be obtained, it might be possible to obtain a fit for the form factor. This would be a valuable supplement to the results currently available.

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¹²We have reduced the results of J. S. O'Connell *et al.* as was indicated in his paper (see Ref. 11).

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