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Effect of reflections in pion-deuteron scattering

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We compare the solutions of the Faddeev equations for pion-deuteron scattering, with and without the reflection terms in which the pion is allowed to scatter back and forth between the two nucleons. We show that in the resonance region, the effect of these terms in the differential cross section is less than 20%. We point out that the Faddeev equations without reflections, reduce to a Lippmann-Schwinger equation with a pion-deuteron effective potential. We also point out that the pole approximation to the nucleon-nucleon T matrix is not a very good approximation.

[NUCLEAR REACTIONS π-d scattering, reflection terms; calculated $d\sigma/d\Omega$, $E=142$, 181, and 228 MeV.]

A great deal of interest in recent years has arisen regarding the effect in pion-nucleus scattering, of the reflection terms in which a pion scatters several times between two nucleons. Thus, it was shown by Keister[1] that in the fixed-nucleon approximation, these terms are much larger than the usual single-scattering term. Similar results were found in the calculation of the contribution of these terms to the Lorentz-Lorenz effect.[2] Recently however, it has been pointed out[3] that nucleon recoil has the effect of reducing drastically the contribution of these terms, although still remaining a sizeable contribution to pion-nucleus scattering. In this paper, we will use the Faddeev equations to calculate exactly the contribution of the reflection terms in the pion-deuteron system.

The amplitude for pion-deuteron elastic scattering is given by

$$F_{el}(\vec{q}_0{}',\vec{q}_0)=\langle \vec{q}_0{}'\phi_0 | T_{el} | \vec{q}_0\phi_0\rangle \ , \tag{1}$$

$$T_{el}=T_2+T_3 \ , \tag{2}$$

where ϕ_0 is the deuteron ground-state wave function, $\vec{q}_0$ and $\vec{q}_0{}'$ are the initial and final pion-deuteron relative momenta, and T_2 and T_3 are the solutions of the Faddeev equations

$$T_1=t_1G_0(T_2+T_3) \ , \tag{3a}$$

$$T_2=t_2+t_2G_0(T_1+T_3) \ , \tag{3b}$$

$$T_3=t_3+t_3G_0(T_1+T_2) \ , \tag{3c}$$

where t_1 is the nucleon-nucleon T matrix, and t_2 and t_3 are pion-nucleon T matrices, while G_0 is the Green's function for three free particles. Equations (1)–(3) are usually derived from a nonrelativistic theory, although they are also valid in the relativistic case.[4] The Green's function G_0 is in the nonrelativistic case,

$$G_0=\frac{1}{E_0+q_0{}^2/2\nu-h_0-H_0+i\epsilon} , \tag{4}$$

where h_0 is the kinetic energy operator for the relative motion of the two nucleons, and H_0 that for the relative motion of the pion with the center of mass of the two nucleons, while E_0 is the (negative) binding energy of the deuteron, $q_0=|\vec{q}_0|=|\vec{q}_0{}'|$ is the magnitude of the on-shell pion-deuteron relative momentum, and ν is the reduced mass of the pion two nucleon system. If we call $\vec{q}$ to the relative momentum between the pion and the two nucleons, then

$$G_0|\vec{q}\rangle=\frac{1}{E_0+q_0{}^2/2\nu-h_0-q^2/2\nu+i\epsilon}|\vec{q}\rangle \ . \tag{5}$$

One usually calls reflections[5–7] to those terms in which the pion interacts consecutively with the two nucleons, such as, for example, $t_2G_0t_3$, $t_2G_0t_3G_0t_2$, etc., which are generated by the term $t_2G_0T_3$ in Eq. (3b) and $t_3G_0T_2$ in Eq. (3c). If one neglects these terms, Eqs. (3) take the simpler form

$$T_1 = t_1 G_0 T_{el} , \quad (6a)$$

$$T_{el} = 2t_2 + 2t_2 G_0 T_1 , \quad (6b)$$

where we have used the fact that particles 2 and 3 are identical.

If one uses a separable potential to represent the nucleon-nucleon interaction, then the T matrix t_1 can be written in the three-body Hilbert space, in the form

$$t_1 = (E_0 - h_0) | \phi_0 \rangle \frac{B}{q_0^2/2\nu - H_0 + i\epsilon} \langle \phi_0 | (E_0 - h_0) , \quad (7)$$

where B is an operator in the same space as H_0, such that

$$B | \vec{q} \rangle = b(E_0 + q_0^2/2\nu - q^2/2\nu) | \vec{q} \rangle , \quad (8)$$

and the function $b(E)$ has a discontinuity across the positive real axis, as well as satisfies

$$b(E_0) = 1 . \quad (9)$$

Using Eq. (7) for t_1 into Eqs. (6), we get that

$$T_1 | \phi_0 \rangle = (E_0 - h_0) | \phi_0 \rangle \frac{B}{q_0^2/2\nu - H_0 + i\epsilon} \times \langle \phi_0 | (E_0 - h_0) G_0 T_{el} | \phi_0 \rangle , \quad (10)$$

where the amplitude

$$\langle \phi_0 | (E_0 - h_0) G_0 T_{el} | \phi_0 \rangle ,$$

satisfies the integral equation

$$\langle \phi_0 | (E_0 - h_0) G_0 T_{el} | \phi_0 \rangle = 2 \langle \phi_0 | (E_0 - h_0) G_0 t_2 | \phi_0 \rangle + 2 \langle \phi_0 | (E_0 - h_0) G_0 t_2 G_0 (E_0 - h_0) | \phi_0 \rangle \times \frac{B}{q_0^2/2\nu - H_0 + i\epsilon} \langle \phi_0 | (E_0 - h_0) G_0 T_{el} | \phi_0 \rangle . \quad (11)$$

If we take matrix elements of Eq. (11) with respect to eigenstates of the operator H_0, we get the Lippmann-Schwinger equation

$$T(\vec{q}\,', \vec{q}_0) = V(\vec{q}\,', \vec{q}_0) + \int d\vec{q}\, V(\vec{q}\,' \vec{q}) \frac{1}{q_0^2/2\nu - q^2/2\nu + i\epsilon} T(\vec{q}, \vec{q}_0) , \quad (12)$$

where

$$T(\vec{q}\,', \vec{q}_0) = \langle \vec{q}\,' \phi_0 | (E_0 - h_0) G_0 T_{el} | \vec{q}_0 \phi_0 \rangle , \quad (13)$$

and the effective potential is

$$V(\vec{q}\,', \vec{q}) = 2 \langle \vec{q}\,' \phi_0 | (E_0 - h_0) G_0 t_2 G_0 (E_0 - h_0) | \vec{q} \phi_0 \rangle b(E_0 + q_0^2/2\nu - q^2/2\nu) . \quad (14)$$

If $\vec{q}_0'$ is an on-shell momentum, we can see from Eq. (5) that the amplitude (13) is the elastic scattering amplitude (1), since

$$T(\vec{q}_0', \vec{q}_0) = \langle \vec{q}_0' \phi_0 | (E_0 - h_0) G_0 T_{el} | \vec{q}_0 \phi_0 \rangle = \langle \vec{q}_0' \phi_0 | T_{el} | \vec{q}_0 \phi_0 \rangle = F_{el}(\vec{q}_0', \vec{q}_0) . \quad (15)$$

We have solved the Faddeev equations with and without reflections as given by Eqs. (3) and (6), respectively, using the fully relativistic formalism described in Ref. 4. We have used the separable T matrices of Ref. 4 to represent the pion-nucleon amplitude t_2 in all the S and P spin and isospin channels, while for the nucleon-nucleon amplitude t_1 we took the $^3S_1$ separable model of Yamaguchi,[8] corrected for relativistic effects by using the prescription of minimal relativity.[9] We show our results in Fig. 1, where we see that the effect of the reflections is at most a 20% correction, which is much smaller than that found in the fixed-nucleon calculations.[1,2] This effect is also smaller than that reported by Banerjee and Wallace[3] for the case of oxygen including nucleon recoil, so that the behavior of the higher-order corrections is different for the deuteron than for heavier nuclei.

If we iterate Eqs. (6), we see that the elastic scattering amplitude without reflections is of the form

$$T_{el} = 2t_2 + 4t_2 G_0 t_1 G_0 t_2 + 8t_2 G_0 t_1 G_0 t_2 G_0 t_1 G_0 t_2 + \cdots , \quad (16)$$

in which the first term is the single-scattering term, the second term is a double scattering term such that between the two pion-nucleon interactions

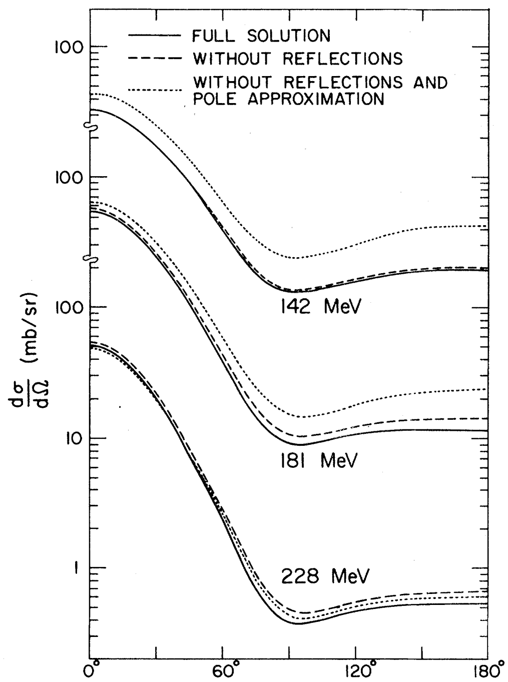

FIG. 1. Pion-deuteron differential cross sections in the c.m. system, for three different laboratory kinetic energies of the pion. The solid lines are the results of the Faddeev equations with reflections given by Eqs. (3), the dashed lines the results of the Faddeev equations without reflections given by Eqs. (6) or (12)−(14), and the dotted lines the results of Eqs. (12)−(14) with $b(E)=1$.

there is a nucleon-nucleon interaction, the third term is a triple scattering term such that between any two consecutive pion-nucleon interactions there is a nucleon-nucleon interaction, and so on. If one writes a spectral representation of the nucleon-nucleon T matrix t_1, one has a term due to the ground state, as well as terms coming from all the excited states, so that if one assumes that the ground-state transitions dominate in the intermediate states, the T matrix t_1 given by Eq. (7), can be written in the approximate form

$$t_1 \approx (E_0-h_0)\,|\phi_0\rangle \frac{1}{{q_0}^2/2\nu - H_0 + i\epsilon} \times \langle\phi_0|\,(E_0-h_0)\,, \qquad (17)$$

in which we have neglected the contribution from all the (continuum) excited states. The approximation (17), is similar in spirit to that used in the formulation of the first-order optical potential, in which one assumes that after each pion-nucleon interaction the nucleus goes back to the ground state (this approximation is usually taken together with the closure approximation, in which the pion-nucleon amplitude in the nuclear medium is replaced by the free pion-nucleon amplitude). Thus, one may get an idea of how important the transitions into nuclear excited states are, by comparing the results obtained using the exact T matrix (7), with those obtained using the pole approximation (17). Therefore, we show in Fig. 1 the results without reflections, using the pole approximation (17), that is taking $b(E)=1$ in Eq. (14), which as we see is not a very good approximation, so that the contribution of the nucleon-nucleon excited states is very important.

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