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Interacting-boson-model and open-shell-Tamm-Dancoff-approximation interpretations of quadrupole collectivity

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The collective *S-D* states used by Otsuka, Arima, and Iachello in their microscopic theory of the interacting boson model are reinterpreted in terms of conventional concepts of collective motion. It is shown that a suitable seniority projected one-body excitation operator acting on a state composed of *S* and *D* pairs excites a new *D* pair. The excitation operator can be calculated using an open-shell Tamm-Dancoff method. The *S-D* states are then pictured as arising from many-quantum excitations of a single collective mode.

[NUCLEAR STRUCTURE IBM shell model subspace constructed; open-shell TDA excitation operator used. Seniority scheme, pairing, and quadrupole degrees of freedom for describing low energy collective motion.]

I. INTRODUCTION

The interacting boson model (IBM) introduced by Arima and Iachello[1] has had encouraging success in phenomenological applications to low lying collective states of even-even nuclei.[2,3] More recently Otsuka, Arima, Iachello, and Talmi,[4] and Otsuka, Arima, and Iachello[5] (OAI), among others, have attempted a microscopic justification of the IBM, raising the possibility that the boson parameters can be calculated from the underlying fermion Hamiltonian. Here we pursue this question following the general lines of the OAI program.

The so called *S-D* basis states mapped by OAI onto boson states are many-fermion states of even particle number that span a subspace of the entire shell model space. This collective subspace contains pairing and quadrupole degrees of freedom appropriate for describing low energy collective motion. The *S-D* states are composed of zero-coupled (*S*) and two-coupled (*D*) fermion pair creation operators. The attempt to describe quadrupole collective motion in terms of pair creation operators is one of the most novel features of the OAI approach. In this picture, a typical collective excitation of a given nucleus is seen as the destruction of a zero-coupled pair and the creation of a two-coupled pair. On the other hand, a conventional picture of collective motion is that an excited state is generated by a one-body excitation operator. This is the viewpoint adopted in closed-shell random phase approximation (RPA) calculations, for example, where the excitation operator creates a particle and a hole. The extent to which these two types of excitations are equivalent is not obvious.

In the present paper we reconcile the pair picture of collective motion with the conventional excitation operator picture, through an equivalence theorem. The theorem shows that there exists a number conserving operator, which, acting on an *S-D* state, simply transforms an *S* pair into a *D* pair. Only one *S-D* state has seniority zero; if this state is chosen as a reference state or "vibrational vacuum," then any desired state of the *S-D* type can be obtained by multiple application of the excitation operator to the reference state.

The excitation operator used in the equivalence theorem is composed of a one-body operator with arbitrary coefficients and seniority projection operators, such that an excitation of a state of good seniority increases the seniority by two. We propose an open-shell Tamm-Dancoff-approximation (TDA) calculation to determine the coefficients in the excitation operator and therefore the structure of the *D* pair. The method is a simplification of Nomura's open-shell RPA method.[6] Because of the correspondence between seniority and quasiparticle number, the method presented here corresponds to TDA using quasiparticles (QTDA), just as Nomura's method corresponds to QRPA. Many successful applications of QTDA and QRPA are reported in the literature.[7]

In view of the equivalence theorem, it is reasonable to interpret the D pair as a collective mode excited by an operator that generalizes the particle-hole excitation operators of conventional closed-shell TDA. In constructing the S-D states, the conventional approach is further extended by allowing multiple excitations of a single collective mode.

In Sec. II we briefly review the OAI program. In Sec. III we present some definitions and discuss some basic aspects of seniority and quasispin. Section IV presents the equivalence theorem, which is proved in the Appendix. We use the equivalence theorem to construct a collective S-D space, in Sec. V. In Sec. VI we present an open-shell TDA method of obtaining the coefficients in the excitation operator. Our conclusions are summarized in Sec. VII.

II. REVIEW OF THE OAI APPROACH

For orientation, we briefly review the microscopic approach of OAI to the interacting boson model for even numbers of identical nucleons.[5] OAI begin by constructing a basis for the S-D subspace by using operators S_+ and $D^\dagger_\mu$. These create specific types of fermion pairs with positive parity and definite angular momentum L: S_+ creates an $L = 0$ pair, while $D^\dagger_\mu$ creates an $L = 2$ pair. Built into $D^\dagger_\mu$ is a seniority projection operator such that $D^\dagger_\mu$ acting on an n-particle state of maximum seniority $(v=n)$ gives a new state of maximum seniority, $v'=v+2$. Beginning with the vacuum $(n=0)$ state, one applies $v/2$ $D^\dagger$ operators followed by $(n-v)/2$ S_+ operators. The resulting state has seniority v and is of the type

$$|\Psi nv\rangle=(S_+)^{(n-v)/2}(D^\dagger_{\mu_1}\cdots D^\dagger_{\mu_{v/2}})|0\rangle\ . \quad (1)$$

The next stage of the OAI construction is to take appropriate linear combinations of states of this form, to get a set of orthonormal states of good angular momentum and seniority which span the collective space. These are then mapped onto corresponding states of $n/2$ mathematical bosons created and destroyed by abstract operators $s^\dagger$, $d^\dagger_\mu$ and s, d_μ, whose only nonvanishing commutators are

$$[s,s^\dagger]=1\ ,\ \ [d_\mu\ ,d^\dagger_{\mu'}]=\delta_{\mu\mu'}\ . \quad (2)$$

As a result of this mapping the boson image of any fermion operator can be constructed so that corresponding boson and S-D matrix elements are equal, although in practice the construction is approximated. Thus all observables, in particular the energy levels, can be calculated in the boson space. In lowest nontrivial order, the boson image of the fermion Hamiltonian has the well known interacting-boson form.

In one way the theory can be seen as a quite straightforward extension of the seniority scheme to include $L = 2$ pairs. However, the novel application of pair creation operators to describe quadrupole collective motion is central to the IBM, and is one of its most original and characteristic features. It is therefore of interest to relate the D pair to a more conventional picture of collective excitation. While it has been pointed out in general terms that states of the S-D type do contain parts with particle-hole collectivity,[8,9] it is not obvious that the IBM picture of collective motion can be reconciled with the traditional "particle-hole" view, in which collective motion is generated by a one-body excitation operator. The present paper attempts such a reconcilation.

The original application of the OAI program was to identical nucleons in a single shell[4,5] of large j. More recently calculations have been done involving many degenerate shells and both protons and neutrons.[9,10] In this "degenerate approximation," the single particle levels are degenerate, and the S and D pairs are taken as favored pairs (i.e., the lowest energy $v=0$, $L=0$, and $v=2$, $L=2$ eigenstates of the surface delta interaction in the two particle space) and states of good total quasispin are constructed. This approach is generalized somewhat when both neutrons and protons are included, but the S and D pairs are still represented by favored pairs. A related approach is used by Duval and Barrett in their nondegenerate j-shell calculations.[11]

III. SENIORITY AND QUASISPIN

We introduce an operator $a^\dagger_{im}$ that creates a spherical shell model single particle state with quantum numbers $i\equiv(\gamma_i,j_i)$ and m. The labels j_i and m refer to angular momentum, while γ_i specifies all other necessary single particle labels, such as the radial quantum number, the parity, and the isospin label (n or p). The spherical tensor fermion destruction operator is

$$\tilde{a}_{im}\equiv(-1)^{i-m}a_{i-m}\ , \quad (3)$$

in terms of $a_{im}\equiv(a^\dagger_{im})^\dagger$.

The operators a_{im} and $\tilde{a}_{im}$ are the $+\frac{1}{2}$ and $-\frac{1}{2}$ components of a quasispin tensor of rank $\frac{1}{2}$. In the ith shell the quasispin vector $\vec{S}_i$ has components defined by

$$S_{i+}=\sqrt{\Omega_i/2}(a_i^\dagger \times a_i^\dagger)_0^0,\ \ S_{i-}=(S_{i+})^\dagger \ , \tag{4}$$

and

$$S_{i0}=(N_i-\Omega_i)/2 \ , \tag{5}$$

where

$$\Omega_i=(2j_i+1)/2 \ , \ \ N_i=\sum_m a_{im}^\dagger a_{im} \ . \tag{6}$$

The generators of the quasispin SU(2) group satisfy commutation relations completely analogous to those between the angular momentum components J_z, $J_\pm=J_x\pm iJ_y$. A state with good quasispin in shell i is by definition an eigenvector of $\vec{S}_i^2$; its eigenvalue is $(\Omega_i-v_i)/2$, where v_i is the seniority in the ith shell.[12]

The zero coupled pair operator used in constructing S-D states is

$$S_+=\sum_i \alpha_i S_{i+} \ . \tag{7}$$

In the special case that all α_i are equal, this is a multiple of the raising operator of the total quasispin operator $\vec{S}=\sum_i \vec{S}_i$.

Each S-D state is required to have good total seniority. Although seniority and quasispin are equivalent in a single j shell, good quasispin is a more restrictive requirement in the multishell case. Thus the S-D states considered here appropriately generalize those of OAI, but are not eigenvectors of $\vec{S}^2$. Instead we define the total seniority to be the sum of the single shell seniorities,

$$v=\sum_i v_i \ , \tag{8}$$

where $v_i=\Omega_i-2S_i$ is the number of particles in shell i that are not coupled to zero. States of different v are, of course, orthogonal.

The D pair is so constructed that an n particle state composed of only D pairs has maximum seniority, i.e., $v=n$. A maximum seniority state has no zero-coupled pairs and is annihilated by $(S_+)^\dagger$ for any choice of α_i. Since S_+ does not change v, the seniority of an S-D state is twice the number of D pairs.

IV. EQUIVALENCE THEOREM

Consider a number-conserving excitation operator of the form:

$$Q^\dagger=\sum_{v'} P_{v'+2}\Omega_{LM}^\dagger P_{v'} \ . \tag{9}$$

Here the orthogonal projection operator P_v deletes all but the seniority v component of a state on which it acts, and

$$\Omega_{LM}^\dagger=\sum_{ij} x_{ij} U_{ij}^{LM} \tag{10}$$

is an arbitrary one-body operator, which is a linear combination (with coefficients x_{ij}) of the elementary one-body tensor operators

$$U_{ij}^{LM}=(a_i^\dagger \times \tilde{a}_j)_M^L \ . \tag{11}$$

Let $Q^\dagger$ act on an n particle state of seniority v, of the form

$$|\Psi nv\rangle=(S_+)^{(n-v)/2}|\Psi vv\rangle \ . \tag{12}$$

The result can be written as follows:

$$Q^\dagger|\Psi nv\rangle=-\tfrac{1}{2}(n-v)(S_+)^{(n-v-2)/2}D_{LM}^\dagger|\Psi vv\rangle \ , \tag{13}$$

where

$$D_{LM}^\dagger=\sum_{v'} P_{v'+2}\sum_{ij}\beta_{ij}(a_i^\dagger \times a_j^\dagger)_M^L P_{v'} \ , \tag{14}$$

and

$$\beta_{ij}=x_{ij}\alpha_j \ . \tag{15}$$

We refer to Eqs. (13)–(15) as the "equivalence theorem." Its proof, along with the outline of a more general version, is given in the Appendix.

As appears in Eq. (13), the action of $Q^\dagger$ has changed an S pair to a D pair of the form given in Eq. (14). Also, the resulting state has seniority $v+2$ because of the seniority projection operators in $Q^\dagger$. This D creation operator is a minor formal improvement over the D operator of OAI, because it has bosonlike commutation relations with the S_+ operator, i.e.,

$$[D_{LM}^\dagger,S_+]=0 \ . \tag{16}$$

This can be seen by applying the commutator to an arbitrary n particle state of seniority v, noting that S_+ commutes with the seniority projection operators contained in $D_{LM}^\dagger$, and recalling that simple pair creation operators commute. While the D operator of Eq. (14) differs from that of OAI, because the seniority projection is expressed in a more general way, there is no practical difference since OAI already specify that the D operator must act only on maximum seniority states. In the case where good total quasispin is required, seniority projection can of course be replaced by quasispin coupling.

An ambiguity in the relation between the coefficients in $Q^\dagger$ and $D_{LM}^\dagger$ arises, because terms like U_{ij}^{LM} and U_{ji}^{LM} in the excitation operator respectively induce terms like $(a_i^\dagger \times a_j^\dagger)_M^L$ and $(a_j^\dagger \times a_i^\dagger)_M^L = (-1)^{i-j-L}(a_i^\dagger \times a_j^\dagger)_M^L$ in the D operator, and so do not give independent contributions to $D_{LM}^\dagger$. To explore this further assume that some convenient ordering of the j shells has been chosen, and that the D operator has been manipulated so that the sum in Eq. (14) is restricted to $i \geq j$. Then for arbitrary x_{ij}, the corresponding β's are

$$\beta_{ii} = x_{ii}\alpha_i \; , \tag{17}$$

for $i = j$, and

$$\beta_{ij} = x_{ij}\alpha_j + (-1)^{i-j-L} x_{ji}\alpha_i \tag{18}$$

for $i > j$. Equations (17) and (18) show that given values for the x's the β's are uniquely determined. However, the converse is not true. Although x_{ii} is uniquely related to β_{ii}, β_{ij} is a linear combination of x_{ij} and x_{ji}. Thus a continuous family of excitation operators exists that all give the same D pair.

To eliminate this ambiguity, a convention can be chosen that restricts the x_{ij}. In Sec. VI we shall use the convention $x_{ij} = 0$ for $i < j$. In this case, Eq. (15) holds and can be solved for x_{ij} in terms of β_{ij} for $i \geq j$. Therefore, this convention results in no loss of generality, assuming of course that α_j never vanishes.

V. CONSTRUCTION OF THE COLLECTIVE SPACE

In this section we show how the S-D states can be obtained using the equivalence theorem, and investigate the role of seniority projection. We do not discuss how to obtain an orthonormal set of collective basis states of good angular momentum since this is discussed by OAI.[5] Also we will usually suppress the angular momentum labels of the D pair operator, as we have done with the $Q^\dagger$ operator. For the description of quadrupole collectivity, $Q^\dagger$ and $D^\dagger$ should have $L = 2$, although formally any value of L is possible. When, for the sake of clarity, the magnetic quantum number of these operators is needed, a subscript μ will be supplied.

It is assumed that a set of α's and β's have been obtained so that S_+ and $Q^\dagger$ are fully defined. The coefficients in $Q^\dagger$ can be obtained as in Sec. VI or by any other method, e.g., they may be taken from the one particle matrix elements of a one-body operator such as the isoscalar electric quadrupole operator. We do not consider how the α's can be obtained; however, the "number operator approximation" of Otsuka and Arima seems appropriate.[13]

The n particle seniority zero S-D state is constructed by applying $n/2$ S_+ operators to the vacuum state. An S-D state of seniority v is obtained by applying $v/2$ $Q^\dagger$ operators to the seniority zero S-D state, i.e.,

$$(Q_{\mu_1}^\dagger \cdots Q_{\mu_{v/2}}^\dagger)(S_+)^{n/2}|0\rangle = K(S_+)^{(n-v)/2}(D_{\mu_1}^\dagger \cdots D_{\mu_{v/2}}^\dagger)|0\rangle \; , \tag{19}$$

where K is a constant. The equality arises from successive applications of the equivalence theorem. The $v = 0$, S-D state is of the form required by the equivalence theorem, i.e., it is composed of $n/2$ S pairs acting on a maximum seniority ($v = n$) state. Then $Q_{\mu_{v/2}}^\dagger$ acting on this state gives a $v = 2$ state with $(n-2)/2$ S pairs and one D pair, which is also of the form required by the equivalence theorem. This process can continue since each application of a $Q^\dagger$ operator gives a state with S_+ operators acting on a maximum seniority state, which is composed of D operators.

Equation (19) shows how any state of the S-D type can be constructed by means of the excitation operator $Q^\dagger$. This prompts the interpretation of the $L = 2$ D pair as the result of a number-conserving quadrupole excitation of the S pairs. In this picture any S-D state can be reached by at most $n/2$ steps of collective excitation starting from the $v = 0$ state. This last statement extends the RPA or TDA picture of collective excitations since the reference state can be excited more than once.

In the excitation operator picture, the simple pair-product structure of the S-D states arises from the form of the seniority zero reference state and the seniority projection operators in $Q^\dagger$. It is the seniority projection that constrains $Q^\dagger$ to convert S pairs into D pairs, but leave D pairs unaffected. To see this, consider the result, which is derived in the Appendix, of an $L \neq 0$ one-body operator acting on the seniority v state of Eq. (12):

$$\Omega_{LM}^{\dagger}\,|\,\Psi n v\,\rangle = (S_+)^{(n-v)/2}\Omega_{LM}^{\dagger}\,|\,\Psi v v\,\rangle - \tfrac{1}{2}(n-v)(S_+)^{(n-v-2)/2} \times \sum_{ij} x_{ij}\alpha_j[a_i^{\dagger}\times a_j^{\dagger}]_M^L\,|\,\Psi v v\,\rangle \ . \quad (20)$$

An excitation by $Q^{\dagger}$ retains only the $v+2$ component of this equation, and therefore eliminates the first term of this equation. Thus $Q^{\dagger}$ leaves preexisting $L\neq 0$ pairs (e.g., D pairs) unaffected.

In the second term new $L\neq 0$ pairs appear. Although the generated pairs have nonzero angular momentum, the antisymmetry of the state induces seniority v and $v-2$ components in addition to the $v+2$ component. These components, which contain indirectly generated $L=0$ pairs, are also eliminated, so that the resulting state has separate monopole and quadrupole (if $L=2$) degrees of freedom.

In contrast to $Q^{\dagger}$ is the excitation operator used by Nomura in his open-shell RPA method.[6] In addition to a seniority raising term, Nomura's operator contains a seniority reducing term; this operator, acting on an *S-D* state of seniority v, gives a state with seniority $v+2$ and $v-2$ components. Inspection of Eq. (20) shows that a state of the *S-D* type does not result.

VI. GENERALIZED TDA CALCULATION

In this section, a generalized TDA calculation is proposed as a way to determine the coefficients in the excitation operator $Q^{\dagger}$, and so extend the vibrational interpretation of the collective *S-D* state along conventional lines. A $Q^{\dagger}$ operator obtained in this way is clearly analogous to those obtained in conventional RPA and TDA. The procedure is a number conserving counterpart to TDA calculations using quasiparticles. Instead of using the Bardeen-Cooper-Schrieffer (BCS) ground state, we incorporate the pairing correlations in the seniority zero *S-D* state, and instead of exciting two quasiparticles, $Q^{\dagger}$ raises the seniority by two.

We use the equations-of-motion formalism of Rowe.[14] In this method one attempts to find the coefficients in an excitation operator of some assumed form, given a Hamiltonian H and a reference state $|\,\hat{0}\rangle$. Rowe uses the equation

$$\langle \hat{0}\,|\,[Q_\delta,H,Q_\gamma^{\dagger}]\,|\,\hat{0}\rangle = E_\gamma \langle \hat{0}\,|\,[Q_\delta,Q_\gamma^{\dagger}]\,|\,\hat{0}\rangle \ , \quad (21)$$

where the subscript γ denotes a particular solution of Eq. (21) with excitation energy, E_γ, and $Q_\delta=(Q_\delta^{\dagger})^{\dagger}$. The symmetrized double commutator is defined as

$$[A,B,C]=\tfrac{1}{2}\{[A,[B,C]]+[[A,B],C]\} \ . \quad (22)$$

In obtaining Eq. (21) Rowe assumes that

$$Q_\delta\,|\,\hat{0}\rangle = 0 \ , \quad (23)$$

which implies that the reference state is a vibrational vacuum.

We take as reference state the seniority zero *S-D* state

$$|\,\hat{0}\rangle = N_0^{-1}(S_+)^{n/2}\,|\,0\rangle \ . \quad (24)$$

The excitation operator $Q_\gamma^{\dagger}$ is that of Eq. (9); however, we rewrite it in the normalized form

$$Q_\gamma^{\dagger} = \sum_{v'} P_{v'+2} \sum_{i\geq j} x_{ij}^{\gamma}(N_{ij})^{-1}[a_i^{\dagger}\times \tilde{a}_j]_M^L P_{v'} \ , \quad (25)$$

where

$$N_{ij} = \langle \hat{0}\,|\,U_{ij}^{LM\dagger}U_{ij}^{LM}\,|\,\hat{0}\rangle^{1/2} \ , \quad (26)$$

and we have used the convention $i\geq j$ mentioned in Sec. IV.

The excited state

$$Q_\gamma^{\dagger}\,|\,\hat{0}\rangle = -\tfrac{1}{2}n(S_+)^{(n-2)/2}\sum_{i\geq j} x_{ij}^{\gamma}\alpha_j(N_{ij})^{-1}P_2(a_i^{\dagger}\times a_j^{\dagger})_M^L\,|\,0\rangle \quad (27)$$

is a sum over the complete set of orthonormal seniority two states allowed in the *S-D* space with amplitudes x_{ij}. The seniority projection operator is necessary only when $L=0$ and $i=j$ since then an $L=0$ pair proportional to S_{i+} has been generated. We ask that the excited states also be orthonormal so that

$$\langle \hat{0}\,|\,Q_\delta Q_\gamma^{\dagger}\,|\,\hat{0}\rangle = \sum_{i\geq j} x_{ij}^{\delta *}x_{ij}^{\gamma} = \delta_{\delta\gamma} \ . \quad (28)$$

[Note that the sum should be restricted to $i>j$ for $L=0$ or odd L. The $L=0$ case has been explained. If L is odd, the terms with $i=j$ in Eq. (27) are zero since $(a_i^\dagger \times a_i^\dagger)_M^L=0$ for L odd.]

With the choice of operator (25) and reference state (24), Eq. (23) is identically satisfied, and we have that

$$\langle \hat{0} | [Q_\delta, Q_\gamma^\dagger] | \hat{0} \rangle = \delta_{\delta\gamma} . \tag{29}$$

Inserting Eq. (25) into (21) and using Eq. (29) gives

$$\sum_{i \geq j} \sum_{k \geq l} x_{kl}^{\delta *} M_{klij} x_{ij}^{\gamma} = E_\gamma \delta_{\delta\gamma} , \tag{30}$$

where

$$M_{klij} = (N_{kl})^{-1} \langle \hat{0} | \left[\sum_{v} P_v U_{kl}^{LM\dagger} P_{v+2}, H, \sum_{v'} P_{v'+2} U_{ij}^{LM} P_{v'} \right] | \hat{0} \rangle (N_{ij})^{-1} . \tag{31}$$

The x's form the unitary matrix that diagonalizes the matrix M defined by (31). Since M is real and Hermitian, the unitary matrix is real orthogonal and the excitation energies, E_γ, are real.

A distinguishing feature of the above calculation is that the excitation operator, Eq. (25), is not purely one-body in contrast to those used in standard RPA and TDA. A related, but more complicated, excitation operator is used in the open-shell RPA method of Nomura,[6] which is designed to accommodate positive parity multipole excitations of open-shell nuclei, assuming a general seniority zero reference state. In that method, Nomura uses an excitation operator that has, among other features, an additional seniority reducing term, which is seen to be analogous to the backward going excitations of standard RPA. Since we include only a seniority raising or "forward going" term, the method above can be called an open-shell TDA calculation.

Another feature of this method is that neither the reference state $|\hat{0}\rangle$ nor the excited state $Q_\gamma^\dagger |\hat{0}\rangle$ is required to be an approximate eigenstate of the Hamiltonian, although they may be for vibrational nuclei. The purpose of the above calculation is to obtain decoupled collective nuclear modes generated by $Q_\gamma^\dagger$. Approximate eigenstates are to result from a diagonalization of H in the collective space generated by the excitation operator.

Each of the excitation operators obtained by solving Eq. (31) generates a distinct D pair and has a distinct excitation energy. In order to construct a set of S-D states with only one type of D pair, one solution must be selected. For low energy collective states, the solution with lowest excitation energy should be selected. The collective modes are assumed to be weakly coupled so that the inclusion of other modes should give a small correction to the approximate eigenstates of H.

VII. CONCLUSIONS

The OAI version of IBM maps the states of the S-D collective fermion subspace onto states of an abstract boson space. As a result of the seniority properties of the S-D basis, the OAI mapping permits one to approximately simulate fermion calculations across a whole range of nuclei. Despite the practical advantages of the mapping, we have concentrated on the structure of the S-D subspace, in which the physical content of the OAI approach must be sought.

The S-D space of OAI has been given a physical interpretation based on an extension of traditional concepts of collective excitations. The use of seniority projection in the excitation operator is crucial in establishing a relation between the TDA picture and the particle-pair picture of OAI. The relation is expressed in the equivalence theorem, Eq. (13), where one sees that a suitably defined number conserving quadrupole operator simply converts an S pair into a D pair. This suggests that a D pair can be interpreted as the result of a number conserving quadrupole excitation of the nucleus. The adaptation of existing equations of motion techniques to obtain the coefficients strengthens this interpretation.

An unconventional feature of the excitation operator is that it is not purely one-body, but also contains seniority projection operators. The seniority projection constrains the excitation to act on S pairs without affecting preexisting D pairs. It is only with this constraint (or truncation) that the boson-number conservation feature of the IBM results. The extent to which this seniority truncation influences the accuracy of the method should be investigated further.

A more general feature not present in standard

RPA and TDA calculations is the use of many-quantum states in a set of basis states. States with different numbers of quadrupole quanta are allowed to interact via a diagonalization of the fermion Hamiltonian in the *S-D* subspace. However, this feature is also found in conventional anharmonic vibrator models,[15] so it cannot be regarded as unique to the IBA. It seems that the IBM is more closely related to conventional methods than appears at first glance.

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APPENDIX: PROOF OF THE EQUIVALENCE THEOREM

The proof of Eq. (13) employs the commutator of the S_+ operator with an elementary one-body operator

$$[S_+ ,(a_i^\dagger \times \tilde{a}_j)_M^L]=\alpha_j(a_i^\dagger \times a_j^\dagger)_M^L \ . \tag{A1}$$

By iterating this equation, one obtains

$$[(S_+)^m ,(a_i^\dagger \times \tilde{a}_j)_M^L]=m(S_+)^{m-1}\alpha_j(a_i^\dagger \times a_j^\dagger)_M^L \ , \tag{A2}$$

where we have also used

$$[S_+ ,(a_i^\dagger \times a_j^\dagger)_M^L]=0 \ . \tag{A3}$$

By multiplying Eq. (A2) by x_{ij} and summing over i and j, one obtains

$$[(S_+)^m ,\Omega_{LM}^\dagger] =m(S_+)^{m-1}\sum_{ij} x_{ij}\alpha_j(a_i^\dagger \times a_j^\dagger)_M^L \ . \tag{A4}$$

Replacing m by $(n-v)/2$ and applying Eq. (A4) to a maximum seniority state with seniority v gives, after some rearrangement,

$$\Omega_{LM}^\dagger \mid \Psi n v\rangle = (S_+)^{(n-v)/2}\Omega_{LM}^\dagger \mid \Psi v v\rangle - \tfrac{1}{2}(n-v)(S_+)^{(n-v-2)/2} \times \sum_{ij} x_{ij}\alpha_j(a_i^\dagger \times a_j^\dagger)_M^L \mid \Psi v v\rangle \ . \tag{A5}$$

The next step is to delete all but the $v+2$ component of Eq. (A5). Since the S_+ operator does not affect the seniority of a state, we need only consider whether $\Omega_{LM}^\dagger \mid \Psi vv\rangle$ and $(a_i^\dagger \times a_j^\dagger)_M^L \mid \Psi vv\rangle$ can have a seniority $v+2$ component. Clearly, the former cannot have such a component since it is a v particle state and it is not possible for it to have $v+2$ particles not coupled to zero. On the other hand $(a_i^\dagger \times a_j^\dagger)_M^L \mid \Psi vv\rangle$ is a $v+2$ particle state and can have a seniority $v+2$ component. Therefore the seniority projection eliminates the first term of Eq. (A5) and retains part of the second term. The result can be expressed as in Eq. (13).

The equivalence theorem can be generalized slightly by allowing each S_+ operator to have a different structure. Let $(S_+)^{(n-v)/2}$ on the left hand side of Eq. (13) be replaced by $\prod_a S_+^a$, where $S_+^a = \sum_i \alpha_i^a S_{i+}$. Then the right hand side of (13) would be replaced by a sum with $(n-v)/2$ terms such that in each term one factor S_+^a is replaced by a corresponding factor $D_{LM}^{a\dagger}$. In this case, the excitation operator generates many different *D*'s because the *S* pairs are not identical.

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