Band structure in ^{66}Ge

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The levels of ^{66}Ge have been investigated using in-beam γ-ray spectroscopy techniques via the ^{58}Ni(^{10}B,pn)^{66}Ge reaction. The energies, relative intensities, angular distributions, and directional correlation ratios of the γ rays were measured. The proposed level scheme exhibits three positive parity bands and one negative parity band. The positive parity levels to the 8^+ yrast and the quasigamma band energies are compared with interacting-boson-model calculations. Two-quasiparticle-plus-rotor model calculations suggest that the (8^+) and (10^+) states at 5534.5 and 6504.8 keV, respectively, are members of the $(g_{9/2})^2$ quasiproton band; the 7^-, (9^-) states are members of a more completely aligned band of $(g_{9/2},f_{5/2})$ quasiproton configuration and the 5^- state at 3684.0 keV is a more partially aligned state of $(g_{9/2},p_{3/2})$ configuration.

NUCLEAR REACTIONS ^{58}Ni(^{10}B,pn), $E(^{10}\text{B})=31$ MeV; measured E_γ, I_γ, $\gamma(\theta)$, DCO; deduced ^{66}Ge levels, J, π and γ branching; performed IBA and two-quasiparticle-plus-rotor model calculations.

I. INTRODUCTION

Systematic studies[1] of even-even nuclei in the $A=70$ region have revealed multiple band structures including ground-state bands, gamma-type vibrational bands, positive and negative parity two-quasiproton and quasineutron bands, and evidence for the coexistence of the spherical and deformed shape in the same nucleus. This region is a rich testing ground of nuclear models. To extend our knowledge further from stability we have investigated the nuclear structure of ^{66}Ge via the ^{58}Ni(^{10}B,pn) reaction using standard in-beam γ-ray spectroscopic techniques. Results of earlier studies as well as those of two other simultaneous investigations[2–7] are compared with our work.

We have observed the (8^+) member of the ground-state band the (8^+) and (10^+) members of another positive-parity band and a γ-type vibrational band. Interacting boson model[8] calculations were made, and they nicely reproduce the positive parity levels to the yrast (8^+) level and members of the γ-type vibrational band to the 5^+ levels. As observed in ^{68}Ge (Ref. 9), decoupling of two protons by Coriolis antipairing force and alignment of the angular momentum of the excited quasiparticles on the $g_{9/2}$ orbital closer to the core angular momentum are believed to be the origin of the positive parity band built on the (8_2^+) state at 5534.5 keV. Two-quasiparticle-plus-rotor model[10] calculations were also made. These calculations support the interpretation of the even-parity band and indicate that the 7^- and (9^-) states have about 95% contribution from the $(g_{9/2},f_{5/2})$ proton configuration, and the 5^- state at 3684.0 keV has about a 97% of $(g_{9/2},f_{3/2})$ proton configuration.

II. EXPERIMENTAL PROCEDURES AND RESULTS

The ^{66}Ge nuclei were produced by the compound nuclear reaction ^{58}Ni(^{10}B,pn) with a ^{10}B bombarding energy of 31 MeV at the Oak Ridge National Laboratory EN tandem accelerator. A thin target of thickness $\sim 200\ \mu\text{g/cm}^2$ with natural Ni backing was used for the reaction. Angular distributions $\gamma(\theta)$, γ-γ coincidence and γ-γ directional correlation measurements were performed at a beam energy of 31.0 MeV. The γ rays were observed with two Ge(Li) detectors with resolution of 2.5 keV at 1330 keV and efficiencies of 18% of a 7.6 cm×7.6 cm NaI detector.

The singles γ-ray spectrum observed at 55° with respect to the beam direction is shown in Fig. 1. Most of the γ rays of ^{66}Ge are labeled with energies in keV; the 511.0 keV γ ray does not belong to ^{66}Ge. It is evident from Fig. 1 that the spectrum is very complex and is contaminated by γ rays from other reaction channels. Theoretical calculations of cross sections for different possible channels predict that the production of ^{66}Ge nuclei is only about 4% of the total cross section.[11] The 338.6-, 886.5-, and 981.1-keV γ rays were highly contaminated, and they could not be resolved in our study. The γ-γ coincidence measurements were made with two Ge(Li) detectors positioned at 0° and 90° to the beam direction with a target to detector distance of 5 cm. The coincidence events were stored on a CDC 3200 computer through the buffer memory of a PDP11 computer as a 1024×1024 matrix. Coincidence spectra were obtained by setting windows on the peaks and on their neighboring background regions. Selected coincidence spectra are shown in Figs. 2(a) and (b).

The γ-γ coincidence relations are expressed in Table I. The energy levels of ^{66}Ge were deduced from these coincidence relations. The energies and relative intensities of the transitions were determined from the γ-γ coincidence and angular distribution measurements.

Gamma-ray angular distribution measurements $\gamma(\theta)$ include data taken at 0°, 55°, and 90° relative to the incident beam direction. A second Ge(Li) detector was placed at 135° to serve as a monitor for normalization purposes. The current integrator values measured using the target assembly as a Faraday cup were also used for the same purpose. In order to extract the angular distribution coefficients A_k, the experimental data were fitted to a polynomial:

$$W(\theta)=\sum_{k=0,2,4} Q_k A_k P_k(\cos\theta) ,$$

where $W(\theta)$ is the normalized γ-ray intensity at the angle θ to the beam direction and Q_k is the solid angle parameter. The angular distribution coefficients were corrected for the finite solid angle of the detectors.

The angular distribution coefficients, the spin sequences, and the multipolarities are summarized in

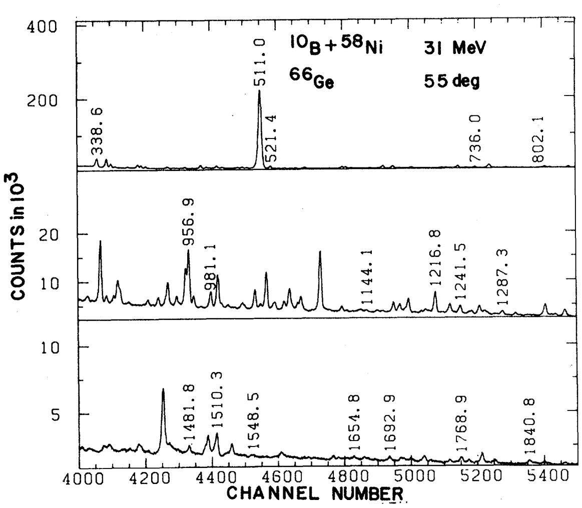

FIG. 1. Singles gamma ray spectrum from the ^{58}Ni(^{10}B,pn) reaction at a beam energy of 31.0 MeV. Energies in keV are given above the peaks of ^{66}Ge (except 511).

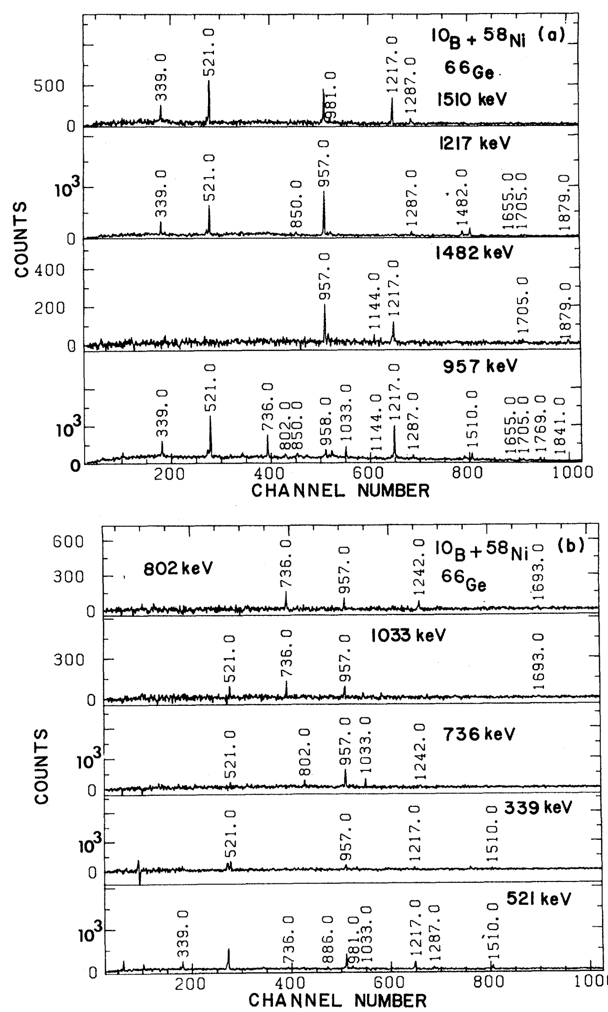

FIG. 2. (a) Selected coincidence spectra corrected for background. (b) Selected coincidence spectra corrected for background.

TABLE I. Coincidence relation for the gamma-ray transitions in $^{66}_{32}$Ge. Underline means strong coincidence lines. ? means not observed because of the weak branching ratio, high background, and/or low efficiency.

Gate (keV)	Energies of gamma-ray peaks found in coincidence spectra (keV)
338.6	<u>521.4</u>, 736.0?, 886.5, 949.4, <u>956.9-957.8</u>, 1032.6, <u>1216.8</u>, <u>1510.3</u>, 1692.9?, 1768.9, 1840.8?
521.4	<u>338.6</u>, 736.0, <u>886.5</u>, 949.4, <u>956.9-957.8</u> <u>981.1</u>, 1032.6, <u>1216.8</u>, <u>1287.3</u>, <u>1510.3</u>, 1692.9, 1768.9, 1840.8
736.0	338.6, <u>521.4</u>, <u>802.1</u>, 949.4, <u>956.9-957.8</u>, 981.1, <u>1032.6</u>, <u>1241.5</u>, 1287.3, 1548.5
802.1	<u>736.0</u>, <u>956.9</u>, <u>1241.5</u>
850.5	<u>956.9</u>, <u>1216.8</u>
886.5	338.6, <u>521.4</u>, 949.4, <u>956.9</u>, 981.1, 1287.3, <u>1840.8</u>
949.4	<u>338.6</u>, <u>521.4</u>, 736.0, 886.5, <u>956.9-957.8</u>, 1032.6?, <u>1216.8</u>, <u>1510.3</u>, 1692.9?, 1768.9?, 1840.8
956.9 } 957.8 }	<u>338.6</u>, <u>521.4</u>, <u>736.0</u>, <u>802.1</u>, <u>850.5</u>, 886.5, 949.4, <u>956.9-957.8</u>, 970.4, <u>981.1</u>, <u>1032.6</u>, 1144.1, <u>1216.8</u>, 1241.5, <u>1287.3</u>, <u>1481.8</u>, <u>1510.3</u>, 1548.5, 1654.8, 1692.9, <u>1705.0</u>, <u>1768.9</u>, <u>1840.8</u>
970.4	<u>956.9</u>, <u>1216.8</u>, <u>1481.8</u>, 1879.0
981.1	<u>521.4</u>, 736.0, 886.5, <u>956.9-957.8</u>, 1032.6?, <u>1216.8</u>, <u>1510.3</u>, 1692.9?, 1768.9?, 1840.8?
1032.6	338.6?, <u>521.4</u>, <u>736.0</u>, 949.4?, <u>956.9-957.8</u>, 981.1?, 1287.3?, 1692.9
1144.1	<u>956.9</u>, <u>1216.8</u>, <u>1481.8</u>, <u>1705.0</u>
1216.8	<u>338.6</u>, <u>521.4</u>, 850.5, 949.4, <u>956.9</u>, 970.4, <u>981.1</u>, 1144.1, <u>1287.3</u>, <u>1481.8</u>, <u>1510.3</u>, 1654.8, <u>1705.0</u>, 1879.0
1241.5	<u>736.0</u>, <u>802.1</u>, <u>956.9</u>, 1692.9
1287.3	<u>521.4</u>, 736.0, 886.5, <u>956.9-957.8</u>, 1032.6, <u>1216.8</u>, <u>1510.3</u>, 1692.9?, 1768.9?, 1840.8
1481.8	<u>956.9</u>, 970.4, 1144.1, <u>1216.8</u>, <u>1705.0</u>, 1879.0,
1510.3	<u>338.6</u>, <u>521.4</u>, 949.4, <u>956.9</u>, <u>981.1</u>, <u>1216.8</u>, <u>1287.3</u>
1548.5	736.0, 956.9
1654.8	956.9, 1216.8
1692.9	338.6?, 521.4, <u>802.1</u>, 949.4?, 957.8, 981.1?, <u>1032.6</u>, 1241.5, 1287.3?, 1548.5?
1705.0	<u>956.9</u>, 1144.1, <u>1216.8</u>, <u>1481.8</u>
1768.9	338.6?, 521.4, 949.4, <u>956.9-957.8</u>, 981.1, 1287.3
1840.8	<u>338.6</u>, <u>521.4</u>, <u>886.5</u>, 949.4?, <u>956.9</u>, <u>981.1</u>, 1287.3
1879.0	<u>956.9</u>, <u>1216.8</u>, <u>1481.8</u>, <u>970.4</u>

Table II. The alignment parameters and the σ/J values are summarized in Table III. The distribution width σ for the population for magnetic substates "m" was defined by assuming the Gaussian distribution.[12]

Directional-correlation ratios from oriented nuclei (DCO) were extracted from the γ-γ coincidence measurements. The experimental DCO ratio is de-

TABLE II. Angular distribution coefficients of γ rays in ^{66}Ge.

E_γ (keV)	A_2	A_4	J_i^π	J_f^π	E/M	δ^a	α_2
521.4	0.341(21)	−0.089(35)	7^-	5^-	$E2$		0.77(5)
736.0	−0.037(19)	−0.026(29)	2_2^+	2_1^+	$E2$, $M1$	$25(^{65}_{11})$	$0.41(^{52}_{11})$
802.1	−0.330(37)	0.066(60)	3^+	2_2^+	$E2$, $M1$	−2.2(2)	0.44(4)
850.5	0.051(32)	0.109(36)	$3^-,5^-$	4^+			
956.9	0.237(13)	−0.066(19)	2^+	0^+	$E2$		0.33(2)
970.4	0.40(15)	−0.01(19)	(10^+)	(8^+)	$(E2)$		$0.96(^{4}_{37})$
1032.6	0.272(19)	−0.071(27)	4_2^+	2_2^+	$E2$		0.53(4)
1216.8	0.256(18)	−0.047(27)	4^+	2^+	$E2$		0.50(4)
1241.5	0.409(46)	−0.245(75)	5^+	3^+	$E2$		0.86(10)
1287.3	0.16(14)	−0.06(15)					
1481.8	0.282(45)	−0.096(46)	6^+	4^+	$E2$		0.62(10)
1510.3	−0.071(24)	0.013(36)	5^-	4^+	$E1$, $M2$	0.10(2)	$0.61(^{32}_{16})$
1768.9	0.26(7)	−0.10(12)	4_2^+	2_1^+	$E2$		0.51(13)
1840.8	−0.14(6)	0.02(9)	3^-	2^+	$E1$, $M2$	0.04(7)	$0.46(^{35}_{14})$
1654.8	−0.35(6)	0.12(9)	$3^-,5^-$	4^+			

$^a\delta^2 = |\langle j_f || \lambda+1 || j_i \rangle / \langle j_f || \lambda || j_i \rangle|^2$.

fined as $R = W(90°,0°)/W(0°,90°)$, where the two angles refer to the first and second members of the γ ray cascade for the respective choice of angles. The theoretical DCO ratios were calculated with α_2, α_4, and δ values obtained from the angular distribution measurements. The experimental and theoretical DCO ratios are given in Table IV.

Spin and parities have been assigned to most of the low energy states, based on the angular distribution measurements; these are consistent with the DCO ratio measurements as well as with the systematics of the $A = 70$ region. The level energies, energies, and intensities of transitions, and the spin-parities are summarized in Fig. 3.

III. SPIN AND PARITY ASSIGNMENT

A. 956.9(1)-keV level

The $2^+ \rightarrow 0^+$ transition energy of ^{66}Ge has been given in Ref. 3 as 957.4 keV. Our analysis of the 957-keV doublet as a single peak yields A_2,A_4 values of 0.237(13) and −0.066(19), respectively.

TABLE III. Attenuation coefficients extracted from the ^{66}Ge γ-ray angular distributions.

Level (keV)	E (keV)	$J_i^\pi \rightarrow J_f^\pi$	α_2 (exp.)	α_4 (exp.)	$(\sigma/J)_2$ (exp.)	$(\sigma/J)_4$ (exp.)	$\alpha_4{}^a$ (calc.)
956.9	956.9	2_1^+ 0^+	0.033(2)	0.038(12)	0.69(3)	0.64(4)	0.028(4)
1692.7	736.0	2_2^+ 2_1^+	$0.41(^{51}_{11})$	$0.06(^{84}_{6})$	$0.60(^{14}_{38})$	$2(^{\infty}_{2})$	$0.05(^{73}_{3})$
2173.7	1216.8	4^+ 2^+	0.50(4)	0.13(8)	0.49(3)	$0.49(^{15}_{8})$	0.13(3)
2495.0	802.1	3^+ 2_2^+	0.44(4)	$0.11(^{17}_{11})$	0.55(4)	$0.52(^{\infty}_{15})$	0.09(2)
2725.6	1032.6	4_2^+ 2_2^+	0.53(4)	0.19(8)	0.47(4)	0.44(7)	0.15(3)
2725.6	1768.9	4_2^+ 2_1^+	0.51(13)	$0.26(^{23}_{26})$	0.47(12)	$0.38(^{\infty}_{38})$	$0.14(^{10}_{7})$
2797.7	1840.8	3^- 2^+	$0.46(^{35}_{14})$	$1(^{0}_{1})$	$0.53(^{15}_{25})$	$0.0(^{\infty}_{0})$	$0.10(^{37}_{6})$
3655.5	1481.8	6^+ 4^+	0.61(10)	0.40(19)	0.39(7)	0.32(9)	$0.24(^{12}_{9})$
3684.0	1510.3	5^- 4^+	$0.61(^{31}_{16})$	$1.00(^{00}_{94})$	$0.40(^{12}_{40})$	$0.0(^{24}_{0})$	$0.23(^{76}_{13})$
3736.5	1241.5	5^+ 3^+	0.86(10)	$0.86(^{14}_{27})$	$0.23(^{7}_{11})$	$0.13(^{10}_{13})$	$0.62(^{26}_{20})$
4205.4	521.4	7^- 5^-	0.77(5)	0.41(7)	0.29(4)	0.31(3)	$0.46(^{10}_{7})$

^{a}Calculated from the α_2 values in the column 4 and assuming a Gaussian distribution for the population of the magnetic substates.

TABLE IV. Experimental and theoretical DCO ratios in ^{66}Ge.

E_{γ_1} (keV)	E_{γ_2} (keV)	Spin sequence	$R_{\text{DCO (exp)}}$	R_{DCO} (theor.)[a]
521.4	1510.3	$7^- \to 5^- \to 4^+$	0.54(7)	0.57(5)
521.4	1216.8	$7^- \to 5^- \to 4^+ \to 2^+$	1.08(13)	1.00
802.1	736.0	$3^+ \to 2_2^+ \to 2_1^+$	1.60(57)	1.58(8)
802.1	956.9	$3^+ \to 2_2^+ \to 2_1^+ \to 0^+$	1.53(58)	1.56(15)
1032.6	736.0	$4_2^+ \to 2_2^+ \to 2^+$	0.63(19)	0.66(6)
1216.8	956.9	$4^+ \to 2^+ \to 0^+$	1.03(13)	1.00
1241.50	802.1	$5^+ \to 3^+ \to 2_2^+$	0.40(22)	0.31(5)
1241.50	736.0	$5^+ \to 3^+ \to 2_2^+ \to 2_1^+$	0.50(30)	0.60(5)
1241.50	956.9	$5^+ \to 3^+ \to 2_2^+ \to 2_1^+ \to 0^+$	0.68(22)	0.60(5)
1287.3	521.4		1.57(38)	
1287.3	1510.3		0.75(35)	
1481.8	1216.8	$6^+ \to 4^+ \to 2^+$	1.00(21)	1.00
1481.8	956.9	$6^+ \to 4^+ \to 2^+ \to 0^+$	1.07(27)	1.00
1510.3	1216.8	$5^- \to 4^+ \to 2^+$	1.63(19)	1.66(7)
1510.3	956.9	$5^- \to 4^+ \to 2^+ \to 0^+$	1.72(21)	1.66(7)

[a]Calculated by using α_2, α_4, and δ values from the angular distribution measurements; uncertainties arise from the uncertainties in the α_2, α_4, and δ values.

These data confirm the earlier 2^+ assignment[3,7] for this state.

Contributions from the 957.8 keV γ rays of the above doublet and the radioactive feeding from ^{66}As to the 2_1^+ level in ^{66}Ge, if any, would have had some influence on the A_2,A_4 coefficients. The evidence indicates that there was little or no radioactive decay feeding from the ^{66}As to 2_1^+ level in ^{66}Ge, as expected from the theoretical cross section, and that the 957.8-keV γ-ray intensity contribution is small.

B. 1692.9(1)-keV level

The 1692.9-keV level is depopulated by a 1692.9-keV transition to the 0^+ level as well as by a 736.0-keV transition to the 2^+ level. The first transition excludes the assignment of 3 and 2^-. The systematics in the $A=70$ region favor a spin of 2^+ for this level. The angular distribution fit for the 736.0-keV transition establishes a spin of 2. For spin 2, two good fits were found at two different

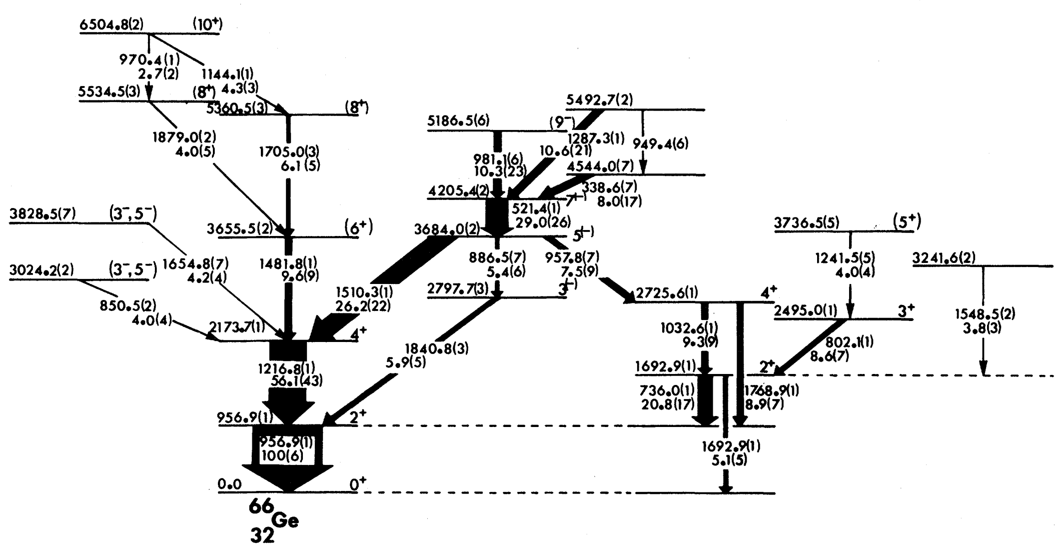

FIG. 3. The energy level diagram of ^{66}Ge obtained from the present experiment. The relative intensities are given below the energies of transitions.

values of δ. The fit found at $\delta=-0.37$ and $\alpha_2=0.70$ was discarded because this α_2 was too large for this level. These results establish a spin-parity of 2^+ for the 1692.9-keV level.

C. 2173.7(1)-keV level

The angular distribution measurement of the 1216.8-keV γ ray, which depopulates this level, allows spins 2 and 4 for this level. But, an assignment of spin 2 requires an α_2 of 0.75, which is inconsistent with the α_2's of the neighboring levels. Therefore, a spin 4 assignment was favored. The experimental DCO ratio strongly supports this assignment, thus confirming the previous assignment[3] of 4^+ to this level.

D. 2495.0(1)-keV level

The positive value of the A_4 coefficient and the large negative A_2 coefficient obtained from the angular distribution measurement of the 802.1-keV γ ray, which depopulates the 2495.0-keV level, indicate that it is a $\Delta J=1$ transition. Even though the statistical uncertainty in the DCO ratio in this case is large, it supports an assignment of spin 3 to this level. Since the mixing ratio of the 802.1-keV $\Delta J=1$ transition was found to be $-2.2(2)$, a positive parity has been assigned to this level.

E. 2725.6(1)-keV level

A chi-square analysis of A_2,A_4 and DCO ratio for the 1032.6-keV transition yields spin 4 for the 2725.6-keV level. This level has been previously tentatively identified as a 4^+ state.[7] Our assignment of spin-parity 4^+ is supported by the angular distribution of the 1768.9-keV γ ray, and as well by the fact that the attenuation coefficient α_2 obtained for this level for the two transitions depopulating it are equal within a relatively small error.

F. 2797.7(3)-keV level

The large negative value of A_2 for the 1840.8-keV γ ray, that deexcites the 2797.7-keV state, eliminates spins 2 and 4, and allows either 1 or 3. A chi-square analysis of the A_2,A_4 values favors very strongly the spin 3, and the systematics in this region agree with this assignment. The best fit was found for a small value of $\delta=0.04(7)$ which indicates $<1\%$ quadrupole contribution. Systematics favor an $E1$ assignment for the 1840.8-keV γ ray and therefore negative parity for the level. Thus, the level at 2797.7-keV has been assigned a spin-parity of 3^-.

G. 3024.2(2)-keV level

The A_2,A_4 values for the 850.5-keV γ ray are in agreement with spin-parities of 3^- and 5^- for the 3024.2-keV state.

H. 3241.6(2)-keV level

No spin assignment could be made for this state because of the large uncertainties in the A_2,A_4 coefficients of the 1548.5-keV γ ray. This γ ray peak was located in a complicated region (see Fig. 1) and was part (the low energy component) of a high intensity contaminant.

I. 3655.5(2)-keV level

Systematics of the medium mass region favor a spin-parity assignment of either 6^+ or 5^- to this level which feeds the 2173.7-keV, 4^+ state. A chi-square analysis of A_2,A_4 values of the 1481.8-keV transition shows that spins 4 [$\delta=1.1(2)$] and 6 are allowed. The two DCO ratio values, neglecting the uncertainty, are strongly in support of spin-parity assignment of 6^+ to this state. If the large uncertainty in the DCO ratios are considered, the 4^+ [DCO ratio$=0.93(6)$] remains as a possibility.

As the A_2,A_4 values and the systematics of this region favor the 6^+ spin-parity assignment very strongly, and the DCO ratios support this assignment, the 3655.5-keV state has been assigned spin-parity 6^+. In Ref. 7 this level was tentatively assigned as 6^+.

J. 3684.0(2)-keV level

This level is depopulated by three transitions with energies of 1510.3, 886.5, and 957.8 keV, respectively, to 4_1^+, 3^-, and 4_2^+ levels. It is unfortunate that angular distribution analysis could not be carried out on the last two of these γ rays for the reasons specified earlier. The γ-decay pattern of this level to levels of known spin-parity exclude spin values 1 and 6 for this level. The A_2,A_4 values for the 1510.3-keV transition allow only spin values of 3 and 5. The DCO-ratio values also were found to agree with the theoretical values for these two spins within the experimental errors. Finally, a chi-square fit was made with A_2,A_4 and the DCO ratios of the 1510.3-keV transition included, as a function of δ. This fit favored spin 5 by a factor of 10 to 1 over spin 3. The best fit was obtained for spin 5 at $\delta=0.10$. This result is in agreement with the systematics in this region. The small δ and systematics support an $E1$ transition. So the level is assigned a spin-parity of 5^-.

K. 3736.5(5)-keV level

The A_2,A_4 values of the 1241.5-keV γ ray that deexcites this level to the 3^+ level at 2495.0 keV allow spins of 3 and 5. The systematics and γ-decay mode strongly favor a 5^+ assignment over $3^\pm$. The DCO ratio measurement with the 1241.5-keV γ ray also supports the 5^+ assignment.

L. 3828.5(7)-keV level

As in the case of the 3024.2-keV state, a unique spin assignment could not be made to this level; 3^- and 5^- assignments are allowed by the A_2,A_4 values of the 1654.8-keV γ ray.

M. 4205.4(2)-keV level

Based on the A_2,A_4 values of the 521.4-keV γ ray that deexcites this state to the 3684.0-keV 5^- level, and the DCO ratio of this transition, spins and parities of 5^- and 7^- are allowed. Systematics favor strongly a spin-parity 7^- over 5^-. Thus, we have assigned 7^- to this state.

N. 4544.0-, 5186.5-, 5360.5-, 5492.7-, 5534.5-, and 6504.8-keV levels

Because the 338.6- and 981.1-keV γ rays were strongly contaminated, the spins of levels at 4544.0 and 5186.5 keV could not be established. Systematics would favor a 9^- assignment for the 5186.5-keV level.

Poor statistics and complications in spectra in the vicinity of the 1705.0-keV γ ray peak (see Fig. 1) hindered the analysis of the angular distribution data. Systematics strongly suggest that the 5360.5-keV level depopulated by this transition should be an 8^+ level, and such an assignment is tentatively made for this level. A tentative assignment of (8^+) to the 5534.5 keV level also was made following the same line of thought.

The complexity in the γ spectrum around the 1287.3-keV peak has introduced very large uncertainty in the A_2,A_4 values. Nevertheless, these values in conjunction with the DCO ratio measurements favor a spin-parity assignment of 7^- to the 5492.7 keV level, but do not exclude 6^- and 8^-. It should be noted that the A_2,A_4 coefficients and the DCO ratio are also consistent, if the large error is considered, with a 9^- assignment; this level is assigned 9^- tentatively in Ref. 7.

Finally, the A_2,A_4 values of the 970.4-keV transition depopulating the 6504.8 keV level suggest that it is an $E2$ transition. The highest level observed, therefore, has been assigned, tentatively, spin-parity of (10^+).

IV. INTERACTING BOSON APPROXIMATION CALCULATIONS

Calculations have been performed for the energy levels of ^{66}Ge based on the interacting boson approximation model (IBA) of collective states.[8] The experimental levels and the corresponding calculated levels are given in Table V, and depicted in Fig.

TABLE V. A comparison of interacting boson approximation model calculated energies with experiment in ^{66}Ge.

Spin	Exp. energy (keV)	Theor. energy (keV)
2_1^+	957	974
2_2^+	1693	1657
3^+	2495	2507
4_1^+	2174	2189
4_2^+	2726	2727
5^+	3737	3750
6^+	3656	3647
8_1^+	5361	5347
(8_2^+)	5535	5845
(10^+)	6505	7290
3^-	2798	2882
5^-	3684	3603
7^-	4205	4294
(9^-)	5187[a], 5493[a]	5788
11^-		7242

[a]Possible candidates.

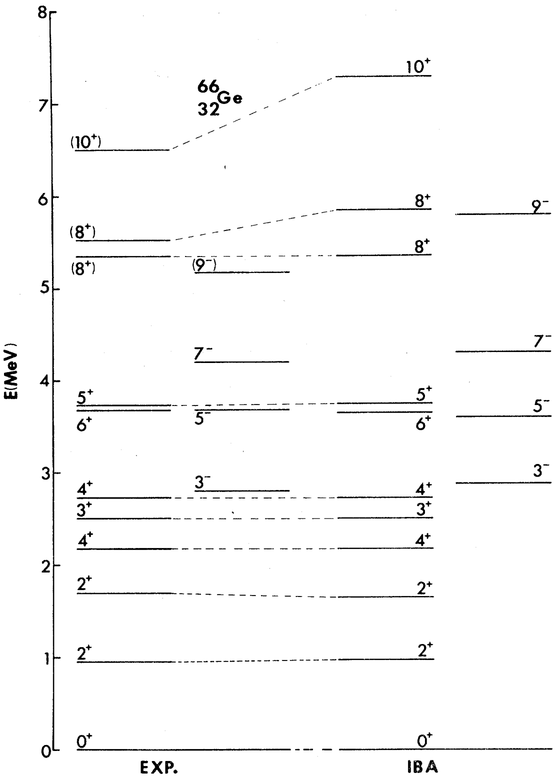

FIG. 4. Interacting boson approximations (IBA) fit to the experimental energy levels in ^{66}Ge.

4. Except for the $(8^+_{1,2})$ and the (10^+) levels, all the other positive parity states were used in the fitting. The IBA parameters obtained for the best fit are given in Table VI. It should be emphasized that the energy of the first excited 0^+ state, had it been observed, would have restricted the c_0 parameter very much; since it is not yet observed, the c_0 parameter was allowed to vary to obtain the best fit for the levels which were believed to be collective in nature.

TABLE VI. IBA parameters for ^{66}Ge.

Positive parity (MeV)	Negative parity (MeV)
$\epsilon=0.97359$	$\epsilon_f=2.88300$
$c_0=-0.87564$	$\chi_1=-0.25300$
$c_2=-0.29022$	$\chi_2=-0.26550$
$c_4=-0.24218$	$\chi_3=-0.13758$
	$\chi_4=0.28475$
	$\chi_5=0.15883$
	$F_3=-0.02803$
	$EPSD=0.40600$

Thus, the energy predicted for the 0^+_1 state should not be considered seriously.

The agreement between the IBA energies and the experimental energies for all the positive parity levels up to the 8^+_1 level is quite good. The energies predicted for the 8^+_2 and 10^+ states are 310 and 784 keV, respectively, higher than the experimental energies. This result clearly indicates that the collective model of interacting s and d bosons does not hold for the experimental 8^+_2 and 10^+ states, which must have another origin.

The plot of the moment of inertia $(2\mathscr{I}/\hbar^2)$ versus the square of the rotational energy[13] for the positive parity levels in ^{66}Ge, Fig. 5, shows that the backbending begins only at the 8^+_2 level. This change supports the interpretation that the 8^+_1 state is a member of the ground band and the 8^+_2 and 10^+ states are not. The experimental 8^+_2 and 10^+ levels can be understood in terms of a rotation aligned band built on a two quasiparticle excitation, as discussed in the next section.

The IBA model not only generates the ground state band, the highest allowed angular momentum states for a given number of d bosons, but also generates the quasigamma vibrational band. In the IBA model, these levels are regarded as allowed angular momentum states which are lower than the highest allowed for a given number of d bosons. The 2^+ bandhead is a two d bosons $(L=2)$ state, while three d bosons generate the 6^+ ground state level and the 3^+, 4^+ levels of the quasigamma vibrational band. The 5^+ level of the quasigamma band cannot be formed from three d bosons $(2n_d-1$ is not allowed); it is a four d bosons state.

Although it appears that the fit of the negative-parity level energy requires eight parameters, they are equivalent, effectively, to only four parameters, ϵ_f, κ, κ', and $EPSD$, where κ is the strength of the

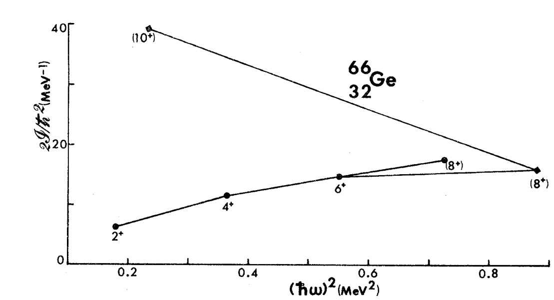

FIG. 5. Plot of the moment of inertia versus square of effective rotational energy for the positive parity cascade in ^{66}Ge.

quadrupole d-boson-quadrupole f-boson $Q_d \cdot Q_f$ interaction; κ' is the strength of the $L_d \cdot L_f$ interaction,[14] where L_d is the angular momentum of a d boson and L_f is that of an f boson; and $EPSD$ corresponds to the higher order term. The parameters $F3$ and $\chi_{1,2,3,4,5}$ in Table VI can be expressed in terms of κ, κ'.

While the energies for the 3^-, 5^-, 7^- levels appear to agree with IBA levels rather well in Fig. 4, in fact, they deviate as much as 90 keV from the experimental levels (see Table V). Moreover, the 9^- level predicted at 5788 keV is far above any of the possible candidates observed, even after including the higher-order term. Actually this should not be surprising. Because only the 3^-, 5^-, and 7^- candidates are available to be used in the IBA fit and there are four parameters, a good fit need not be meaningful. However, failure to fit the levels in such a case is serious. The large 3^--5^- energy gap compared to the smaller 5^--7^- gap suggests that the 5^- state and the state above it are members of a different band. Thus, an IBA fit to this unusual 3^--5^--7^- energy spacing may be expected to be poor and to predict wrong energies for the higher states.

One would expect a negative-parity two-quasiparticle band beginning at 5^-, based on the observation[9] of such bands in ^{68}Ge. Our interpretation is that the 5^- level at 3684.0 keV is, in fact, the beginning of a two quasiparticle aligned band that includes one particle in a $g_{9/2}$ orbital as described in the next section.

As seen above, the interacting boson approximation model reproduces excellently the low-spin, positive parity, low-energy levels which are highly collective in nature. However, it is known that there are additional levels that are not included in this model at present that involve single-particle configuration. For a description of all the states, the IBA should incorporate the single particle Hamiltonian as well.

V. TWO-QUASIPARTICLE-PLUS-ROTOR CALCULATIONS

The observations that the second (8^+) and the (10^+) levels are well below the energies predicted by the IBA collective model and that the transition energy of 970.4 keV between these levels is very close to the yrast $2^+ \rightarrow 0^+$ transition energy, indicates that, as in ^{68}Ge, the Coriolis decoupling of paired particles and the alignment of the angular momentum of these quasiparticles with the core angular momentum plays an important role in the high-spin region. Thus calculations were carried out for ^{66}Ge in the two-quasiparticle-plus-rotor model. First, the ground-state band of ^{66}Ge was fitted with a variable moment of inertia model calculation.[15] As shown in Fig. 6, the energies to 8_1^+ are very well reproduced for values of $\mathscr{I}_0 = 0.731 \times 10^{-3}$ keV^{-1} and $C = 0.402 \times 10^8$ keV^3. The Nilsson levels, as a function of deformation, for protons and neutrons are given in Figs. 7 and 8, respectively. The spherical shell model quantum numbers are given, in these figures, at zero deformation ($\delta = 0$) and the asymptotic quantum numbers are given at $\delta = 0.20$. It is reasonable to assume that $\delta = 0.1$ for ^{66}Ge, since $\delta = 0.10$ had been determined for ^{68}Ge through lifetime measurements of its states.[9]

The Fermi level energies in the Nilsson level plot for protons and neutrons systems are located at about 45.1 and 44.5 MeV, respectively. It is also easy to note that the energies of the $g_{9/2}$ levels for protons, on the average, are about 0.3 MeV closer to the Fermi level than for neutrons. Furthermore, as ^{66}Ge is more neutron deficient, the neutron pairing energy is about 0.1 MeV less than that for protons.[16] These facts are reflected in the two-quasineutron-plus-rotor calculations which predict the 8_2^+ and 10^+ levels at more than 500 keV above the experimental levels.

Our calculations indicate that the (8_2^+) (5535 keV) and (10^+) (6505 keV) states are $(g_{9/2})^2$ two quasiproton levels with calculated 8_2^+ bandhead at 5566 keV and 10^+ level at 6479 keV. In order to fit the data, a gap energy of 0.54 MeV was used in our calculation. The effect of the Coriolis antipairing force, which increases rapidly with increase of angular momentum, can be seen clearly from the reduction of the gap energy (measure of pairing energy) to 0.54 MeV from the ground-state pairing energy of $(\Delta = 12/\sqrt{A}) = 1.48$ MeV. The following Nilsson $g_{9/2}$ configurations were used to construct the basis states: $\frac{9}{2}^+(404)$, $\frac{7}{2}^+(413)$, $\frac{5}{2}^+(422)$,

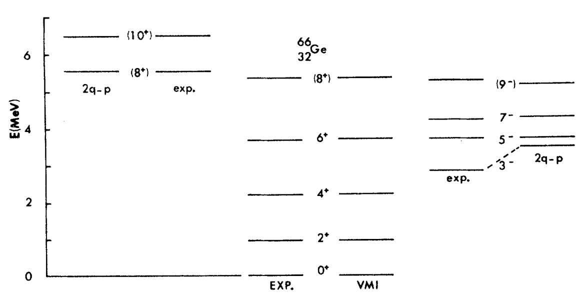

FIG. 6. Two-quasiproton-plus-rotor model levels with the ground state band fit of VMI model.

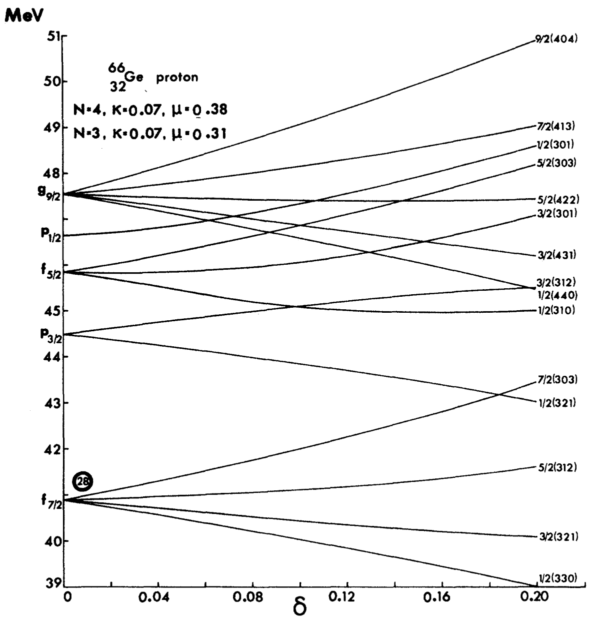

FIG. 7. Proton Nilsson levels for ^{66}Ge nucleus.

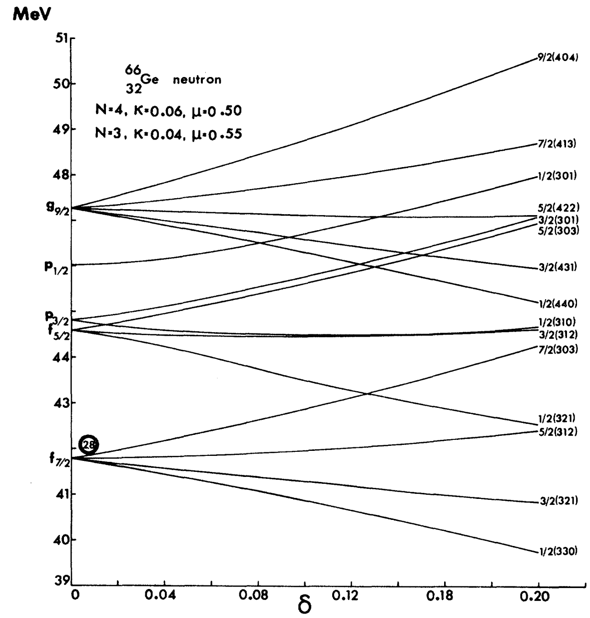

FIG. 8. Neutron Nilsson levels for ^{66}Ge nucleus.

$\frac{3}{2}^+(431)$, and $\frac{1}{2}^+(440)$.

It is very important to note that quantum mechanically, single-particle angular momenta cannot be perfectly aligned to that of the core, because nucleons carry integral spin of $\frac{1}{2}$. The only possible way to create $K=0$ states would be to have two particles in the same Nilsson level. But, the Coriolis antipairing force would be acting to break this pair by exciting one particle to a higher Nilsson level. Since $\frac{1}{2}^+(440)$ and $\frac{3}{2}^+(431)$ states are closer to the Fermi level than the other states, the basis states of $K=1$ and 2 would also contribute significantly to the above 8_2^+ and 10^+ states. Our calculation of the wave functions shows that the $K=0,1,2$ basis state contributions are, respectively, about 23%, 39%, and 23%.

The energy of the 981.1-keV transition to the 7^- level suggests that the level at 5186.5 keV is a 9^- state and the 7^- level is a more completely aligned member of a two-quasiproton negative-parity band.

TABLE VII. Comparison of two-quasiproton calculated levels with experiment in ^{66}Ge.

Spin	Theor. energy (keV)	Exp. energy (keV)
3^-	3439	2798
5^-	3682	3684
6^-	4022	
7^-	4253, 4509	4205, 4544[a]
8^-	4964	
9^-	5141, 5640	5187[a], 5493[a]
10^-	6154	
11^-	6296	
8^+	5566	5535
10^+	6479	6505
12^+	7661	
14^+	9058	

[a]Possible candidates.

TABLE VIII. Two-quasiproton level parameters for ^{66}Ge.

Parameter	Positive parity	Negative parity
Fermi energy	45.07 MeV	45.07 MeV
Gap energy	0.54 MeV	0.54 MeV
$\mathscr{I}_0$	0.731×10^{-3} keV^{-1}	0.92×10^{-3} keV^{1}
C	0.402×10^{8} keV^{3}	0.33×10^{8} keV^{3}

The 5^- level could result from the same particles with less alignment being coupled to the core.

The difference in energy between the Nilsson levels and the Fermi level was considered in choosing the following Nilsson configurations for the construction of the basis states:

$$\tfrac{5}{2}^+(422),\ \tfrac{3}{2}^+(431),\ \tfrac{1}{2}^+(440)\quad g_{9/2}\ \text{levels}$$

$$\tfrac{5}{2}^-(303),\ \tfrac{3}{2}^-(301),\ \tfrac{1}{2}^-(310)\quad f_{5/2}\ \text{levels}$$

$$\tfrac{3}{2}^-(312),\ \tfrac{1}{2}^-(321)\quad p_{3/2}\ \text{levels}\ .$$

In fitting the negative-parity band starting at 5^-, the parameters $\mathscr{I}_0$ and C were allowed to vary as much as 25% from the values obtained from the VMI fit of the ground-state band, since promotion of a pair of particles can change the moment of inertia of this band compared to the ground band. The energies obtained from the two-quasiproton-plus-rotor model calculations are given in Table VII and the parameters are given in Table VIII.

Two-quasiproton-plus-rotor calculation reproduces the 5^-, 7^- levels, and predicts our tentatively assigned 9^- level, very satisfactorily. This agreement is displayed in Fig. 6. Furthermore, it predicts a second less aligned band with 7^- and 9^- levels at 4509 and 5640 keV, respectively. These energies deviate by 35 and 147 keV, respectively, from the second experimental band energies of 4544.0 and 5493.7 keV. However, the quasineutron level calculation, in fact, also reproduces these levels nearly as well with 7^- and 9^- states at energies 4588 and 5452 keV. In the above model, cross transitions from proton states to neutron states or vice versa, would be expected to be very weak, unless there is reasonable amount of mixing. The strong branching from this upper level to the lower band which is assigned as a two quasiproton band indicates that the second band is either a proton band or is a π-ν mixed band. Thus, the actual nature of these levels can be interpreted only after comparing the transition matrix elements and the experimental results.

In the fit to the 5^-, 7^-, and (9^-) levels, the wave-function calculations show that, indeed, the 5^- level is less aligned than the 7^- or (9^-) ones. Alignment in these states can be expressed by the contributions from the basis states of $K=0,1,2$. For the 7^- and (9^-) states the contributions of $K=0,1,2$ basis states are, respectively, about 31%, 45%, and 19%, and that for 5^- are 16%, 41%, and 32%. Furthermore, the calculation reveals that about 86% of the 7^-, (9^-) states arise from the $g_{9/2}+f_{5/2}$ configuration and 97% of the 5^- state arises from the $g_{9/2}+p_{3/2}$ configuration.

In light of these results, we suggest that the 7^- and (9^-) states are members of a more completely aligned $(g_{9/2},f_{5/2})$ two-quasiproton band, and the 5^- state is more partially aligned $(g_{9/2},p_{3/2})$ two quasiproton state.

It is clear that the experimental levels involve both collective and single-particle motions. For a model to explain all of the band structure seen in ^{66}Ge, it must include collective and single-particle descriptions.

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