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Microscopic distorted wave theory of inelastic scattering

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An exact microscopic distorted wave theory of inelastic scattering is formulated which contains the physical picture usually associated with distorted wave approximations without the usual redundancy. This formulation encompasses the inelastic scattering of two fragments, elementary or composite (both with or without the full complexity of inter-fragment Pauli symmetries). The fact that these considerations need not be based upon elementary potential interactions is an indication of the generality of the approach and supports its applicability to inelastic meson scattering. The theory also maintains a description of inelastic scattering which is a natural extension of the description of elastic scattering and it provides a general basis for obtaining truncation models with an explicit distorted wave structure. The distorted wave impulse approximation is presented as an example of a particular truncation/approximation encompassed by this theory and the nature of the distorted waves is explicated.

[NUCLEAR REACTIONS Distorted wave theory, inelastic scattering, multiple scattering, spectator expansion, Pauli exclusion principle, composite particles, unitarity structure.]

I. INTRODUCTION

Applications of the distorted wave approach (DWA) to the calculation of nuclear scattering processes have dominated the literature of nuclear reaction theory for over two decades.[1–40] A DWA has, in fact, been used in calculations of reactions involving nucleonic,[1–18] pionic,[19–30] kaonic,[31–34] and composite[35–40] nuclear probes in various kinematic domains and for reactions as diverse as elastic, inelastic, knockout, and rearrangement scattering. The ubiquity of this method is a consequence of its relative computational simplicity, its very real practical successes, and most importantly the simple but general underlying physical picture. What is lacking is a complete and unambiguous theoretical underpinning.

The physical picture associated with the DWA enjoys nearly universal appreciation and it can be summarized very straightforwardly.[41] Namely, as the projectile approaches the target: (1) it scatters elastically in the "average" field of the target, and this process is described within the framework of a two-body optical potential formalism where the two fragments retain their separate identities; (2) a complicated sequence of scatterings occurs which entails the full complexity of a many-body system and which ultimately yields the (possibly new) projectile-target system in the final (observed) configuration; and (3) the final projectile-target system elastically scatters as the fragments separate and this process is again treated within the framework of an optical potential description. The literature is replete with instances[1–40] of successful applications of the DWA in a variety of physical circumstances. As a concrete example, we note the results[17,18] concerning the excitation of certain states via inelastic proton scattering at bombarding energies ≈ 130 MeV, where it is shown that the plane wave limit of the DWA, together with only the grossest characteristics of a DWA, provides a clear and semiquantitative understanding of the reaction. In view of the legitimate successes of distorted wave approximations, we believe there to be ample motivation for an exact theoretical formalism with an intrinsic distorted wave structure.

In view of the numerous applications, the legitimate successes, and the appealing physical picture of the DWA, it is surprising to find that there exists no satisfactory and general microscopic formalism which is constructed to take appropriate advantage of the DWA. Indeed, the theoretical underpinning of such calculations is most often referenced[4–40] as arising from the Watson[2] or Kerman, McManus, and Thaler (KMT) (Ref. 3) formalism in multiple scattering applications, the Feshbach formalism[42,43] in resonance or doorway applications, or as a "result" of the use of the two-potential formula.[15,44] Each of these contains shortcomings in regard to the establishment of a natural formalism which possesses an intrinsic DWA structure and which provides a means, within this context, to investigate quantitative theoretical questions concerning the origin of the successes and limitations found in the pragmatic computational applications of distorted wave approximations. In particular, the KMT approach is microscopic, but it is explicitly based upon a potential theory of distinguishable particles and, furthermore, it introduces the "excited" distorted wave in an *ad hoc* fashion. The unsymmetrized Feshbach projection operator approach is also based on a potential theory of distinguishable particles, which because of the definition of the projectors, corresponds more closely to a coupled channel approach than to a DWA. The symmetrized version of the Feshbach approach treats Pauli effects, but it is not an optical potential formulation.[45–50] Finally, the use of the two-potential formula does not automatically produce a microscopic theory and theoretical considerations based upon this method have tended to be heuristic, albeit physically motivated.[15,44]

In this paper we restrict ourselves to the consideration of the simplest nonelastic process, namely, inelastic scattering of two fragments. The special characteristic of inelastic scattering which is central to our considerations is that the asymptotic behavior of the initial and final states is determined, even in the fully antisymmetrized circumstance, by the same channel Green's function, so that the initial and final asymptotic channel states are orthogonal eigenstates of the same channel Hamiltonian. An equivalent statement of this feature is that the separation of the full Hilbert space into the spaces spanned by the relative momentum eigenstates of the two fragments built upon their initial and final internal states, respectively, and the complement of these spaces defines an orthogonal decomposition whose three projectors commute with the asymptotic channel Green's function. In this paper, we obtain a general microscopic distorted wave formulation of inelastic scattering which is exact and encompasses: (1) the physical picture associated with DWA; (2) potential scattering of distinguishable projectiles, elementary or composite; (3) mesonic scattering, since it need not be based fundamentally upon potentials; (4) Pauli symmetries arising from the presence of indistinguishable particles; (5) a description of inelastic scattering which is a natural extension of elastic scattering and which intertwines the description of the two processes; and (6) a convenient framework for obtaining truncation models, e.g., multiple scattering, collective, resonance, and doorway models, *within an explicit DWA context.*

The microscopic framework presented here provides a basis through which a better understanding of the DWA as a reaction mechanism may be obtained. This is of particular current interest in inelastic nucleon-nucleus and pion-nucleus reactions, for example, since the information contained in the high precision experimental data cannot be properly disentangled without a better understanding of the underlying theoretical framework.

II. POTENTIAL SCATTERING WITH DISTINGUISHABLE PARTICLES

In this section we restrict ourselves to the elastic and inelastic scattering of two nuclear[51] fragments, which consist of an elementary or composite "projectile" and a nuclear "target," in the circumstance that the elementary interactions between the two fragments are specified by potentials and the constituent particles of one of the fragments are distinguishable from those of the other fragment. These restrictions are relaxed in later sections. The treatment we present in this section is very detailed so that we may relax these restrictions in a completely straightforward manner without losing sight of the essential features.

The transition operator for elastic and inelastic scattering is

$$T^\alpha = V^\alpha + V^\alpha G V^\alpha = V^\alpha G G_\alpha{}^{-1} = G_\alpha{}^{-1} G V^\alpha , \quad (2.1)$$

where V^α is the external channel interaction between the two fragments (that is, the sum of the elementary interactions between the particles in the target and those in the projectile), while the full and channel Green's functions are given by the usual definitions

$$G^{-1}=E-H+i\epsilon , \quad (2.2)$$

and

$$G_\alpha{}^{-1}=G^{-1}+V^\alpha=E-H_\alpha+i\epsilon , \quad (2.3)$$

respectively. In Eqs. (2.2) and (2.3), E is the energy, H is the full Hamiltonian of the system, and H_α is the α channel Hamiltonian, which we obtain by deleting from H the interaction *between* the two fragments. Thus H_α (or G_α) governs the asymptotic behavior of the systen when the fragments are far apart. The transition amplitude for elastic or inelastic scattering from an initial internal state (i) of the two fragments to a final internal state (f) is then given by

$$T_{fi}=\langle \phi_f(\vec{k}\,') \mid T^\alpha \mid \phi_i(\vec{k}) \rangle , \quad (2.4)$$

where $\vec{k}$ and $\vec{k}\,'$ specify the initial and final relative momenta of the two fragments and

$$(E-H_\alpha) \mid \phi_f(\vec{k}\,') \rangle = 0 = (E-H_\alpha) \mid \phi_i(\vec{k}) \rangle . \quad (2.5)$$

The vectors $\mid \phi_j(\vec{k}) \rangle$ are thus eigenstates of H_α; they describe the asymptotic configuration of the system with the two fragments in their respective bound states (labeled j) and their relative plane wave motion (labeled $\vec{k}$). We note that since the internal states of the two fragments are eigenstates of the same Hamiltonian we may write

$$\langle \phi_f(\vec{k}\,') \mid \phi_i(\vec{k}) \rangle = \delta^3(\vec{k}\,'-\vec{k})\delta_{f,i} . \quad (2.6)$$

In fact, since the internal states of the fragments may be taken to be orthonormal, we can decompose the full Hilbert space of the problem, $\mathscr{H}$, into orthogonal subspaces defined through these states. For example, the projector P_j,

$$P_j = \int d^3k \mid \phi_j(\vec{k}) \rangle \langle \phi_j(\vec{k}) \mid , \quad (2.7)$$

projects onto the subspace of $\mathscr{H}$ spanned by the vectors of $\mathscr{H}$ which have the fragments in the internal states j and arbitrary relative momentum of the fragments. It is easy to show from Eqs. (2.6) and (2.7) that

$$P_iP_j=P_jP_i=P_i\delta_{i,j} . \quad (2.8)$$

In the following, we denote by P and $\mid \phi(\vec{k}) \rangle$ without any subscript, the case where the fragments are in their respective ground states and by $j=f$ the final (observed) configuration of the fragments. The complements of the projectors P and P_f are denoted as

$$Q=1-P , \quad (2.9)$$

and

$$Q_f=1-P_f , \quad (2.10)$$

respectively.

We now return to Eq. (2.4) and note that Eq. (2.1) implies

$$\langle \phi_f(\vec{k}\,') \mid T^\alpha \mid \phi(\vec{k}) \rangle = \langle \phi_f(\vec{k}\,') \mid V^\alpha \mid \psi_\alpha \rangle , \quad (2.11)$$

where

$$\mid \psi_\alpha \rangle \equiv GG_\alpha{}^{-1} \mid \phi(\vec{k}) \rangle \quad (2.12)$$

is the exact outgoing scattering eigenfunction of the full Hamiltonian H, which evolves from the asymptotic incident state $\mid \phi(\vec{k}) \rangle$. Evidently $\mid \psi_\alpha \rangle$ contains all the complexity inherent in the many-body problem and so our concern is to find a microscopically truncatable reexpression of Eq. (2.11). We note that the resolvent identity,

$$G=G_\alpha+G_\alpha V^\alpha G , \quad (2.13)$$

together with Eq. (2.12) implies that

$$\mid \psi_\alpha \rangle = \mid \phi(\vec{k}) \rangle + G_\alpha V^\alpha \mid \psi_\alpha \rangle . \quad (2.14)$$

Since we want descriptions of elastic and inelastic scattering which are compatible, we introduce the previously defined projection operators into Eq. (2.14) and observe that

$$P \mid \psi_\alpha \rangle = \mid \phi(\vec{k}) \rangle + G_\alpha P V^\alpha \mid \psi_\alpha \rangle \quad (2.15a)$$

and

$$Q \mid \psi_\alpha \rangle = G_\alpha Q V^\alpha \mid \psi_\alpha \rangle . \quad (2.15b)$$

The bra $P \mid \psi_\alpha \rangle$ is the projection of the complete wave function onto the elastic channel, so that it represents the *exact* portion of the wave function in the asymptotic region.[42] Thus precise knowledge of $P \mid \psi_\alpha \rangle$ implies precise knowledge of the elastic scattering and, conversely, elastic scattering data can be used to check approximations to $P \mid \psi_\alpha \rangle$. Now, we note that $Q \mid \psi_\alpha \rangle$ can be eliminated from Eqs. (2.15). Equation (2.15a) implies that

$$P \mid \psi_\alpha \rangle = \mid \phi(\vec{k}) \rangle + G_\alpha P V^\alpha (P+Q) \mid \psi_\alpha \rangle , \quad (2.16)$$

while from Eq. (2.15b) we have similarly

$$Q \mid \psi_\alpha \rangle = [1-G_\alpha Q V^\alpha]^{-1} G_\alpha Q V^\alpha P \mid \psi_\alpha \rangle . \quad (2.17)$$

Upon noting that

$$|\psi_\alpha\rangle = P|\psi_\alpha\rangle + Q|\psi_\alpha\rangle , \tag{2.18}$$

and using Eq. (2.17), we find that

$$|\psi_\alpha\rangle = [1+(1-G_\alpha QV^\alpha)^{-1}G_\alpha QV^\alpha]P|\psi_\alpha\rangle \tag{2.19}$$

or

$$|\psi_\alpha\rangle = (1-G_\alpha QV^\alpha)^{-1}P|\psi_\alpha\rangle . \tag{2.20}$$

Thus the transition amplitude of Eq. (2.11) can be rewritten as

$$T_{fi} = \langle \phi_f(\vec{k}')|U^\alpha|\chi\rangle , \tag{2.21}$$

where we have defined the optical potential operator U^α by

$$\begin{aligned} U^\alpha &\equiv V^\alpha(1-G_\alpha QV^\alpha)^{-1} \\ &= V^\alpha + V^\alpha Q(G_\alpha{}^{-1} - V^\alpha Q)^{-1}V^\alpha , \end{aligned} \tag{2.22}$$

and the distorted wave $|\chi\rangle$ by

$$|\chi\rangle = P|\chi\rangle \equiv P|\psi_\alpha\rangle . \tag{2.23}$$

If we use Eq. (2.17) in Eq. (2.16), we also infer that

$$|\chi\rangle = |\phi(\vec{k})\rangle + G_\alpha P[V^\alpha(1-G_\alpha QV^\alpha)^{-1}]P|\chi\rangle , \tag{2.24}$$

or

$$|\chi\rangle = |\phi(\vec{k})\rangle + G_\alpha \mathscr{V}_{\rm opt}|\chi\rangle , \tag{2.25}$$

where the optical potential $\mathscr{V}_{\rm opt}$ is defined as

$$\mathscr{V}_{\rm opt} = PU^\alpha P . \tag{2.26}$$

Equation (2.21) involves the matrix element of the complicated operator U^α between a plane wave two-fragment state vector on the left and a distorted wave on the right. Given $\mathscr{V}_{\rm opt}$, the distorted wave is calculated from the two-body relation, Eq. (2.25). In the case where we are interested in elastic scattering, the operator U^α in the transition amplitude of Eq. (2.21) also becomes $\mathscr{V}_{\rm opt}$.

All of the preceding is, of course, very well known. However, we now proceed to convert Eq. (2.21) to the form of the DWA, in which a microscopically meaningful distorted wave appears on the left side of the matrix element in Eq. (2.21) in place of the plane wave. This will, of course, necessarily require a replacement of the operator U^α in Eq. (2.21).

Let us begin by noting that Eq. (2.21) can be rewritten as

$$T_{fi} = \langle \phi_f(\vec{k}')|(1-V^\alpha QG_\alpha)^{-1}V^\alpha|\chi\rangle , \tag{2.27}$$

or

$$T_{fi} = \langle \hat{\psi}_f^{(-)}|V^\alpha|\chi\rangle , \tag{2.28}$$

in which we have defined

$$\langle \hat{\psi}_f^{(-)}| \equiv \langle \phi_f(\vec{k}')|G_\alpha{}^{-1}(G_\alpha{}^{-1}-V^\alpha Q)^{-1} , \tag{2.29}$$

and have explicitly noted the incoming wave boundary conditions. We note that if $\langle \hat{\psi}_f^{(-)}|$ were not such a complicated object, in particular, if it were true that $\langle \hat{\psi}_f^{(-)}| = \langle \hat{\psi}_f^{(-)}|P_f$, then Eq. (2.28) would be in the DWA form and the distorted wave Born approximation (DWBA) would be exact. However, $\langle \hat{\psi}_f^{(-)}|$ is a very complicated state vector and comparison of Eqs. (2.28) and (2.29) with Eqs. (2.11) and (2.12) suggests that we proceed to reduce Eq. (2.29) in analogy with Eqs. (2.12)–(2.23). In fact, it is easily seen in this manner that the analog of Eq. (2.20) is

$$\langle \hat{\psi}_f^{(-)}| = \langle \hat{\psi}_f^{(-)}|P_f[1-V^\alpha QQ_f G_\alpha]^{-1} . \tag{2.30}$$

Thus in place of Eq. (2.21), we have

$$T_{fi} = \langle \chi_f^{(-)}|W^\alpha|\chi\rangle , \tag{2.31}$$

where

$$W^\alpha \equiv [1-V^\alpha QQ_f G_\alpha]^{-1}V^\alpha \tag{2.32}$$

and

$$\langle \chi_f^{(-)}| = \langle \chi_f^{(-)}|P_f \equiv \langle \hat{\psi}_f^{(-)}|P_f . \tag{2.33}$$

In analogy to Eq. (2.24), the final distorted wave $\langle \chi_f^{(-)}|$ satisfies the optical model equation

$$\langle \chi_f^{(-)}| = \langle \phi_f(\vec{k}')| + \langle \chi_f^{(-)}|\mathscr{W}_{\rm opt}G_\alpha , \tag{2.34}$$

where the final state optical potential $\mathscr{W}_{\rm opt}$ is given by

$$\mathscr{W}_{\rm opt} = P_f W^\alpha P_f \tag{2.35}$$

$$= P_f\{V^\alpha + V^\alpha(QQ_f)[G_\alpha{}^{-1} - V^\alpha(QQ_f)]^{-1}V^\alpha\}P_f . \tag{2.36}$$

Equations (2.30)–(2.36), together with Eqs. (2.21)–(2.26), constitute the principal results of this section; they represent an *exact* restatement of the inelastic transition amplitude of Eq. (2.4) with an explicit DWA structure.

Let us review the content of Eqs. (2.30)–(2.36). The DWA matrix element is defined by Eq. (2.31) in terms of the distorted waves $|\chi\rangle$ and $\langle \chi_f^{(-)}|$ and the optical potential operator W^α. The sequence of analysis has led to the replacement of T^{α}, first by U^{α} and then by W^{α}, which is defined by Eq. (2.32). Evidently these replacements are a result of the segregation of some of the physical scattering processes which occurs in going from plane waves to the distorted waves, $|\chi\rangle$ and $\langle\chi_f^{(-)}|$, whose calculation is specified by Eqs. (2.24)–(2.26) and Eqs. (2.33)–(2.36). It is essential to recognize that

$$\langle\chi_f^{(-)}| = \langle\hat{\psi}_f^{(-)}|P_f \neq \langle\phi_f(\vec{k}\,')|G_\alpha^{-1}GP_f \,. \qquad (2.37)$$

Thus the final distorted wave is *not* simply related to true elastic scattering in the final channel, in contrast to the corresponding characteristic of the initial distorted wave. In view of Eqs. (2.32)–(2.36), it is evident that this is a result of the occurrence of the projector Q in Eq. (2.29), which arises so that processes implicit in the calculation of $|\chi\rangle$ are not "double counted" in Eq. (2.31). Thus we see that Eq. (2.37) is a result of a *microscopic and nonredundant* formalism. We note that Eq. (2.37) does not represent an undesirable feature of the formalism, since elastic scattering from excited fragments is not generally accessible experimentally.

In the next section we recast the formulation above from a different but equivalent viewpoint which is useful both in explicating certain important features of the treatment and in generalizing it, as well as for practical applications. Before so doing, however, we digress to note some important aspects of the approach which are made especially evident by the construction of this section. In particular, we examine the graphical structure of Eq. (2.31). If we expand Eq. (2.1) by repeated application of the resolvent identity, Eq. (2.13), we obtain the Born series

$$T^{\alpha} = V^{\alpha}[1 + G_\alpha V^{\alpha} + G_\alpha V^{\alpha} G_\alpha V^{\alpha} + \cdots] \,, \qquad (2.38a)$$

$$= [1 + V^{\alpha} G_\alpha + V^{\alpha} G_\alpha V^{\alpha} G_\alpha + \cdots] V^{\alpha} \,. \qquad (2.38b)$$

On the other hand, expansion of the denominators of Eqs. (2.22) and (2.32) yields

$$U^{\alpha} = V^{\alpha}[1 + QG_\alpha V^{\alpha} + QG_\alpha V^{\alpha} QG_\alpha V^{\alpha} + \cdots] \qquad (2.39)$$

and

$$W^{\alpha} = [1 + V^{\alpha} QQ_f G_\alpha + V^{\alpha} QQ_f G_\alpha V^{\alpha} QQ_f G_\alpha + \cdots] V^{\alpha} \,. \qquad (2.40)$$

Consider now the DWA matrix element of Eq. (2.31). Beginning on the right, we encounter first the distorted wave $|\chi\rangle$ which is obtained by solving Eq. (2.25). Combining Eqs. (2.24) and (2.39), we see that $|\chi\rangle$ is a result of all possible scattering processes which occur through the bracketed quantity in Eq. (2.38a) and which *end* (on the left) in the P space. Intermediate scatterings to the Q space are included in U^{α} [cf. Eq. (2.39)], while intermediate scatterings to the P space are obtained upon solving Eq. (2.24). We encounter next the operator W^{α} which, in view of Eq. (2.40), contains all the intermediate scatterings of Eqs. (2.38) except those to the P and P_f spaces. Finally, we see upon combining Eqs. (2.34) and (2.40) that the distorted wave $\langle\chi_f^{(-)}|$ is a result of all possible intermediate scattering processes except those to the P space, which occur in the bracketed quantity in Eq. (2.38b), and which *end* (on the right) in the P_f space. Scattering to the space $(1-P-P_f)$ is included in W^{α}, while scattering to the P_f space results from solution of Eq. (2.34).

We can now state the microscopic content of the separation of the transition amplitude of Eq. (2.4) into the DWA form of Eq. (2.31) if we insert

$$P + P_f + QQ_f = 1 \qquad (2.41)$$

next to each propagator in Eq. (2.38) [note that each of the projectors in Eq. (2.41) commutes with G_α]. Consider then an arbitrary term (or "graph") of the resulting expression which has the generic form

$$[\ldots V^{\alpha} QG_\alpha V^{\alpha} QG_\alpha \ldots V^{\alpha} G_\alpha] P_f [V^{\alpha} QQ_f G_\alpha \ldots V^{\alpha} QQ_f G_\alpha V^{\alpha}] P [G_\alpha \ldots V^{\alpha} G_\alpha V^{\alpha} \ldots V^{\alpha}] \,. \qquad (2.42)$$

The brackets in Eq. (2.42) correspond to the way this term is segregated in Eq. (2.31) and in Fig. 1. In particular, if we trace such an arbitrary term then the division is as follows. Starting from the left, follow the term until the first P is encountered; then the rest of the term is contained in the right-hand distorted wave. Now backtrack to the left until the first P_f is encountered; then the part of the term still to the left is contained in the left-hand distorted wave. Finally, the part of the term which lies between the two stopping points is a term in the expansion of W^{α}. In view of the above discussion, we draw the following conclusions. The segregation into three factors of Eq. (2.31) represents a complete, concise, and nonredundant explicit DWA formalism which is characterized by a classification

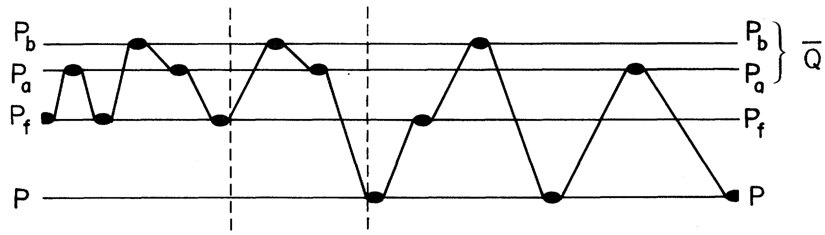

FIG. 1. Graphical presentation of a term in the inelastic matrix element T_{fi}^{inel} for nucleon-nucleus scattering. The term diagrammed is

$$\langle \phi_f(\vec{k}\,') | V^\alpha G_\alpha P_a V^\alpha G_\alpha P_f V^\alpha G_\alpha P_b V^\alpha G_\alpha P_a V^\alpha G_\alpha P_f V^\alpha G_\alpha P_b V^\alpha G_\alpha P_a V^\alpha G_\alpha P V^\alpha G_\alpha P_f V^\alpha G_\alpha P_b V^\alpha G_\alpha P V^\alpha G_\alpha P_a V^\alpha | \phi(\vec{k}) \rangle \ .$$

Each horizontal line indicates a target state projector and is labeled by the projector corresponding to the state. A lozenge which appears on a horizontal line stands for a propagator. The truncated lozenges to the extreme right and left represent $|\phi(\vec{k})\rangle$ and $\langle \phi_f(\vec{k}\,')|$, respectively. An inclined line which joins two lozenges stands for the external interaction V^α. Thus reading from left to right we immediately see how this diagram can be viewed as a graphical representation of the above matrix element. The vertical dashed lines show the segregation of this diagram into three distinct parts. That part of the graph which appears between the two vertical dashed lines is treated as part of the transition operator in the distorted wave matrix element. That part of the graph which appears to the right of both vertical dashed lines is absorbed into the initial distorted wave. That part of the graph which appears to the left of both vertical dashed lines is absorbed into the final distorted wave.

and grouping of the intermediate scatterings in terms of the subspaces of the full Hilbert space into which the scattering occurs. Furthermore, the two distorted wave problems contain an explicit decomposition of the full Hilbert space into orthogonal subspaces and this divides the problem of determining the distorted waves into two components. First one obtains the optical potential operator and then one solves the appropriate optical model equations; these separate problems are confined to disjoint subspaces. In the following sections we refer to the results of this paragraph as specifying the structural and segregational characteristics of the formalism.

III. REFORMULATION AND UNITARITY AND HERMITICITY STRUCTURE

In this section we derive an alternative treatment to that of the previous section. The two formulations are completely equivalent within the restrictions of the previous section, but the approach obtained below allows us to remove these restrictions straightforwardly. Furthermore, it is advantageous for the discussion of the unitarity and Hermiticity structure of our approach and this leads to further insights.

We begin by recalling that [cf. Eqs. (2.1), (2.22), and (2.32)]

$$T^\alpha = V^\alpha + V^\alpha G V^\alpha \ , \tag{3.1}$$

$$U^\alpha = V^\alpha + U^\alpha G_\alpha Q V^\alpha \tag{3.2}$$

$$= V^\alpha + V^\alpha G_\alpha Q U^\alpha \tag{3.3}$$

and

$$W^\alpha = V^\alpha + V^\alpha G_\alpha \bar{Q} W^\alpha \tag{3.4}$$

$$= V^\alpha + W^\alpha G_\alpha \bar{Q} V^\alpha \ , \tag{3.5}$$

where for convenience we have defined

$$\bar{Q} = Q Q_f = 1 - P - P_f \equiv 1 - \bar{P} \ . \tag{3.6}$$

Application of the resolvent identity Eq. (2.13) to Eq. (3.1) yields the familiar Lippmann-Schwinger equation for T^α,

$$T^\alpha = V^\alpha + V^\alpha G_\alpha T^\alpha \ . \tag{3.7}$$

We note from Eqs. (3.2) and (3.5) that

$$[1 + U^\alpha G_\alpha Q] V^\alpha = U^\alpha \tag{3.8}$$

and

$$[1 + W^\alpha G_\alpha \bar{Q}] V^\alpha = W^\alpha \ , \tag{3.9}$$

so that if we multiply Eq. (3.7) on the left by the bracketed quantities in these expressions we obtain

$$T^\alpha = U^\alpha + U^\alpha P G_\alpha T^\alpha \tag{3.10}$$

and

$$T^\alpha = W^\alpha + W^\alpha \bar{P} G_\alpha T^\alpha \ , \tag{3.11}$$

respectively. We also note that similar multiplication of Eq. (3.3) by the bracketed quantity in Eq. (3.9) yields

$$U^\alpha = W^\alpha + W^\alpha G_\alpha P_f U^\alpha \tag{3.12a}$$

$$=W^{\alpha}+U^{\alpha}G_{\alpha}P_{f}W^{\alpha} . \tag{3.12b}$$

Evidently, if we regard Eqs. (3.10) and (3.11) as the *definitions* of the operators U^{α} and W^{α}, then these equations represent a concise alternative formulation of the considerations of Sec. II. Equations (3.12) then provide the connection between the fundamentally important operators of the DWA which was constructed in the previous section.

In order to emphasize the completeness of the formulation based directly upon Eqs. (3.10)–(3.12), we outline the derivation of the DWA result of Sec. II, where T^{α} is given by Eq. (3.1). It follows immediately from Eq. (3.10) that

$$T^{\alpha}|\phi(\vec{k})\rangle=U^{\alpha}P|\psi_{\alpha}\rangle=U^{\alpha}|\chi\rangle , \tag{3.13}$$

where

$$|\chi\rangle=P|\psi_{\alpha}\rangle\equiv[1+PG_{\alpha}T^{\alpha}]|\phi(\vec{k})\rangle , \tag{3.14}$$

and it is easily seen from Eqs. (2.1) and (2.12) that this definition of $P|\psi_{\alpha}\rangle$ coincides with that of Sec. II. Equation (3.13) yields the expression for the transition amplitude given in Eq. (2.21), while the optical model equation for $P|\psi_{\alpha}\rangle$, Eq. (2.25), is obtained upon insertion of Eq. (3.13) into Eq. (3.14). In analogy to the use of Eq. (3.10) to introduce the right-hand distorted wave $|\chi\rangle$, we then employ Eq. (3.12b) in order to introduce the left-hand distorted wave $\langle\chi_f^{(-)}|$. In particular, we find from Eq. (3.12b) that

$$\langle\phi_f(\vec{k}\,')|U^{\alpha}=\langle\hat{\psi}^{(-)}|P_fW^{\alpha}$$
$$=\langle\chi_f^{(-)}|W^{\alpha} , \tag{3.15}$$

where

$$\langle\chi_f^{(-)}|=\langle\hat{\psi}^{(-)}|P_f$$
$$\equiv\langle\phi_f(\vec{k}\,')|[1+U^{\alpha}P_fG_{\alpha}] , \tag{3.16}$$

and it is easily seen that this definition of $\langle\hat{\psi}^{(-)}|P_f$ is equivalent to that of Sec. II. Combination of Eqs. (2.21) and (3.15) then yields the DWA expression of the transition amplitude given in Eq. (2.31), and the use of Eq. (3.15) in Eq. (3.16) yields the optical model equation for $\langle\hat{\psi}^{(-)}|P_f$, Eq. (2.34). Thus it is evident that the alternative approach of Eqs. (3.10)–(3.12) is completely equivalent to that of Sec. II in the case where the transition operator is given by Eq. (3.1). However, this alternative approach to the formulation of an exact DWA for the inelastic scattering problem is more general than that of Sec. II since, as is evident from Eqs. (3.10)–(3.12), it is not predicated on the assumption of a specific form for the transition operator. In fact, it is clear that the form of the rest of the approach, Eqs. (3.13)–(3.16) and their consequences, does not depend on the specific form of the transition operator. In particular, the interaction between the constituent particles of the two fragments need not be specified by potentials, so that the present formulation appears to be advantageous for applications to inelastic meson scattering. Furthermore, the introduction of interfragment Pauli symmetries into the formalism is facilitated by this approach, as will be shown in the next section.

Let us consider now the unitarity and Hermiticity properties of the DWA of this section in the particular case where T^{α} is given by Eq. (3.1). The properties to be established provide a restatement of the important structural and segregational aspects discussed in Sec. II. The insights so obtained generalize to the next section where they play a central role when the full complexity of interfragment antisymmetrization is incorporated into the formalism. We begin by generalizing our notation to make the dependence of the various operators on the (complex) parametric energy, z, explicit, for example,

$$G^{-1}(z)=z-H . \tag{3.17}$$

We also adopt the notational form

$$G(+)\equiv G(E+i\epsilon)=G , \tag{3.18}$$

$$G(-)\equiv G(E-i\epsilon)=G^{\dagger} , \tag{3.19}$$

which makes the outgoing or incoming wave boundary conditions, respectively, inherent in the various operators explicit. In Eq. (3.18) we have noted that the (+) operators are understood in the absence of an explicit label, in agreement with the previous notation. Since

$$T^{\alpha}(z)=V^{\alpha}+V^{\alpha}G(z)V^{\alpha} , \tag{3.20}$$

we have

$$T^{\alpha}(+)=V^{\alpha}+V^{\alpha}G(+)V^{\alpha} \tag{3.21}$$

and

$$T^{\alpha}(-)=V^{\alpha}+V^{\alpha}G(-)V^{\alpha} . \tag{3.22}$$

From Eqs. (3.21) and (3.22) we see that $T^{\alpha}(z)$ is Hermitian analytic, viz.,

$$T^{\alpha}(+)=T^{\alpha}(-)^{\dagger} , \tag{3.23}$$

due to the fact that $G(z)$ possesses this characteristic. Furthermore, it follows from Eqs. (3.21) and

(3.22) that T^α satisfies

$$T^\alpha(+)-T^\alpha(-)=-2\pi i V^\alpha \delta(E-H)V^\alpha \quad (3.24)$$

$$=-2\pi i \sum_n V^\alpha | \psi_n^{(+)}\rangle \times\delta(E-E_n)\langle \psi_n^{(+)} | V^\alpha , \quad (3.25)$$

where we have introduced a complete set $\{ | \psi_n^{(+)}\rangle \}$ of eigenstates of the full Hamiltonian with outgoing wave boundary conditions. The sum in Eq. (3.25) implies integration in the case of continuum solutions. To each eigenstate of H in Eq. (3.25) we can associate an asymptotic configuration governed by an asymptotic channel Hamiltonian H_γ, so that we find that T^α satisfies the off-shell discontinuity relation

$$T^\alpha(+)-T^\alpha(-)=\sum_\gamma T^{\alpha\gamma}(+)\Delta_\gamma[T^{\alpha\gamma}(+)]^\dagger , \quad (3.26)$$

where we have introduced

$$\Delta_\gamma \equiv -2\pi i \mathscr{P}_\gamma \delta(E-H_\gamma) . \quad (3.27)$$

In Eq. (3.26) the sum runs over all possible asymptotic channels, γ, and $T^{\alpha\gamma}(+)$ is the transition operator appropriate to the description of scattering from a physical asymptotic state in channel γ to a state in channel α, i.e.,

$$T^{\alpha\gamma}(+)=V^\alpha G G_\gamma{}^{-1} . \quad (3.28)$$

In Eq. (3.27), $\mathscr{P}_\gamma$ projects onto the space spanned by the bound cluster eigenstates of H_γ, that is to say, each channel Hamiltonian H_γ describes a partitioning, γ, into groups of particles which interact only with members of the same group and $\mathscr{P}_\gamma$ projects onto the spaces spanned by the bound states of these groups. We note that bound states of the entire system are ignored in Eq. (3.26). The physical interpretation of the off-shell discontinuity relation Eq. (3.26) is that the cut structure of the operator $T^\alpha(z)$ is a unitarity reflection of the scattering into each of the energetically allowed channels. In fact, the Hermitian analyticity of $T^\alpha(z)$, Eq. (3.23), implies that Eq. (3.26) is equivalent to the off-shell unitarity relation

$$T^\alpha(+)-T^\alpha(+)^\dagger=\sum_\gamma T^{\alpha\gamma}(+)\Delta_\gamma[T^{\alpha\gamma}(+)]^\dagger . \quad (3.29)$$

We now employ these properties of $T^\alpha(z)$ in order to examine the corresponding properties of the operators $U^\alpha(z)$ and $W^\alpha(z)$. These operators satisfy the equations

$$U^\alpha(z)=[1-U^\alpha(z)PG_\alpha(z)]T^\alpha(z) \quad (3.30)$$

$$=T^\alpha(z)[1-PG_\alpha(z)U^\alpha(z)] \quad (3.31)$$

and

$$W^\alpha(z)=[1-W^\alpha(z)\bar{P}G_\alpha(z)]T^\alpha(z) \quad (3.32)$$

$$=T^\alpha(z)[1-\bar{P}G_\alpha(z)W^\alpha(z)] . \quad (3.33)$$

In the following we restrict our analysis to the operator $U^\alpha(z)$; the properties of $W^\alpha(z)$ are obtained by the replacements: $U^\alpha \to W^\alpha$, $P\to\bar{P}$, and $Q\to\bar{Q}$. First, we note that the adjoints of Eqs. (3.30) and (3.31) are

$$U^\alpha(z)^\dagger=T^\alpha(z)^\dagger[1-PG_\alpha(z)^\dagger U^\alpha(z)^\dagger] \quad (3.34)$$

and

$$U^\alpha(z)^\dagger=[1-U^\alpha(z)^\dagger PG_\alpha(z)^\dagger]T^\alpha(z)^\dagger , \quad (3.35)$$

respectively. If we now multiply Eq. (3.29) on the left and on the right by the bracketed quantities in Eqs. (3.30) and (3.34), respectively, with $z=E+i\epsilon$, we obtain

$$U^\alpha(+)-U^\alpha(+)^\dagger+U^\alpha(+)[G_\alpha(+)-G_\alpha(-)]U^\alpha(+)^\dagger=U^\alpha(+)-U^\alpha(+)^\dagger-2\pi i U^\alpha(+)P\delta(E-H_\alpha)U^\alpha(+)^\dagger$$
$$=\sum_\gamma U^{\alpha\gamma}(+)\Delta_\gamma U^{\alpha\gamma}(+)^\dagger . \quad (3.36)$$

In Eq. (3.36) we have defined

$$U^{\alpha\gamma}(z)\equiv[1-U^\alpha(z)PG_\alpha(z)]T^{\alpha\gamma}(z) , \quad (3.37)$$

so that

$$U^{\alpha\alpha}(z)=U^\alpha(z) . \quad (3.38)$$

Upon rearrangement of Eq. (3.36) with the use of Eq. (3.38), we see that $U^\alpha(+)$ satisfies the unitarity relation

$$U^\alpha(+)-U^\alpha(+)^\dagger = U^\alpha(+)Q\Delta_\alpha U^\alpha(+)^\dagger + \sum_{\gamma\neq\alpha} U^{\alpha\gamma}(+)\Delta_\gamma U^{\alpha\gamma}(+)^\dagger \ . \tag{3.39}$$

The projector Q in Eq. (3.39) implies that the unitarity relation for the optical potential $U^\alpha(+)$ contains no *elastic* unitarity cut. Furthermore, we note that if we set $z=E+i\epsilon$ in Eqs. (3.30) and (3.31) and $z=E-i\epsilon$ in Eqs. (3.34) and (3.35), then comparison of Eqs. (3.30) and (3.35) or Eqs. (3.31) and (3.34) shows that $U^\alpha(z)$ is Hermitian analytic,

$$U^\alpha(+)=U^\alpha(-)^\dagger \ , \tag{3.40}$$

since both $G_\alpha(z)$ and $T^\alpha(z)$ possess this property. It then follows from Eqs. (3.39) and (3.40) that $U^\alpha(z)$ satisfies the discontinuity relation

$$U^\alpha(+)-U^\alpha(-) = U^\alpha(+)Q\Delta_\alpha U^\alpha(+)^\dagger + \sum_{\gamma\neq\alpha} U^{\alpha\gamma}(+)\Delta_\gamma U^{\alpha\gamma}(+)^\dagger \ , \tag{3.41}$$

so that the optical potential operator $U^\alpha(z)$ is free of the elastic unitarity cut. Corresponding results for the operator $W^\alpha(z)$ are obtained. The analogs of Eqs. (3.39)–(3.41) are

$$W^\alpha(+)-W^\alpha(+)^\dagger = W^\alpha(+)\bar{Q}\Delta_\alpha W^\alpha(+)^\dagger + \sum_{\gamma\neq\alpha} W^{\alpha\gamma}(+)\Delta_\gamma W^{\alpha\gamma}(+)^\dagger \ , \tag{3.42}$$

$$W^\alpha(+)=W^\alpha(-)^\dagger \ , \tag{3.43}$$

and

$$W^\alpha(+)-W^\alpha(-) = W^\alpha(+)\bar{Q}\Delta_\alpha W^\alpha(+)^\dagger + \sum_{\gamma\neq\alpha} W^{\alpha\gamma}(+)\Delta_\gamma W^{\alpha\gamma}(+)^\dagger \ , \tag{3.44}$$

where

$$W^{\alpha\gamma}(z)\equiv[1-W^\alpha(z)\bar{P}G_\alpha(z)]T^{\alpha\gamma}(z) \ . \tag{3.45}$$

Evidently both the unitarity relation Eq. (3.42) and the discontinuity relation Eq. (3.44) for the operator $W^\alpha(z)$ are free of the elastic unitarity cut and of the inelastic cut corresponding to the particular excited state of interest.

The results of Eqs. (3.39)–(3.44) imply an alternative to the statements of Sec. II concerning the important structural and segregational characteristics of the DWA as we have defined it. In particular, the fact that there is no intermediate scattering to the P space intrinsic to U^α follows from Eqs. (3.39) and (3.41), since these equations imply the removal of the elastic unitarity cut. Similarly, the fact that there is no intermediate scattering to the $\bar{P}$ space intrinsic to W^α follows from Eqs. (3.42) and (3.44). The segregational and structural features of the exact DWA matix element Eq. (2.31) are thus concisely expressed by Eqs. (3.39)–(3.44). Briefly, the operator W^α contains no intermediate scattering to the $\bar{P}$ space, and this also implies that the distorted wave $\langle\chi_f^{(-)}|$ contains intermediate scatterings to all but the $\bar{P}$ space [cf. Eqs. (2.34) and (2.35)]. Thus $\langle\chi_f^{(-)}|$ contains all scattering processes which end in the P_f space, except intermediate scattering to the P space. Similarly, the distorted wave $|\chi\rangle$ contains all scattering processes [cf. Eqs. (2.25) and (2.26)] subject only to the constraint that they end in the P space. In this manner we see that the DWA formalism is characterized by a classification and grouping of the intermediate scatterings according to the subspaces into which the intermediate scattering occurs. Furthermore, the two distorted wave problems contain a division into two problems according to a decomposition of the full space into orthogonal subspaces.

The concise expression of the structural and segregational characteristics of the DWA by means of the unitarity and discontinuity relations satisfied by the optical potential operators U^α and W^α is important because it generalizes to the case where the full complexity of interfragment Pauli symmetries is incorporated into the formalism. This is discussed in the next section, in anticipation of which we remark on the crucial features of the foregoing formalism with regard to the unitarity and discontinuity relation for U^α and W^α. The essential elements are that the transition operator be Hermitian analytic and satisfy a unitarity relation of the form of Eq. (3.29).

IV. INTERFRAGMENT PAULI SYMMETRIES

In this section we extend the exact DWA developed in Secs. II and III to the circumstance wherein the two fragments have identical fermions in common. This extension is easily accomplished through the methods of Sec. III in part because elastic scattering formalisms have recently been developed which are based upon the definition of the optical potential by Eq. (3.10) even in the presence of identical fermions.[45–50] In particular, unitarity, discontinuity, and Hermiticity characteristics of the antisymmetrized elastic scattering optical potential operator are explicated in Refs. 45 and 46; multiple scattering treatments are obtained in Refs. 47 and 49; a resonance formalism is derived in Refs. 48 and 49; and finally, the relationship between this approach and the Feshbach (antisymmetrized projection operator) approach is detailed in Refs. 49 and 50. Although a number of questions yet remain to be addressed in this new elastic scattering formalism, we regard the viability of this approach as established. An interesting flexibility is associated with the new antisymmetrized scattering formalism[45–50] in that the detailed properties of the optical potential operator depend upon the *choice* of off-shell extension of the symmetrized transition operator through which it is defined. These alternatives are also a characteristic of the exact antisymmetrized DWA obtained below.

To maintain an exact DWA for inelastic scattering which is an natural extension of an elastic scattering formalism, we define the *antisymmetrized* optical potential operators $\mathscr{U}^{\alpha}$ and $\mathscr{W}^{\alpha}$, in analogy with Eqs. (3.10) and (3.11), by

$$\mathscr{T}^{\alpha}=\mathscr{U}^{\alpha}+\mathscr{U}^{\alpha}PG_{\alpha}\mathscr{T}^{\alpha} \tag{4.1}$$

$$=\mathscr{U}^{\alpha}+\mathscr{T}^{\alpha}PG_{\alpha}\mathscr{U}^{\alpha} \tag{4.2}$$

and

$$\mathscr{T}^{\alpha}=\mathscr{W}^{\alpha}+\mathscr{W}^{\alpha}\bar{P}G_{\alpha}\mathscr{T}^{\alpha} \tag{4.3}$$

$$=\mathscr{W}^{\alpha}+\mathscr{T}^{\alpha}\bar{P}G_{\alpha}\mathscr{W}^{\alpha}\ . \tag{4.4}$$

In Eqs. (4.1)–(4.4) the operator $\mathscr{T}^{\alpha}$ refers to an arbitrary, but definite, choice of off-shell extension for the fully antisymemtrized transition operator. The only restriction on the operator $\mathscr{T}^{\alpha}$ is that it yield the properly anitsymmetrized transition amplitude,

$$\mathscr{T}_{fi}=\langle\phi_f(\vec{k}\,')\,|\,\mathscr{T}^{\alpha}\,|\,\phi(\vec{k})\rangle\ , \tag{4.5}$$

fully on shell, and it is easily seen by the methods of Sec. III that *any* choice of $\mathscr{T}^{\alpha}$ which satisfies Eq. (4.5) necessarily leads to an exact antisymmetrized DWA. In particular, if we combine Eqs. (4.2) and (4.3) and similarly Eqs. (4.1) and (4.4) in order to eliminate $\mathscr{T}^{\alpha}$ from each of these sets of equations we obtain

$$\mathscr{U}^{\alpha}=\mathscr{W}^{\alpha}+\mathscr{W}^{\alpha}P_fG_{\alpha}\mathscr{U}^{\alpha} \tag{4.6}$$

and

$$\mathscr{U}^{\alpha}=\mathscr{W}^{\alpha}+\mathscr{U}^{\alpha}P_fG_{\alpha}\mathscr{W}^{\alpha}\ . \tag{4.7}$$

Thus it follows, as in Sec. III, that the antisymmetrized inelastic transition amplitude of Eq. (4.5) can be rewritten as

$$\mathscr{T}_{fi}=\langle\chi_f^{(-)}\,|\,\mathscr{W}^{\alpha}\,|\,\chi\rangle\ , \tag{4.8}$$

as long as we *define* the distorted waves $|\chi\rangle$ and $\langle\chi_f^{(-)}|$ by

$$|\chi\rangle=P\,|\,\chi\rangle\equiv[1+PG_{\alpha}\mathscr{T}^{\alpha}]\,|\,\phi(\vec{k})\rangle\ , \tag{4.9}$$

$$\langle\chi_f^{(-)}|=\langle\chi_f^{(-)}|\,P_f \equiv\langle\phi_f(\vec{k}\,')\,|\,[1+\mathscr{U}^{\alpha}G_{\alpha}P_f]\ . \tag{4.10}$$

Furthermore, in view of these definitions and Eqs. (4.1) and (4.7), the distorted waves satisfy the equations

$$|\chi\rangle=|\,\phi(\vec{k})\rangle+G_{\alpha}P\mathscr{U}^{\alpha}P\,|\,\chi\rangle \tag{4.11}$$

and

$$\langle\chi_f^{(-)}|=\langle\phi(\vec{k}\,')\,|+\langle\chi_f^{(-)}|\,P_f\mathscr{W}^{\alpha}P_fG_{\alpha}\ . \tag{4.12}$$

In a formal sense, we are now finished. Given *any* choice for $\mathscr{T}^{\alpha}$, the formalism above defines a completely consistent and general DWA which is applicable to the description of the inelastic scattering of two composite fragments with all of the complexity associated with the Pauli principle properly incorporated. However, important practical questions remain, especially in regard to the options permitted by the choice of $\mathscr{T}^{\alpha}$. In particular, we seek motivation for a specific choice of $\mathscr{T}^{\alpha}$. We may also seek to preserve in the antisymmetrized case the structural features which characterized the unsymmetrized formalism. These questions require us to treat the antisymmetrized formalism explicitly.

It is convenient to define several operator antisymmetrizers with which we will work. Up to this point we have not taken explicit note of the fact that the antisymmetrized problem is inherently a multichannel one since we have needed only a single channel (α) which represents the choice of an unsymmetrized elastic channel.[45–50] We may, however, define the set $\hat{\alpha}$ of channels to be the set of all channels which are related to the α channel by the exchange of identical fermions *between* the two α channels fragments. That is, the set $\hat{\alpha}$ consists of all the channels which are indistinguishable from the α channel by virtue of the Pauli principle. We can then define a complete antisymmetrizer for the problem by[52]

$$\mathscr{A}(\hat{\alpha})=\sum_{\alpha'\epsilon\hat{\alpha}}U_{\alpha',\alpha}R_{\alpha}\ , \tag{4.13}$$

where $U_{\alpha',\alpha}$ is the parity-weighted unitary operator which accomplishes the exchange of identical particles between the two fragments such that $\alpha \to \alpha'$, and R_α is the projection operator ($R_\alpha{}^2 = R_\alpha$) which antisymmetrizes the two fragments *separately.* It can be shown[52] that $\mathscr{A}(\hat{\alpha})$ is proportional to the *projector* Λ onto totally antisymmetric vectors,

$$\mathscr{A}(\hat{\alpha}) = C(\hat{\alpha})\Lambda \ , \tag{4.14}$$

where the proportionality constant $C(\hat{\alpha})$ depends only on the set $\hat{\alpha}$. A simple example of these definitions is afforded by the case of a nucleon (0) scattering from a nucleus composed of A identical nucleons. In this case

$$\mathscr{A}(\hat{\alpha}) = \left[1 - \sum_{i=1}^{A} E_{0i} \right] R_\alpha \ , \tag{4.15}$$

where E_{0i} is the exchange operator[47] for nucleons 0 and i, and

$$C(\hat{\alpha}) = A + 1 \ . \tag{4.16}$$

The definition

$$\hat{\mathscr{A}}(\hat{\alpha}) \equiv \mathscr{A}(\hat{\alpha}) - R_\alpha = [\mathscr{A}(\hat{\alpha}) - 1] R_\alpha \ , \tag{4.17}$$

will also prove useful.

Let us now examine the structure of the properly antisymmetrized DWA of this section. Equation (4.9) defines the right-hand distorted wave for an arbitrary choice of $\mathscr{T}^\alpha$. However, from a practical point of view, we would like to restrict our choices so that the distorted wave is

$$|\chi\rangle = P |\tilde{\psi}_\alpha\rangle \ , \tag{4.18}$$

where $|\tilde{\psi}_\alpha\rangle$ is the fully antisymmetric complete wave function for the system, viz.,

$$|\tilde{\psi}_\alpha\rangle = \mathscr{A}(\hat{\alpha}) G G_\alpha{}^{-1} |\phi(\vec{k})\rangle \ . \tag{4.19}$$

The reason for this is that Eq. (4.18) guarantees that the distorted wave has the same asymptotic behavior in the elastic channel as $|\tilde{\psi}_\alpha\rangle$ itself, and this asymptotic behavior is directly related to the elastic scattering data. Thus, Eq. (4.18) allows us to maintain a close connection to elastic scattering, both experimentally and theoretically. It is therefore useful to combine Eqs. (4.18) and (4.19) in order to rewrite Eq. (4.18) in the form of Eq. (4.9). We find

$$|\chi\rangle \equiv P\mathscr{A}(\hat{\alpha}) G G_\alpha{}^{-1} |\phi(\vec{k})\rangle \tag{4.20}$$

$$= [1 + P\{ \mathscr{A}(\hat{\alpha}) G G_\alpha{}^{-1} - 1 \}] |\phi(\vec{k})\rangle \tag{4.21}$$

$$= [1 + PG_\alpha \{ G_\alpha{}^{-1} \mathscr{A}(\hat{\alpha}) G G_\alpha{}^{-1} - G_\alpha{}^{-1} \}] |\phi(\vec{k})\rangle \ . \tag{4.22}$$

Thus, it follows immediately that if we take $\mathscr{T}^\alpha$ to be

$$\tilde{\mathscr{T}}^\alpha \equiv G_\alpha{}^{-1} \mathscr{A}(\hat{\alpha}) G G_\alpha{}^{-1} - G_\alpha{}^{-1} \ , \tag{4.23}$$

then we necessarily have Eq. (4.18). The operator $\tilde{\mathscr{T}}^\alpha$ is, in fact, the properly symmetrized transition operator[45–50] based upon the Alt, Grassberger, and Sandhas[53] off-shell extension of the multichannel transition operators. Since Eq. (4.22) is half on the energy shell, this transition operator is not unique in yielding Eq. (4.18). The symmetrized prior form of $\mathscr{T}^\alpha$, defined to be

$$\hat{\mathscr{T}}^\alpha \equiv V^\alpha \mathscr{A}(\hat{\alpha}) G G_\alpha{}^{-1} \ , \tag{4.24}$$

also yields[47,48] Eq. (4.18). The transition operators of Eqs. (4.23) and (4.24) are related by

$$\tilde{\mathscr{T}}^\alpha = \hat{\mathscr{T}}^\alpha + \hat{\mathscr{A}}(\hat{\alpha}) G_\alpha{}^{-1} \ . \tag{4.25}$$

In the following we limit ourselves to these two choices for $\mathscr{T}^\alpha$ which have proven useful in the antisymmetrized elastic scattering theory.[45–50] These considerations provide at least a partial answer to our first question, namely, if we require that the distorted wave $|\chi\rangle$ obey Eq. (4.18) then Eq. (4.9) restricts the choice of $\mathscr{T}^\alpha$. The forms $\tilde{T}^\alpha$ and $\hat{T}^\alpha$ of Eqs. (4.23) and (4.24) satisfy this criterion.

We now turn to the consideration of the structural and segregational characteristics of the foregoing antisymmetrized DWA. We observe that the operator $\tilde{\mathscr{T}}^\alpha$ is Hermitian analytic,

$$\tilde{\mathscr{T}}^\alpha(+) = \tilde{\mathscr{T}}^\alpha(-)^\dagger \ , \tag{4.26}$$

which is obvious from Eq. (4.23). Recalling the salient features of Sec. III, we are prompted to examine the discontinuity and unitarity relations satisfied by $\tilde{\mathscr{T}}^\alpha(z)$. However, it is convenient to obtain the discontinuity and unitarity relations for $\hat{\mathscr{T}}^\alpha(z)$ first. If we rewrite Eq. (4.24) as

$$\hat{\mathscr{T}}^\alpha(z) = V^\alpha \mathscr{A}(\hat{\alpha}) + V^\alpha \mathscr{A}(\hat{\alpha}) G(z) V^\alpha \ , \tag{4.27}$$

then it is easy to see that

$$\hat{\mathscr{T}}^\alpha(+) - \hat{\mathscr{T}}^\alpha(-) = -2\pi i C(\hat{\alpha}) \sum_n V^\alpha |\tilde{\psi}_n^{(+)}\rangle \times \delta(E - E_n) \langle \tilde{\psi}_n^{(+)}| V^\alpha \ . \tag{4.28}$$

Equation (4.28) is analogous to Eq. (3.25) except for the fact that the sum in Eq. (4.28) is limited to the set of *completely antisymmetrized* outgoing wave eigenstates of the full Hamiltonian. Each of these properly normalized states may then be characterized (nonuniquely, but equivalently) in terms of an unsymmetrized asymptotic incident γ channel vector $|\phi_\gamma\rangle$, from which it evolves, i.e.,

$$|\tilde{\psi}_n^{(+)}\rangle = C(\hat{\gamma})^{-1/2}\mathscr{A}(\hat{\gamma})GG_\gamma^{-1}|\phi_\gamma\rangle . \quad (4.29)$$

Thus we find that the discontinuity relation satisfied by $\hat{\mathscr{T}}^\alpha$ is

$$\hat{\mathscr{T}}^\alpha(+)-\hat{\mathscr{T}}^\alpha(-)=\sum_{\hat{\gamma}}\hat{\mathscr{T}}^{\alpha\gamma}(+)\Delta_\gamma\hat{\mathscr{T}}^{\alpha\gamma}(+)^\dagger , \quad (4.30)$$

where

$$\hat{\mathscr{T}}^{\alpha\gamma}(+)\equiv\left[\frac{C(\hat{\alpha})}{C(\hat{\gamma})}\right]^{1/2}V^\alpha\mathscr{A}(\hat{\gamma})GG_\gamma^{-1} , \quad (4.31)$$

so that

$$\hat{\mathscr{T}}^{\alpha\alpha}(+)=\hat{\mathscr{T}}^\alpha(+) , \quad (4.32)$$

and, as in Sec. III,

$$\Delta_\gamma\equiv -2\pi i\mathscr{P}_\gamma\delta(E-H_\gamma) . \quad (4.33)$$

It is important to emphasize that the sum in Eq. (4.30) is over the *sets* composed of Pauli equivalent channels and that $\gamma\in\hat{\gamma}$ is an arbitrary, but definite, element of $\hat{\gamma}$. The operator $\hat{\mathscr{T}}^\alpha$ is not Hermitian analytic but instead satisfies

$$\hat{\mathscr{T}}^\alpha(-)=\hat{\mathscr{T}}^\alpha(+)^\dagger+[V^\alpha,\mathscr{A}(\hat{\alpha})] . \quad (4.34)$$

Combination of Eqs. (4.30) and (4.34) yields

$$\hat{\mathscr{T}}^\alpha(+)-\hat{\mathscr{T}}^\alpha(+)^\dagger=\sum_{\hat{\gamma}}\hat{\mathscr{T}}^{\alpha\gamma}(+)\Delta_\gamma\hat{\mathscr{T}}^{\alpha\gamma}(+)^\dagger +[V^\alpha,\mathscr{A}(\hat{\alpha})] . \quad (4.35)$$

Let us now compare Eqs. (4.30), (4.34), and (4.35) with their analogs in the unsymmetrized formalism of Sec. III, Eqs. (3.26), (3.23), and (3.29), respectively. In this manner we infer that the transition operator $\hat{\mathscr{T}}^\alpha$ does not possess the characteristics necessary in order for us to extend the structural and segregational features of the unsymmetrized DWA formalism to the fully antisymmetrized circumstance. In the case of the discontinuity relation, Eq. (4.30), this is a result of the fact that $\hat{\mathscr{T}}^\alpha$ is not Hermitian analytic, while in the case of the unitarity relation, Eq. (4.35), the second term on the right-hand side precludes the generalization of the treatment of Sec. III. These considerations suggest that we investigate the antisymmetrized DWA formalism based upon $\tilde{\mathscr{T}}^\alpha$.

If we combine Eqs. (4.25), (4.30), and (4.35), then we find that the discontinuity and unitarity relations satisfied by $\tilde{\mathscr{T}}^\alpha$ are

$$\tilde{\mathscr{T}}^\alpha(+)-\tilde{\mathscr{T}}^\alpha(-)=\sum_{\hat{\gamma}}\hat{\mathscr{T}}^{\alpha\gamma}(+)\Delta_\gamma\hat{\mathscr{T}}^{\alpha\gamma}(+)^\dagger , \quad (4.36)$$

and

$$\tilde{\mathscr{T}}^\alpha(+)-\tilde{\mathscr{T}}^\alpha(+)^\dagger=\sum_{\hat{\gamma}}\hat{\mathscr{T}}^{\alpha\gamma}(+)\Delta_\gamma\hat{\mathscr{T}}^{\alpha\gamma}(+)^\dagger , \quad (4.37)$$

respectively. Furthermore, if we separate off the (antisymmetrized) $\hat{\alpha}$ channel cuts in Eqs. (4.36) and (4.37) and make use of the on-shell δ function in Δ_α, then these equations may be rewritten as[54]

$$\tilde{\mathscr{T}}^\alpha(+)-\tilde{\mathscr{T}}^\alpha(-)=\tilde{\mathscr{T}}^\alpha(+)\Delta_\alpha\tilde{\mathscr{T}}^\alpha(+)^\dagger +\sum_{\hat{\gamma}\neq\hat{\alpha}}\hat{\mathscr{T}}^{\alpha\gamma}(+)\Delta_\gamma\hat{\mathscr{T}}^{\alpha\gamma}(+)^\dagger \quad (4.38)$$

and

$$\tilde{\mathscr{T}}^\alpha(+)-\tilde{\mathscr{T}}^\alpha(+)^\dagger=\tilde{\mathscr{T}}^\alpha(+)\Delta_\alpha\tilde{\mathscr{T}}^\alpha(+)^\dagger +\sum_{\hat{\gamma}\neq\hat{\alpha}}\hat{\mathscr{T}}^{\alpha\gamma}(+)\Delta_\gamma\hat{\mathscr{T}}^{\alpha\gamma}(+)^\dagger , \quad (4.39)$$

respectively. If we now compare Eqs. (4.26), (4.38), and (4.39) with Eqs. (3.23), (3.26), and (3.29) then we immedately see that the operator $\tilde{\mathscr{T}}^\alpha$ *does* possess the features of the unsymmetrized transition operator which were important in obtaining unitarity and discontinuity relations for the optical potential operators of the form of Eqs. (3.39), (3.41), (3.42), and (3.44). It is now easy to infer the resulting relations for the antisymmetrized optical potential operators $\tilde{\mathscr{U}}^\alpha$ and $\tilde{\mathscr{W}}^\alpha$, which arise from the use of $\tilde{\mathscr{T}}^\alpha$ in the definitions of Eqs. (4.1)–(4.4), since they follow in a straightforward manner from the method of Sec. III. We have the unitarity relations

$$\tilde{\mathscr{U}}^\alpha(+)-\tilde{\mathscr{U}}^\alpha(+)^\dagger=\tilde{\mathscr{U}}^\alpha(+)Q\Delta_\alpha\tilde{\mathscr{U}}^\alpha(+)^\dagger +\sum_{\hat{\gamma}\neq\hat{\alpha}}\tilde{\mathscr{U}}^{\alpha\gamma}(+)\Delta_\gamma\tilde{\mathscr{U}}^{\alpha\gamma}(+)^\dagger \quad (4.40)$$

and

$$\tilde{\mathscr{W}}^{\alpha}(+)-\tilde{\mathscr{W}}^{\alpha}(+)^{\dagger}=\tilde{\mathscr{W}}^{\alpha}(+)\bar{Q}\Delta_{\alpha}\tilde{\mathscr{W}}^{\alpha}(+)^{\dagger}+\sum_{\hat{\gamma}\neq\hat{\alpha}}\tilde{\mathscr{W}}^{\alpha\gamma}(+)\Delta_{\gamma}\tilde{\mathscr{W}}^{\alpha\gamma}(+)^{\dagger}\ , \tag{4.41}$$

where, in analogy to Eqs. (3.37) and (3.45), we have defined

$$\tilde{\mathscr{U}}^{\alpha\gamma}(z)=[1-\tilde{\mathscr{U}}^{\alpha}(z)PG_{\alpha}(z)]\hat{\mathscr{T}}^{\alpha\gamma}(z)\ , \tag{4.42}$$

and

$$\tilde{\mathscr{W}}^{\alpha\gamma}(z)=[1-\tilde{\mathscr{W}}^{\alpha}(z)\bar{P}G_{\alpha}(z)]\hat{\mathscr{T}}^{\alpha\gamma}(z)\ . \tag{4.43}$$

It also follows, as in Sec. III, that the optical potential operators are Hermitian analytic,

$$\tilde{\mathscr{U}}^{\alpha}(+)=\tilde{\mathscr{U}}^{\alpha}(-)^{\dagger} \tag{4.44}$$

and

$$\tilde{\mathscr{W}}^{\alpha}(+)=\tilde{\mathscr{W}}^{\alpha}(-)^{\dagger}\ , \tag{4.45}$$

so that from Eqs. (4.40) and (4.41) we obtain the discontinuity relations

$$\tilde{\mathscr{U}}^{\alpha}(+)-\tilde{\mathscr{U}}^{\alpha}(-)=\tilde{\mathscr{U}}^{\alpha}(+)Q\Delta_{\alpha}\tilde{\mathscr{U}}^{\alpha}(+)^{\dagger}+\sum_{\hat{\gamma}\neq\hat{\alpha}}\tilde{\mathscr{U}}^{\alpha\gamma}(+)\Delta_{\gamma}\tilde{\mathscr{U}}^{\alpha\gamma}(+)^{\dagger} \tag{4.46}$$

and

$$\tilde{\mathscr{W}}^{\alpha}(+)-\tilde{\mathscr{W}}^{\alpha}(-)=\tilde{\mathscr{W}}^{\alpha}(+)\bar{Q}\Delta_{\alpha}\tilde{\mathscr{W}}^{\alpha}(+)^{\dagger}+\sum_{\hat{\gamma}\neq\hat{\alpha}}\tilde{\mathscr{W}}^{\alpha\gamma}(+)\Delta_{\gamma}\tilde{\mathscr{W}}^{\alpha\gamma}(+)^{\dagger}\ . \tag{4.47}$$

From Eqs. (4.40)−(4.47) we see that the antisymmetrized DWA formalism based upon $\tilde{\mathscr{T}}^{\alpha}$ preserves the structural and segregational features of the unsymmetrized formalism of Sec. III. In contrast, this is not the case for the formalism based upon $\hat{\mathscr{T}}^{\alpha}$.

The results of this section may be summarized as follows. We have derived an exact DWA formulation of inelastic scattering which is applicable to the circumstance in which the full complexity of Pauli symmetries is present. The optical potential operators are dependent upon the choice of off-shell extension for the symmetrized transition operator. The only restriction on this choice is that it gives the correct scattering amplitude fully on shell. However, if we wish to maintain a close relationship to elastic scattering, then this choice can be restricted to those forms for which Eq. (4.9) implies Eq. (4.18). The choices $\tilde{\mathscr{T}}^{\alpha}$ and $\hat{\mathscr{T}}^{\alpha}$ satisfy this restriction. Furthermore, if we wish to maintain the structural and segregational features of the unsymmetrized formalism, then the use of $\tilde{\mathscr{T}}^{\alpha}$, as opposed to $\hat{\mathscr{T}}^{\alpha}$, is necessary. However, as we shall see in the next section, other considerations may favor the use of $\hat{T}^{\alpha}$ rather than $\tilde{T}^{\alpha}$.

V. MULTIPLE SCATTERING—SPECTATOR EXPANSION

In this section we investigate the implications of the foregoing exact DWA with regard to multiple scattering formalisms of the KMT (Ref. 3) or Watson[2] types. It is not our purpose here to discuss the relative merits of a particular approach to the derivation of multiple scattering expansions,[55,56] but only to indicate the applicability of the DWA obtained in this paper to such considerations. In fact, the exact DWA advocated herein is sufficiently general to accommodate any of the standard multiple scattering formalisms. We employ the spectator expansion[57,58,47,49] method for our considerations and, for the sake of simplicity, we restrict ourselves to the case of an elementary projectile scattering from a target which consists of A identical nucleons.

We first suppose that a spectator expansion of the transition operator has been obtained which is of the generic form

$$T^{\alpha}=\sum_{i=1}^{A}T_{i}^{\alpha}+\sum_{i<j}S_{ij}^{\alpha}+\cdots \tag{5.1}$$

$$=\sum_{i=1}^{A}T_{i}^{\alpha}+\sum_{i<j}(T_{ij}^{\alpha}-T_{i}-T_{j})+\cdots\ . \tag{5.2}$$

In an unsymmetrized formalism the operators in Eqs. (5.1) and (5.2) have a well-defined characterization.[49] Namely, T_i^{α} consists of all terms of the expansion of T^{α} [cf. Eqs. (2.38) and (3.7)] in which the projectile interacts only with target particle i. Similarly, S_{ij}^{α} consists of all the terms of the expansion in which the projectile interacts with *both* target particles i and j, and no others, and so on. Thus the spectator expansion is an expansion in the number of target particles with which the projectile interacts. The separation of the transition operator in this manner is necessarily somewhat ambiguous in the case of identical particles but, nevertheless, an appealing generalization of these ideas survives the extension to the fully antisymmetrized circumstance.[47,49] We do not need to consider the content of Eqs. (5.1) and (5.2) in detail, herein; we simply note that expansions of this form are easily obtained in the unsymmetrized case[47,49] and, for the properly symmetrized operator $\hat{\mathscr{T}}^{\alpha}$ of Sec. IV, in the fully antisymmetrized case.[47] Thus the multiple scattering considerations of this section are applicable to both of these circumstances. In the case of the transition operator $\tilde{\mathscr{T}}^{\alpha}$ of Sec. IV, however, a generalization of the spectator expansion is necessary,[49] and so the treatment below is not immediately applicable to the DWA based upon this operator. We refer the interested reader to Refs. 47, 49, 57, and 58, where treatments of the spectator expansion may be found. We do remark, however, that whereas the formulation based on $\tilde{\mathscr{T}}^{\alpha}$ seems better suited to treatment of the consequences of unitarity, the formulation based on $\hat{\mathscr{T}}^{\alpha}$ makes closer contact with the unsymmetrized multiple scattering theory.

The spectator expansion of the optical potential operator $\mathscr{U}^{\alpha}$ is defined, in analogy to Eqs. (5.1) and (5.2), by[58,47,49]

$$U^{\alpha}=\sum_{i=1}^{A} U_i^{\alpha}+\sum_{i<j} U_{ij}^{\alpha}+\cdots , \tag{5.3}$$

and we define the spectator expansion of W^{α} similarly:

$$W^{\alpha}=\sum_{i=1}^{A} W_i^{\alpha}+\sum_{i<j} W_{ij}^{\alpha}+\cdots . \tag{5.4}$$

These definitions are completed by substitution of Eqs. (5.1), (5.3), and (5.4) into the defining equations of the optical potential operators, Eqs. (4.1)–(4.4). We obtain in this manner

$$\left[\sum_{i=1}^{A} T_i^{\alpha}+\sum_{i<j} S_{ij}^{\alpha}+\cdots\right]=\left[\sum_{i=1}^{A} U_i^{\alpha}+\sum_{i<j} U_{ij}^{\alpha}+\cdots\right]\left[1+PG_{\alpha}\left\{\sum_{i=1}^{A} T_i^{\alpha}+\sum_{i<j} S_{ij}^{\alpha}+\cdots\right\}\right] \tag{5.5}$$

and

$$\left[\sum_{i=1}^{A} T_i^{\alpha}+\sum_{i<j} S_{ij}^{\alpha}+\cdots\right]=\left[\sum_{i=1}^{A} W_i^{\alpha}+\sum_{i<j} W_{ij}^{\alpha}+\cdots\right]\left[1+\bar{P}G_{\alpha}\left\{\sum_{i=1}^{A} T_i^{\alpha}+\sum_{i<j} S_{ij}^{\alpha}+\cdots\right\}\right], \tag{5.6}$$

respectively. It is also convenient to express the spectator expansion of W^{α} in terms of that of U^{α} and, upon employing Eq. (4.7), we find that

$$\left[\sum_{i=1}^{A} U_i^{\alpha}+\sum_{i<j} U_{ij}^{\alpha}+\cdots\right]=\left[1+\left\{\sum_{i=1}^{A} U_i^{\alpha}+\sum_{i<j} U_{ij}^{\alpha}+\cdots\right\}P_f G_{\alpha}\right]\left[\sum_{i=1}^{A} W_i^{\alpha}+\sum_{i<j} W_{ij}^{\alpha}+\cdots\right]. \tag{5.7}$$

We can now readily obtain the lowest order term of the spectator expansions of the operators U^{α} and W^{α}, which we label $\bar{U}^{\alpha}$ and $\bar{W}^{\alpha}$, by means of term by term identification in Eqs. (5.5)–(5.7). This procedure may be thought of as an identification in the sense of the "connectivity" structure of the different terms.[59,60] In this way we obtain

$$T_i^{\alpha}=U_i^{\alpha}[1+PG_{\alpha}T_i^{\alpha}] , \tag{5.8}$$

$$T_i^{\alpha}=W_i^{\alpha}[1+\bar{P}G_{\alpha}T_i^{\alpha}] , \tag{5.9}$$

and

$$U_i^{\alpha}=[1+U_i^{\alpha}P_f G_{\alpha}]W_i^{\alpha} , \tag{5.10}$$

respectively. These relations determine $\bar{U}^{\alpha}$ and $\bar{W}^{\alpha}$ since we have from Eqs. (5.8) and (5.10),

$$\bar{U}^{\alpha}=\sum_{i=1}^{A} U_i^{\alpha}=\sum_{i=1}^{A} T_i^{\alpha}[1+PG_{\alpha}T_i^{\alpha}]^{-1} \tag{5.11}$$

and

$$\bar{W}^{\alpha}=\sum_{i=1}^{A} W_i^{\alpha}=\sum_{i=1}^{A} [1+U_i^{\alpha}P_f G_{\alpha}]^{-1}U_i^{\alpha} . \tag{5.12}$$

Let us now return to the exact DWA matrix element of Eq. (2.31), which we write as

$$T_{fi}=\langle \chi_f^{(-)} | P_f W^{\alpha} P | \chi \rangle . \tag{5.13}$$

We denote by $T_{fi}^{(1)}$ the first order multiple scattering approximation to Eq. (5.13) which is defined as follows. We replace the operator W^{α} by $\bar{W}^{\alpha}$ and the optical potentials used in calculating the distorted waves in Eq. (5.13), $P_f W^{\alpha} P_f$ and $PU^{\alpha}P$, by $P_f \bar{W}^{\alpha} P_f$ and $P\bar{U}^{\alpha}P$, respectively. If we utilize Eqs. (5.11) and (5.12) we have

$$P_f \bar{W}^{\alpha} P=[1+P_f U_1^{\alpha} P_f G_{\alpha}]^{-1} A P_f U_1^{\alpha} P , \tag{5.14}$$

$$P_f\overline{W}^\alpha P_f=[1+P_fU_1^\alpha P_fG_\alpha]^{-1}AP_fU_1^\alpha P_f\ , \tag{5.15}$$

and

$$P\overline{U}^\alpha P=APT_1^\alpha P[1+G_\alpha PT_1^\alpha P]^{-1}\ . \tag{5.16}$$

In obtaining Eqs. (5.14)–(5.16) we have used the fact that the target states are antisymmetrized[47] (the A target particles have been assumed to be identical), so that the projected operators are independent of the index i and we require only one label (which we have chosen to be that of particle 1). The approximate distorted waves, which we denote by $|\overline{\chi}\rangle$ and $\langle\overline{\chi}_f^{(-)}|$, are obtained through the replacement of the optical potentials in Eqs. (2.25) and (2.34) by their first order spectator expansions. Thus we have

$$|\overline{\chi}\rangle=[1-G_\alpha P\overline{U}^\alpha P]^{-1}|\phi(\vec{k})\rangle \tag{5.17}$$

and

$$\langle\overline{\chi}_f^{(-)}|=\langle\phi_f(\vec{k}')|[1-P_f\overline{W}^\alpha P_fG_\alpha]^{-1}\ . \tag{5.18}$$

If we now insert Eq. (5.16) into Eq. (5.17) and Eq. (5.15) into Eq. (5.18), we obtain

$$|\overline{\chi}\rangle=(1+G_\alpha PT_1^\alpha P)\times[1-G_\alpha(A-1)PT_1^\alpha P]^{-1}|\phi(\vec{k})\rangle\ , \tag{5.19}$$

and

$$\langle\overline{\chi}_f^{(-)}|=\langle\phi_f(\vec{k}')|[1-(A-1)P_fU_1^\alpha P_fG_\alpha]^{-1}\times(1+P_fU_1^\alpha P_fG_\alpha)\ , \tag{5.20}$$

respectively. Upon combining Eqs. (5.14), (5.19), and (5.20) in the first order DWA matrix element

$$T_{fi}^{(1)}\equiv\langle\overline{\chi}_f^{(-)}|P_f\overline{W}^\alpha P|\overline{\chi}\rangle\ , \tag{5.21}$$

we find, with the aid of Eq. (5.8), that this matrix element is equivalent to

$$T_{fi}^{(1)}=\langle\mathscr{N}_f^{(-)}|AP_fT_1^\alpha P|\mathscr{N}\rangle\ , \tag{5.22}$$

where we have defined new distorted waves, $\langle\mathscr{N}_f^{(-)}|$ and $|\mathscr{N}\rangle$, by

$$\langle\mathscr{N}_f^{(-)}|\equiv\langle\phi_f(\vec{k}')|[1-(A-1)P_fU_1^\alpha P_fG_\alpha]^{-1}\ , \tag{5.23}$$

and

$$|\mathscr{N}\rangle\equiv[1-G_\alpha(A-1)PT_1^\alpha P]^{-1}|\phi(\vec{k})\rangle\ , \tag{5.24}$$

respectively.

The first order DWA defined by Eqs. (5.22)–(5.24) and Eq. (5.8) constitutes a principal result of this section. It is to be noted, however, that further approximation, in particular to the projections of the operator T_1^α, is necessary in order to reduce the DWA to the level of calculational feasibility. This is a characteristic feature of the spectator method,[47,49,60] and we shall consider this problem, as well as the problem of providing computational prescriptions, in the following paper.[61] However, let us note the implications of the distorted wave approximation obtained by the replacement[47,49] of the operator T_1^α (or $\hat{T}^\alpha$) by the appropriate two-particle transition operator, t_1, for the (free) projectile-target nucleon system. In this case, we see that the operator which mediates the transition in Eq. (5.22) is simply At_1 and this is the familiar approximation commonly employed in current distorted wave impulse calculations. Furthermore, the distorted wave $|\mathscr{N}\rangle$ is then just the familiar KMT distorted wave.[3,47,49] In constrast, $\langle\mathscr{N}_f^{(-)}|$ differs substantively from the usual approximations employed for the left-hand distorted wave. We note upon comparison of Eqs. (5.23) and (5.24) that $\langle\mathscr{N}_f^{(-)}|$ also has the form of a KMT distorted wave; the difference is that U_1^α appears in place of T_1^α. In the approximation in which T_1^α becomes t_1, U_1^α becomes u_1, which according to Eq. (5.8) is given by

$$u_1=t_1-t_1G_\alpha Pu_1\ . \tag{5.25}$$

Of course, we recognize u_1 as the operator τ_1 of Watson.[2] The one-body operator of interest to us is $P_fu_1P_f$, which we can obtain from Eq. (5.25) by solving the pair of coupled integral equations

$$P_fu_1P_f=P_ft_1P_f-P_ft_1PG_\alpha Pu_1P_f\ , \tag{5.26a}$$

$$Pu_1P_f=Pt_1P_f-Pt_1PG_\alpha Pu_1P_f\ , \tag{5.26b}$$

or equivalently by evaluating the formal solution of Eqs. (5.26) which is

$$P_fu_1P_f=P_ft_1P_f-P_ft_1P\frac{1}{G_\alpha{}^{-1}+Pt_1P}Pt_1P_f\ . \tag{5.27}$$

In view of the structure of Eq. (5.25), it is evident that the difference under discussion arises as an approximation to the removal of intermediate scatterings to the P space which are included in the right-hand distorted wave, $|\mathscr{N}\rangle$. We note once again that at the level of approximation of Eq. (5.25), the constraint of the Pauli principle reduces to the simple requirement, originally proposed by Takeda and

Watson,[62] that t_1 be a properly antisymmetrized two-body operator.

VI. DISCUSSION

We have constructed a microscopic theory of inelastic scattering with an intrinsic distorted wave structure such that the transition matrix element is given by

$$T_{fi}^{\text{inel}}=\langle\chi_f^{(-)}\,|\,W^\alpha\,|\,\chi\rangle\ . \tag{6.1}$$

This result along with the definitions of the ingredients of that equation is first presented in Sec. II [cf. Eq. (2.31)]. The distorted wave $|\,\chi\rangle$ is taken to be the standard, microscopically based, distorted wave appropriate to elastic scattering, viz.,

$$|\,\chi\rangle=P\,|\,\psi_\alpha\rangle\ , \tag{6.2}$$

where $|\,\psi_\alpha\rangle$ is the exact many-body outgoing wave scattering vector. As in the theory of elastic scattering we therefore find that

$$|\,\chi\rangle=|\,\phi(\vec{k})\rangle+G_\alpha PU^\alpha P\,|\,\chi\rangle\ , \tag{6.3}$$

where the optical potential operator U^α is

$$U^\alpha=V^\alpha+V^\alpha G_\alpha QU^\alpha\ . \tag{6.4}$$

The effective transition operator, W^α, in Eq. (6.1) is defined by the equation

$$W^\alpha=V^\alpha+V^\alpha G_\alpha QQ_f W^\alpha\ , \tag{6.5}$$

and the final distorted wave is given by

$$\langle\chi_f^{(-)}\,|=\langle\phi_f(\vec{k}\,')\,|+\langle\chi_f^{(-)}\,|\,P_fW^\alpha P_f G_\alpha\ . \tag{6.6}$$

We recognize that in this formulation, given in Sec. II and epitomized in Eqs. (6.1)–(6.6), the distorted wave $\langle\chi_f^{(-)}|$ is not the distorted wave appropriate to elastic scattering from the target in the excited state, as might have been assumed. Rather the potential which generates this distorted wave is given by $P_fW^\alpha P_f$, which differs from the optical potential for elastic scattering from the final (excited) state by the appearance of QQ_f in place of Q_f in Eq. (6.5). This implies that W^α corresponds to a transition operator in which all intermediate scatterings to states in the P and P_f spaces are specifically excluded. The absence of intermediate scatterings to states in the P_f space arises for exactly the same reason as the exclusion of intermediate scatterings to states in P in the definition of U^α. These intermediate scatterings are excluded because they reappear upon iteration of the optical model Lippmann-Schwinger equation for elastic scattering. This exclusion thus is necessary to avoid "double-counting."

The distorted wave matrix element, Eq. (6.1), has a definite built-in asymmetry in the treatment of the initial and final distorted waves. This asymmetry is related to the more familiar asymmetry in the theory of elastic scattering. There we begin with $T^{\text{el}}=\langle\phi(\vec{k}\,')|V^\alpha\,|\,\psi_\alpha\rangle$ and, by expressing, $|\,\psi_\alpha\rangle$ in terms of the distorted wave $P\,|\,\psi_\alpha\rangle\equiv|\,\chi\rangle$ $[\,|\,\psi_\alpha\rangle=[1-G_\alpha QV^\alpha]^{-1}\,|\,\psi_\alpha\rangle]$, transform this matrix element into the form $T^{\text{el}}=\langle\phi(\vec{k}\,')\,|\,U^\alpha\,|\,\chi\rangle$. We are accustomed to the lack of fore-aft symmetry in this expression for the elastic transition matrix element. It would obviously be a mistake to replace the undistorted wave by a distorted wave in the elastic scattering matrix element.

We emphasize this point very strongly, because not only is this of theoretical signficance, but it is also of great practical importance. The importance of the exclusion of a single state is illustrated by a comparison of $P_fU^\alpha P_f$ and $P_fW^\alpha P_f$, which differ only in that intermediate scatterings to states in P_f are excluded from W^α but not from U^α. The explicit relation is

$$P_fU^\alpha P_f=P_fW^\alpha P_f+P_fW^\alpha P_fG_\alpha P_fU^\alpha P_f\ . \tag{6.7}$$

Clearly the plane wave matrix elements of $P_fU^\alpha P_f$ represent the elastic scattering for the target in the excited state f which would result from the potential $P_fW^\alpha P_f$. Since experience has indicated that substantial differences are to be expected between the inhomogeneous term in Eq. (6.7) and its solution, we may conclude that distinction between Q, Q_f, and QQ_f cannot be ignored in practical numerical work. As yet there are no numerical calculations to indicate where these effects are largest and how large they can be.

In Sec. III of this paper an alternate derivation of the results of Sec. II is put forth and the analytic structure of this distorted wave theory is studied. One advantage of the treatment given in Sec. III is that the restriction to potential scattering is relaxed. A further attribute of this presentation is that it makes evident the unitarity properties of the theory and relates the cut structure of the important operators of the formalism to the physical scattering channels. This is a very useful accomplishment since it elucidates the structure of the theory in the fully antisymmetrized case, where many properties of the unsymmetrized discussion do not lend themselves to unambiguous generalization.

The segregational properties of the formalism are also treated at length, so that the topological structure of the theory becomes evident. Thus we are able immediately to see which parts of a given perturbation diagram belong to which elements of the theory. A figure which illustrates this point is included as an aid toward visualization of this very simple segregation property.

In Sec. IV, the question of the Pauli principle is taken up. It is shown that the theory presented earlier is readily generalized to take account of all antisymmetry effects in a manner very little different from that employed for elastic scattering. We observe that the details of the theory we obtain depend on the choice of the off-shell extension of the symmetrized transition operator $\mathscr{T}^{\alpha}$, and that if we wish to maintain the closest possible contact with the structural features of the unsymmetrized theory, then a particular choice of the off-shell extention is favored.

In Sec. V the application of the formalism toward a workable, *truncatable* theory is investigated. In particular, we present the spectator expansion of each of the three quantities of the DWA theory. We observe that for a specific choice of off-shell extension of T, different from that which seemed most favorable in the study of the analytic structure, the fully antisymmetrized theory leads to the same spectator expansion as does the unsymmetrized theory. The only restriction which the Pauli principle introduces is the requirement that each of the low-order T matrices, which are the ingredients of the spectator expansion, be antisymmetrized in the variables representing the active participants in the reaction.

The result of this paper is that a rigorous microscopic distorted wave theory of inelastic scattering exists. This theory will be seen[61] to be readily truncated so as to be computationally practicable over a wide region of physical circumstances. In Sec. V, we show that the lowest order impulse approximation for nucleon-nucleus inelastic scattering [cf. Eq. (5.22) *et seq*] is given by

$$T_{fi}^{\text{inel}} = \langle \eta_f^{(-)} | \sum_i t_i | \eta \rangle \; . \tag{6.8}$$

The transition operator in Eq. (6.8) is At, where t is the free nucleon-nucleon transition operator. This implies that the distorted waves $\langle \eta_f^{(-)} |$ and $| \eta \rangle$ must be given by Eqs. (5.23) and (5.24) so that $| \eta \rangle$ is the distorted wave generated by the potential $PU'P$, with

$$PU'P \equiv \frac{A-1}{A} P \sum_i t_i P \tag{6.9}$$

and $\langle \eta_f^{(-)} |$ is the distorted wave generated by the potential $P_f W' P_f$, with

$$P_f W' P_f = \frac{A-1}{A} P_f \sum_i u_i P_f \; , \tag{6.10}$$

where

$$u_i = t_i - t_i G_\alpha P u_i \; . \tag{6.11}$$

The distorted wave impulse approximation result given above, and in Sec. V, has an initial state distortion which is just that of the first order KMT form, as may be seen from Eq. (6.9). The final distorted wave is also in the KMT form where u plays the role of t. The purpose of this very brief excursion into the impulse approximation was simply to indicate how this theory can lead to practical prescriptions which differ from the standard distorted wave impulse approximation and are preferable on theoretical grounds. Some of the more practical aspects of the formulation of the theory of inelastic scattering are discussed in the following paper.[61]

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