Implications of anomalous isospin violation for the low energy nucleon-nucleon interaction

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Quark mass differences are used to derive an anomalous (i.e., due to the strong interactions) isospin violating potential. The potential includes the effects of pseudoscalar (π-η-η') and vector meson (ρ-ω) mixing. The finite ρ width is included. Low energy nn, pp, and np phase shift differences are calculated in terms of the fundamental coupling constants. Despite considerable uncertainty in the coupling constants, it is possible to understand the very small nn and pp scattering length differences. Furthermore, it appears that effects in the $l>0$ partial waves at finite energies are large enough to be observable in the next few years. Estimates of effects in bound nuclei are presented. These include calculations of the contributions to mirror nuclei binding energy differences and to isospin mixing for a few characteristic cases. In most of those cases the anomalous isospin breaking contributions are of roughly the same size as observed discrepancies. However, inclusion of short range correlations substantially reduces the calculated effects in these cases.

[NUCLEAR REACTIONS pp, pn, and nn elastic scattering, $E=0-350$ MeV, calculated phase shift differences due to anomalous isospin breaking. Effects in finite nuclei estimated.]

I. INTRODUCTION

Isospin was postulated as a quantum number for nucleons by Heisenberg[1] essentially immediately following the discovery of the neutron in 1932.[2] It was quickly realized that the weak interaction, which turns neutrons into protons and vice versa, requires nuclear wave functions that are antisymmetrized among all A particles rather than among protons and neutrons separately.[3] This led to *de facto* use of isospin as a nuclear quantum number.[4] By 1936 isospin conservation in the strong interaction was being explicitly endorsed on both aesthetic[5] and empirical[6] grounds. Since then it has been generally believed that the strong interactions are exactly invariant under isospin rotations and that all isospin breaking is due to the electromagnetic and weak interactions. This view, however, encounters two serious difficulties. The first is that it does not seem to work phenomenologically. For example, estimates of the electromagnetic contribution to the proton-neutron mass difference using either the Cottingham formula[7] or the quark model[8] generally yield the result $(m_p-m_n)^{\rm em}\simeq 1$ MeV, which is the right order of magnitude but the wrong sign. Similarly, for mirror nuclei with $A>1$ there have been persistent difficulties in explaining what were believed to be Coulomb energy differences.[9] A second difficulty is that exact strong interaction isospin invariance is not a natural feature of the modern gauge theories of the strong, weak, and electromagnetic interactions. The Lagrangian for quantum chromodynamics[10] (QCD), the candidate gauge theory of the strong interactions, is of the form

$$L^{\rm QCD}=L_0-m_u\bar{u}u-m_d\bar{d}d-m_s\bar{s}s -m_c\bar{c}c-m_b\bar{b}b-\cdots\ , \tag{1}$$

where L_0 (which contains the quark and gluon kinetic energy and interaction terms) is invariant under chiral $SU_N\times SU_N$ transformations (N is the number of quark flavors). The bare mass parameters m_u, m_d, $\cdots$ are presumably generated by the Higgs mechanism (or some other spontaneous symmetry scheme) in the weak $SU_2\times U_1$ sector of the theory, but can be regarded as bare masses as far as the strong interactions are concerned. $L^{\rm QCD}$ is invariant under isospin rotations only if $m_d=m_u$.

However, there is in general no more reason to expect $m_d = m_u$ than $m_s = m_d = m_u$ (exact SU_3 symmetry) or $m_c = m_s = m_d = m_u$ (exact SU_4 symmetry). In fact, quark masses generated by the Higgs mechanism will in general be different unless extra symmetries are added to the $SU_2 \times U_1$ model to force a degeneracy.

In general, therefore, isospin violation can be described by a Lagrangian

$$L^{\Delta SU_2} = L^{\text{tad}} + L^{\text{em}} , \tag{2}$$

where L^{em} describes the conventional electromagnetic interactions (any small weak interaction effects can be included in L^{em}), and

$$L^{\text{tad}} = -\tfrac{1}{2}(m_d - m_u)(\bar{d}d - \bar{u}u) \tag{3}$$

is the anomalous strong interaction term associated with quark mass differences. The superscript is motivated by the fact that L^{tad} is a concrete realization of the old Coleman-Glashow tadpole model.[11] L^{tad} is analogous to the term $-(2m_s - m_u - m_d) \times (2\bar{s}s - \bar{u}u - \bar{d}d)/6$, which breaks SU_3 symmetry.

Before proceeding, a number of comments concerning quark masses are in order[12]: (a) The quark masses are renormalized by both the strong and electromagnetic interactions. However, the details of the renormalization prescription drop out of the mass ratios, which are all that we will use. (b) Divergences in the matrix elements of L^{em} can be absorbed into the electromagnetic renormalization of $m_d - m_u$, so that $(m_p - m_n)^{\text{em}}$, for example, is a finite quantity. This is one reason why L^{tad} must be included in the theory.[13] However, once L^{tad} is introduced to absorb the electromagnetic divergences it can have a finite piece as well. This is the origin of anomalous isospin breaking. (c) It is believed that the axial vector generators of $SU_N \times SU_N$ in QCD are dynamically broken.[14] Hence, in the limit $m_u = m_d = m_s = 0$, the $SU_3 \times SU_3$ chiral symmetry would be dynamically broken down to SU_3. (The c and b quarks are sufficiently massive that it is not very useful to consider the limits $m_c = 0$, $m_b = 0$, etc.) Manifestations of this breaking would include eight Goldstone bosons (π, K, η), a nonzero pion decay constant, and a large dynamical (nonperturbative) mass $M^u = M^d = M^s \equiv M$ for the u, d, and s. The Lagrangian masses m_u, m_d, and m_s are referred to as current algebra masses to distinguish them from M. The current algebra masses explicitly break the $SU_3 \times SU_3$ symmetry[15,16]; they give masses to the π, K, and η and break SU_3 and SU_2. The mass parameters that occur in phenomenological models of hadrons are referred to as constituent masses. They are presumably a linear combination of the dynamical and current algebra masses.

There have been many studies of the ratios of the current algebra quark masses. The ratio $r_+ \equiv (m_d + m_u)/2m_s$ is determined from the pseudoscalar mass spectrum.[17–20] [Experimental and theoretical uncertainties in the pion-nucleon sigma term are too large to allow a useful determination of r_+ (Refs. 21 and 19).] The isospin violating quantity $r_- \equiv (m_d - m_u)/2m_s$ is determined from the baryon mass splittings,[18,19,22–25] the $K^+ - K^0$ mass splitting,[17–19,23–25] the $\eta \to 3\pi$ decay rate,[18,23,26–30] and ρ-ω mixing.[31,32] Additional existing or proposed tests of anomalous isospin violation in particle physics include the decay[33] $\psi' \to J/\psi\, \pi^0$, meson decays,[32] nonleptonic hyperon decays,[34] and $\Sigma \to \Lambda$ beta decay.[35,36] All of these determinations are in good agreement with each other. In this article we will use the typical values[31]

$$\begin{aligned} r_{\mp} &= \frac{m_d + m_u}{2m_s} = 0.031 \pm 0.007 , \\ r_- &= \frac{m_d - m_u}{2m_s} = 0.011 \pm 0.002 , \end{aligned} \tag{4}$$

from which one derives

$$\begin{aligned} \frac{m_u}{m_d} &= 0.47 \pm 0.11 , \\ \frac{m_d}{m_s} &= 0.042 \pm 0.007 . \end{aligned} \tag{5}$$

From (4) and (5) one concludes that $m_u \neq m_d$. Furthermore, the large size of the anomalous isospin violation $(m_u/m_d \simeq \frac{1}{2})$ suggests that it is intrinsic to the strong interactions and is not itself somehow induced by electromagnetism [which would yield $m_u/m_d \simeq 1 + O(\alpha)$].

The absolute scale of the current quark masses is more difficult to obtain and depends somewhat on the renormalization prescription.[37] A typical estimate would be $m_s \simeq 150$ MeV, $m_d \simeq 6$ MeV, and $m_u \simeq 3$ MeV. We will not utilize the absolute scale in any way but present it only to illustrate the fact that m_d, m_u, and (to a lesser extent) m_s are small compared to other hadronic mass scales, such as the dynamical mass $M \simeq m_p/3$. (In contrast, the current masses of the heavier quarks are much larger than the dynamical masses.) Hence, m_d and m_u are expected to play a very small role in hadron dynamics. It is this fact and not a near degeneracy of m_d and m_u that leads to the approximate isospin invariance of the strong interactions.

With this understanding of why the isospin breaking intrinsic to the strong interaction is expected to be small, it is of interest to attempt to calculate observable effects in the two-body and many body systems. First, let us review some terminology.

Charge independent or isospin conserving forces are those with no dependence on the isospin coordinates other than the isoscalar $\vec{\tau}_A \cdot \vec{\tau}_B$. (In terms of the classification scheme of Henley and Miller[38] these are Class I forces.) The presence of either isovector or isotensor forces will introduce charge dependence. Isotensor forces (Class II) lead to no differences between *nn* and *pp* observables, but contribute to differences between the $T=1$ *np* and the average of the *nn* and *pp* values. Charge symmetry breaking (CSB) or isovector forces lead to differences in *nn* and *pp* observables if the operator is symmetric in τ_A and τ_B (Class III). This is the most commonly treated type of CSB force. In addition, there are (CSB) operators antisymmetric in τ_A and τ_B (Class IV) which lead to mixing between $T=1$ and $T=0$ states of the *np* system. These operators, however, have no effect on the *nn* or *pp* systems.

In this paper we consider the effects of the isospin mixing of pseudoscalar (π-η-η') and vector (ρ-ω) mesons. The first is due essentially entirely to quark mass differences and is a purely strong interaction effect. In the case of ρ-ω mixing the strong interaction induced mixing is modified (downward) by 20% due to electromagnetic effects. There may well be other effects arising from the quark mass difference; some of these are discussed briefly in Sec. II.

The meson mixing contributions to anomalous isospin breaking considered here are purely isovector. There are also (apparently larger) isotensor forces resulting from the π^+-π^0 mass difference. As this difference is electromagnetic in nature and its effects have been well studied,[39] we do not include it here. For similar reasons, we do not consider the other various Coulomb or Coulomb-strong contributions to isospin breaking.

There have been many previous calculations[39,40] of isospin breaking effects in both low energy two-body scattering and nuclear structure. These are well reviewed in the articles by Henley and Miller[38] and by Shlomo.[9] Past works have generally assumed the isospin breaking to be due to meson mixing which was Coulombic in origin. However, since phenomenological mixing from $\eta \to 3\pi$ and $\omega \to 2\pi$ decays rather than *a priori* estimates were used in most previous calculations, the recognition that the mixing is due to the strong interaction does not lead to substantial changes in the final results.

The new features of these calculations which do affect the final results are the inclusion of the η' meson in the pseudoscalar meson mixing and the incorporation of the finite ρ-meson width in the vector meson calculations, and the recognition of the sensitivity of the results to coupling constant and short distance uncertainties. In addition, past calculations have generally focused on the $^1S_0$ scattering length,[41] whereas we have expanded the scope to finite energies and finite l values. Perhaps most important, the results for both scattering and bound states are now calculated within a systematic framework with excellent theoretical underpinnings. The calculations presented here are carried out using a nonrelativistic reduction of the isospin breaking diagrams into local potentials. This is not necessary, but substantially simplifies incorporation of the effects into standard nucleon-nucleon and nuclear structure treatments.

We have calculated the differences among *nn*, *pp*, and *np* phase shifts for *S*, *P*, and *D* waves from 0 to 350 MeV. In the $^1S_0$, $^3P_0$, and $^3P_1$ channels modest effects (of order $\frac{1}{2}°$) are found. Effects in the $^3P_2$ and $^1D_2$ phase shifts are very small. In the cases where the phase shift differences are appreciable, there are large uncertainties arising from uncertainties in coupling constants, and so results have been presented, whenever possible, in terms of these uncertain parameters. The most interesting results of the two-body calculations are an explanation of why the *S*-wave scattering length measurements are consistent with no charge symmetry breaking despite such terms in the potential, and indications that charge symmetry breaking effects are large compared to the uncertainties one may expect in independently determined $T=1$ *pp* and *np* finite energy phase shifts within the next few years.

In finite nuclei, CSB potentials induce mixing between nuclear states of different total isospin and also split the binding energies of mirror nuclei. We present two estimates of isospin mixing in finite nuclei: among the 1^- states in ^{4}He and between the 1^+ states in ^{12}C. We find reasonably large effects in the first case and small effects in the second. In general, effects will be larger for odd parity 1 particle-1 hole states. For states which are rearrangements within a shell we estimate the anomalous isospin effect to be $\frac{1}{10}$ that of the

Coulomb interaction.

The situation for binding energy differences is unique in that the dominant terms are attractive at short range and repulsive at long range for *nn* relative to *pp*. This leads to dramatic sensitivity to the very short range part of the nucleon-nucleon wave functions. A proper treatment of isospin breaking in the many-body problem will therefore require careful attention to these matters. We have made simple estimates which show that a naive treatment of the nuclear structure (uncorrelated single particle wave functions) leads to contributions from the isospin breaking large enough to remove the observed discrepancies in light mirror nuclei. Because of the short range of the CSB interaction, the observed increase with A is not reproduced. We have also shown that very modest phenomenological short range correlations can dramatically reduce the results for both mixing and energy shifts.

The theoretical isospin breaking potential is derived in Sec. II. In Sec. III we present results for the two-body system which explain the very small ${}^1S_0$ scattering length difference, and indicate the possibility of observing charge symmetry breaking in finite energy experiments. Our qualitative results concerning effects in bound systems are presented in Sec. IV. A summary and conclusions are given in Sec. V.

II. THE ISOSPIN VIOLATING POTENTIAL

In this section we construct an effective potential for the anomalous isospin-violating part of the low energy nucleon-nucleon interaction. It is of course beyond our technical ability to derive the exact implications of the quark operator L^{tad} in (3) for physical hadron interactions, so we must resort to approximations. In this we are guided by two observations: (a) one boson exchange potentials (OBEP), as shown in Fig. 1(a), have been reasonably successful in describing the long range part of the low energy NN interaction[42]; (b) a generally successful prescription for handling the effects of SU_3 breaking in particle physics has been to use physical masses when computing phase space and to include SU_3 breaking mixing effects (e.g., ω-ϕ, η-η', and f-f' mixing), but to ignore SU_3 breaking in amplitudes and vertex functions. In analogy, we will assume that the dominant effect of L^{tad} is to induce mixings between mesons relevant to an OBEP. This procedure is valid where the OBEP is a good description of the nucleon-nucleon interaction, at medium and long ranges. We apply it as well for nucleon-nucleon separations down to zero. This seems to us a reasonable prescription, but it is not rigorously justifiable and could possibly contain significant errors. In particular, we will consider π^0-η, π^0-η' [Fig. 1(b)], and ρ-ω [Fig. 1(c)] mixing effects. Of course there are other important isospin-violating effects that must be included in any comparison with experimental results. These include Coulomb (and other electromagnetic) interactions and kinematic (mass) effects, such as those associated with π^+-π^0 and p-n mass differences (m_p-m_n is assumed to be due in part to L^{tad}, but it is conventional to treat its effects separately).

There are many other possible effects which are difficult to evaluate but are hopefully small. These include (a) mixing effects between 0^+ mesons, which we ignore because of the highly confusing situation concerning these states[43]; (b) mixing between tensor mesons, which are usually ignored in OBEP models (and for which the relevant couplings are not well known); (c) a difference between the M_{nn}^0 and M_{pp}^0 coupling constants, where $M_0=\pi^0$ or ρ^0 (part of the difference is included in the π^0-η-η' and ρ-ω mixing effects. In fact, one can use a soft pion argument to show that π^0-η-η' mixing dominates the difference between $g_{\pi^0 pp}$ and $g_{\pi^0 nn}$ at zero momentum transfer); (d) multibody forces, such as those associated with the $\omega \to 2\pi$ or $\eta \to 3\pi$ vertices, with the mesons attached to different nucleons; and (e) the ρ^0-ρ^+ mass difference.

A. The pseudoscalar potential

The pseudoscalar isospin-violating potential[44] V_{PS} describes the effects of π^0-η and π^0-η' mixing. (The latter is surprisingly important.) The relevant diagrams in Fig. 1(b) correspond to an NN amplitude

$$T_{\mathrm{PS}} = -g_\pi g_\eta \mu_{\pi\eta}{}^2(\tau_A{}^3 I_B + I_A \tau_B{}^3) \times \frac{\bar{u}_2\gamma_5 u_1 \bar{u}_4 \gamma_5 u_3}{(t-\mu_\pi{}^2)(t-\mu_\eta{}^2)} + (\eta \to \eta') , \qquad (6)$$

where $\mu_{\pi\eta}{}^2 = -\langle \pi | L^{\mathrm{tad}} | \eta \rangle$ is the $\pi^0\eta$ mixing parameter, $t=(p_2-p_1)^2$ is the four-momentum transfer squared, $\tau_{A,B}{}^3$ and $I_{A,B}$ are Pauli and identity matrices acting on the nucleon isospin indices at vertices A and B, and the pseudoscalar coupling constants are defined by

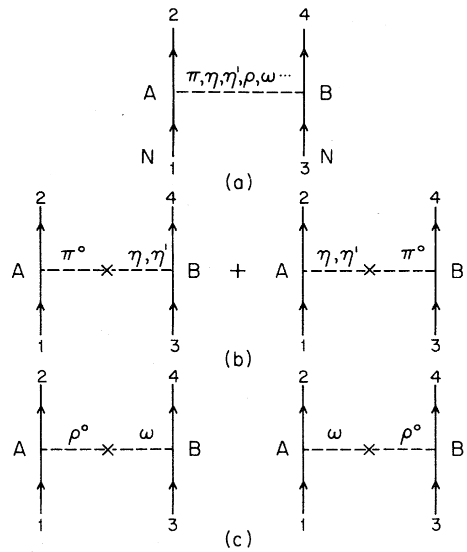

FIG. 1. (a) Meson exchange contributions to V_{NN}. (b) Pseudoscalar meson mixing contributions to the charge symmetry breaking part of V_{NN}. (c) Same as (b) but for vector mesons.

$$\begin{aligned}L_{\text{Yukawa}}=&-ig_\eta(\bar p\gamma_5 p+\bar n\gamma_5 n)\eta\\&-ig_{\eta'}(\bar p\gamma_5 p+\bar n\gamma_5 n)\eta'\\&-ig_\pi[(\bar p\gamma_5 p-\bar n\gamma_5 n)\pi^0\\&\quad+\sqrt{2}\bar p\gamma_5 n\pi^+ +\sqrt{2}\bar n\gamma_5 p\pi^-]\,. \qquad (7)\end{aligned}$$

We use the γ matrix and spinor conventions of Bjorken and Drell[45] throughout.

In (6) the SU_2 breaking mixing effects are treated perturbatively. To emphasize the more familiar mixing angle formalism, it is useful to rewrite (6) as

$$\begin{aligned}T_{\text{PS}}=&g_\pi g_\eta(\tau_A{}^3 I_B+I_A\tau_B{}^3)\bar u_2\gamma_5 u_1\bar u_4\gamma_5 u_3\\&\times\frac{\mu_{\pi\eta}{}^2}{\mu_\eta{}^2-\mu_\pi{}^2}\left[\frac{1}{t-\mu_\pi{}^2}-\frac{1}{t-\mu_\eta{}^2}\right]+(\eta\to\eta')\,.\end{aligned} \qquad (8)$$

The $(t-\mu_\pi{}^2)^{-1}$ terms represent the exchange of a physical pion

$$|\pi^0\rangle\simeq|\pi_B^0\rangle-\chi_{\pi\eta}|\eta_B\rangle-\chi_{\pi\eta'}|\eta'_B\rangle\,, \qquad (9)$$

where π_B, η_B, and η'_B are the unmixed states of definite isospin, and the mixing angles are

$$\chi_{\pi\eta}\equiv\frac{\mu_{\pi\eta}{}^2}{\mu_\eta{}^2-\mu_\pi{}^2};\quad \chi_{\pi\eta'}\equiv\frac{\mu_{\pi\eta'}{}^2}{\mu_{\eta'}{}^2-\mu_\pi{}^2}\,. \qquad (10)$$

Similarly, the $(t-\mu_\eta{}^2)^{-1}$ and $(t-\mu_{\eta'}{}^2)^{-1}$ terms represent the exchange of the physical particles

$$\begin{aligned}|\eta\rangle&\simeq|\eta_B\rangle+\chi_{\pi\eta}|\pi_B^0\rangle\,,\\|\eta'\rangle&\simeq|\eta'_B\rangle+\chi_{\pi\eta'}|\pi_B^0\rangle\,.\end{aligned} \qquad (11)$$

To derive a potential from T_{PS}, define the center of mass three momenta $\vec p$ and $\vec k$ by

$$\begin{aligned}\vec p_1&=-\vec p_3=\vec p-\vec k/2\,,\\\vec p_2&=-\vec p_4=\vec p+\vec k/2\,,\end{aligned} \qquad (12)$$

so that $t=-\vec k^2$. The nonrelativistic potential V_{PS} corresponding to T_{PS} is the Fourier transform of MT_{PS}/E evaluated in the limit $\vec p\to 0$, where M is the nucleon mass and $E=\sqrt{M^2+\vec k^2/4}$ is the center of mass (c.m.) energy of each nucleon.

From (8) one has

$$\begin{aligned}V_{\text{PS}}(r)=&-(\tau_A{}^3 I_B+I_A\tau_B{}^3)\frac{g_\pi g_\eta}{4\pi}\chi_{\pi\eta}\\&\times[v(\mu_\pi,r)-v(\mu_\eta,r)]+(\eta\to\eta')\,,\end{aligned} \qquad (13)$$

where (to leading order in the recoil factor μ^2/M^2)

$$v(\mu,r)\equiv\frac{\mu^3}{4M^2}[\tfrac{1}{3}\vec\sigma_A\cdot\vec\sigma_B\phi(\mu r)+S_{AB}\chi(\mu r)]\,, \qquad (14)$$

with

$$\begin{aligned}\phi(x)&\equiv\frac{e^{-x}}{x}\,,\\\chi(x)&\equiv\tfrac{1}{3}\left[1+\frac{3}{x}+\frac{3}{x^2}\right]\phi(x)\,.\end{aligned} \qquad (15)$$

In (14), σ_A and σ_B are the Pauli matrices acting on the spin indices at vertices A and B and S_{AB} is the tensor operator

$$S_{AB}\equiv 3\vec\sigma_A\cdot\hat r\vec\sigma_B\cdot\hat r-\vec\sigma_A\cdot\vec\sigma_B\,. \qquad (16)$$

The isospin factor $\tau_A{}^3 I_B+I_A\tau_B{}^3$ in (13) is equal to $+2$, 0, and -2 for pp, pn, and nn, respectively.

For the mixing angles in (13) we use

$$\begin{aligned}\chi_{\pi\eta}&=\frac{\mu_{\pi\eta}{}^2}{\mu_\eta{}^2-\mu_\pi{}^2}=-0.013\,,\\\chi_{\pi\eta'}&=\frac{\mu_{\pi\eta'}{}^2}{\mu_{\eta'}{}^2-\mu_\pi{}^2}=-0.0039\,,\end{aligned} \qquad (17)$$

which correspond to

$$\mu_{\pi\eta}{}^2 = -0.0036\ \text{GeV}^2\ ,$$
$$\mu_{\pi\eta'}{}^2 = -0.0035\ \text{GeV}^2\ . \tag{18}$$

Equations (18) are derived using

$$\mu_{\pi 8}{}^2 = \frac{-2}{\sqrt{3}}(\mu_K{}^2 - \mu_\pi{}^2)r_- = -0.0029\ \text{GeV}^2 \tag{19}$$

for the π^0-octet mixing, which follows from lowest order SU_3 breaking;

$$\mu_{\pi 1}{}^2 = \sqrt{2}\mu_{\pi 8}{}^2 = -0.0041\ \text{GeV}^2 \tag{20}$$

for the π^0-singlet mixing, which follows from a Zweig-rule type ansatz that L^{tad} will not induce mixing between π^0 and the $\bar{s}s$ state; and the assumption of an η-η' mixing angles $\theta_{\text{PS}} \simeq -10.4°$, where

$$| \eta_B \rangle = \cos\theta_{\text{PS}} | 8 \rangle - \sin\theta_{\text{PS}} | 1 \rangle\ ,$$
$$| \eta'_B \rangle = \sin\theta_{\text{PS}} | 1 \rangle + \cos\theta_{\text{PS}} | 8 \rangle\ . \tag{21}$$

This value follows from the quadratic mass formula or from plausible theoretical assumptions.[46] [Equation (17) would be changed to $\chi_{\pi\eta} = -0.010$ and $\chi_{\pi\eta'} = -0.0046$ if η-η' mixing were ignored. In our previous Letter[47] we set $\theta_{\text{PS}} = 0$ and used an earlier value $r_- = 0.014$.] Equation (17) is in reasonable agreement with the values $\chi_{\pi\eta} = -0.02$, $\chi_{\pi\eta'} = -0.003$ obtained by Isgur[25,32] from constituent quark arguments.

The largest uncertainties in V_{PS} are in the coupling constants g_η and $g_{\eta'}$. We have used two types of estimates which yield considerably different values for g_η but similar ratios $g_{\eta'}/g_\eta$. One approach is to combine SU_3 with the Zweig rule assumption that the $\bar{s}s$ state does not couple to nucleons. SU_3 predicts

$$g_8 = \sqrt{3}(1 - \tfrac{4}{3}\alpha)g_\pi\ , \tag{22}$$

where g_8 is the octet-nucleon coupling, $\alpha = D/(D+F)$, where F and D are the strengths of the antisymmetric and symmetric meson-baryon couplings, and $g_\pi{}^2/4\pi \simeq 14.01$ is the pion-nucleon coupling.[42] The Zweig rule ansatz implies

$$g_1 = \sqrt{2}g_8 \tag{23}$$

for the SU_3 singlet coupling, so that

$$g_\eta = \sqrt{3}(1 - \tfrac{4}{3}\alpha) \times (\cos\theta_{\text{PS}} - \sqrt{2}\sin\theta_{\text{PS}})g_\pi\ ,$$
$$g_{\eta'} = \sqrt{3}(1 - \tfrac{4}{3}\alpha) \times (\sin\theta_{\text{PS}} + \sqrt{2}\cos\theta_{\text{PS}})g_\pi\ . \tag{24}$$

Using the SU_6 prediction

$$\alpha = 0.60 \tag{25}$$

along with $\theta_{\text{PS}} = -10.4°$, (24) implies

$$\frac{g_\pi g_\eta^{\text{TH}}}{4\pi} = 6.01\ ,$$
$$\frac{g_\pi g_{\eta'}^{\text{TH}}}{4\pi} = 5.87\ , \tag{26}$$

where the superscript TH refers to the theoretical predictions for g_η and $g_{\eta'}$. The SU_3, SU_6, and Zweig rule assumptions on which (26) is based are generally believed to be fairly reliable. However, there is a near cancellation between F and D couplings in (24) (the $1 - 4\alpha/3$ term). Modest SU_3, SU_6, and Zweig rule breaking effects might not respect this cancellation and could possibly lead to a substantial modification of (26).

An alternate approach is to use the values g_η^{OB} and $g_{\eta'}^{\text{OB}}$ obtained from fits[42,44] of the OBEP to the nucleon-nucleon and hyperon-nucleon scattering data (the latter is needed to separate the η and η' contributions). This type of determination, although based directly on experimental results, relies heavily on the validity of specific OBEP models for the isospin conserving potential. One danger is that the parameters extracted include the effects of particle exchanges that have not been explicitly included in the model.

We will use the results of Nagels, Rijken, and deSwart,[44] who assume the SU_3 relation (22) but do not assume the Zweig rule [Eq. (23)] or SU_6 [Eq. (25)]. In an early fit utilizing $\theta_{\text{PS}} = -10.4°$ they obtain $g_\pi g_\eta/4\pi = 9.99$, $g_\pi g_{\eta'}/4\pi = 14.2$, $\alpha = 0.515$, and $g_1 = 2.2g_8$. However, these values of g_η and $g_{\eta'}$ may include effects of scalar exchanges, which were not included in the model. A later fit included a scalar nonet but used the mixing angle $\theta_{\text{PS}} = -23°$ obtained from a linear mass formula. The result was

$$\frac{g_\pi g_\eta^{\text{OB}}}{4\pi} = 10.32\ ,$$
$$\frac{g_\pi g_{\eta'}^{\text{OB}}}{4\pi} = 11.12\ , \tag{27}$$

with $\alpha = 0.59$ and $g_1 = 2.8g_8$. [Both (26) and (27) imply $g_\eta \sim g_{\eta'}$. The larger ratio g_1/g_8 and the linear mixing angle compensate in (27).] We will choose (27) as a typical OBEP result.

We will not attempt to argue whether the values $g_{\eta,\eta'}^{\text{TH}}$ in (26) or the larger values $g_{\eta,\eta'}^{\text{OB}}$ in (27) are superior. Rather, we will present results for both sets of parameters and consider the deviations to be an indication of the uncertainties involved.

B. The vector potential

The isospin violating potential[44] V_V associated with ρ-ω mixing [Fig. 1(c)] is calculated in a similar way. (We ignore ρ-ϕ mixing because both $\mu_{\rho\phi}{}^2$ and $g_{\phi NN}$ are Zweig suppressed for an ideally mixed (pure $\bar{s}s$)ϕ.] V_V is the Fourier transform of MT_V/E in the $\vec{p}\to 0$ limit, where (ignoring for now the finite ρ width)

$$T_V=-g_\rho g_\omega \frac{\mu_{\rho\omega}{}^2}{(t-\mu_\rho{}^2)(t-\mu_\omega{}^2)} \times[J^\rho_\mu(A)J^{\omega\mu}(B)+J^\omega_\mu(A)J^{\rho\mu}(B)] \,. \quad (28)$$

The current matrix elements J^ρ_μ and J^ω_μ are defined by

$$J^\rho_\mu=\bar{u}(\vec{p}_F)\tau^3\left[\gamma_\mu+\frac{iK^\rho}{2M}\sigma_{\mu\nu}(p_F-p_i)^\nu\right]u(\vec{p}_i)\,,$$

$$J^\omega_\mu=\bar{u}(\vec{p}_F)I\left[\gamma_\mu+\frac{iK^\omega}{2M}\sigma_{\mu\nu}(p_F-p_i)^\nu\right]u(\vec{p}_i)\,, \quad (29)$$

where τ^3 and I act on the nucleon isospin indices, and $K^\rho(K^\omega)$ is the ratio of the $\rho(\omega)$ magnetic to charge coupling. g_ρ and g_ω are defined by the $\rho^0 NN$ and ωNN vertices $-ig_\rho\epsilon^\mu_\rho J^\rho_\mu$ and $-ig_\omega\epsilon^\mu_\omega J^\omega_\mu$.

It is straightforward but tedious to derive V_V. It turns out that accurate results can only be obtained if recoil $(\vec{k}^2/M^2)$ corrections[48] are kept to order $(\vec{k}^2/M^2)^2$. To this order, the result is

$$V_V=(\tau_A{}^3 I_B+I_A\tau_B{}^3)\frac{g_\rho g_\omega}{4\pi}\frac{\mu_{\rho\omega}{}^2}{\mu_\rho{}^2-\mu_\omega{}^2} \times[v_+(\mu_\rho,r)-v_+(\mu_\omega,r)] +(\tau_A{}^3 I_B-I_A\tau_B{}^3)\frac{g_\rho g_\omega}{4\pi}\frac{\mu_{\rho\omega}{}^2}{\mu_\rho{}^2-\mu_\omega{}^2} \times[v_-(\mu_\rho,r)-v_-(\mu_\omega,r)] \,. \quad (30)$$

$v_\pm(\mu,r)$ are defined as

$$\begin{aligned} v_+(\mu,r)\equiv{}&\mu\left[\left[1+\frac{\mu^2}{8M^2}\right]+K^\Sigma\frac{\mu^2}{4M^2}+K^\pi\frac{\mu^4}{16M^4}\right]\phi(\mu r)-\mu\left[\frac{3\mu^2}{2M^2}+K^\Sigma\frac{\mu^2}{M^2}+K^\pi\frac{3\mu^4}{8M^4}\right]\vec{L}\cdot\vec{S}\lambda(\mu r)\\ &+\mu\left[\frac{\mu^2}{4M^2}+K^\Sigma\frac{\mu^2}{4M^2}+K^\pi\frac{\mu^2}{4M^2}\left[1+\frac{\mu^2}{8M^2}\right]\right][\tfrac{2}{3}\vec{\sigma}_A\cdot\vec{\sigma}_B\phi(\mu r)-S_{AB}\chi(\mu r)]\\ &+3\mu\left[\frac{\mu^4}{16M^4}+K^\Sigma\frac{\mu^4}{4M^4}+K^\pi\frac{\mu^4}{2M^4}\right]Q_{AB}\frac{\chi(\mu r)}{\mu^2 r^2} \end{aligned} \quad (31)$$

and

$$v_-(\mu,r)\equiv\frac{1}{2}\frac{\mu^3}{M^2}(K^\omega-K^\rho)(\vec{\sigma}_A-\vec{\sigma}_B)\cdot\vec{L}\lambda(\mu r)\,. \quad (32)$$

In (31) and (32),

$$K^\Sigma\equiv K^\omega+K^\rho\,,\qquad K^\pi\equiv K^\omega K^\rho\,, \quad (33)$$

$$\lambda(x)\equiv\left[\frac{1}{x}+\frac{1}{x^2}\right]\phi(x)\,, \quad (34)$$

and

$$Q_{AB}\equiv\tfrac{1}{2}[\vec{\sigma}_A\cdot\vec{L}\vec{\sigma}_B\cdot\vec{L}+\vec{\sigma}_B\cdot\vec{L}\vec{\sigma}_A\cdot\vec{L}]\,. \quad (35)$$

ϕ, χ, and S_{AB} are defined in (15) and (16).

So far we have neglected the finite ρ width. However, μ_ω-μ_ρ is small compared to Γ_ρ, so the effects of Γ_ρ should be included in (30) (ω width effects are negligible). We follow the prescription of Nagels *et al.*[44] in replacing the propagator $(\mu_\rho{}^2\simeq -t)^{-1}\simeq(\vec{k}^2+\mu_\rho{}^2)^{-1}$ by

$$P(\vec{k}^2)\equiv\left[\vec{k}^2+\mu_\rho{}^2+\mu_\rho\Gamma_\rho\frac{\vec{k}^2}{\mu_\rho{}^2}\left[\frac{\vec{k}^2+4\mu_\pi{}^2}{\mu_\rho{}^2-4\mu_\pi{}^2}\right]^{3/2}\right]^{-1}, \quad (36)$$

which has the correct $\vec{k}^2\to -4\mu_\pi{}^2$ threshold behavior and the correct limit

$$P(\vec{k}^2)\underset{\vec{k}^2\to-\mu_\rho{}^2}{\longrightarrow}(\vec{k}^2+\mu_e{}^2-i\mu_\rho\Gamma_\rho)^{-1}\,. \quad (37)$$

$P(\vec{k}^2)$ can be rewritten as

$$P(\vec{k}^2)=\int_{4\mu_\pi^2}^{\infty}\frac{\sigma(\mu'^2)d\mu'^2}{\vec{k}^2+\mu'^2}\ , \tag{38}$$

with the spectral function[49]

$$\sigma(\mu'^2)\equiv\frac{1}{\pi\mu_\rho}\frac{\alpha(\lambda)}{(\lambda-1)^2+\alpha^2(\lambda)}\ , \tag{39}$$

where

$$\lambda\equiv\mu'^2/\mu_\rho^2 \tag{40}$$

and

$$\alpha(\lambda)\equiv\lambda\frac{\Gamma_\rho}{\mu_\rho}\left[\frac{\lambda-4\mu_\pi^2/\mu_\rho^2}{1-4\mu_\pi^2/\mu_\rho^2}\right]^{3/2}\ . \tag{41}$$

Hence, the effect of Γ_ρ is to replace $V_V(\mu_\rho,\mu_\omega)$ in (30) by

$$\int_{4\mu_\pi^2}^{\infty}\sigma(\mu'^2)V_V(\mu',\mu_\omega)d\mu'^2\ . \tag{42}$$

We include the ρ width via (42) in all of our calculations. (The μ' integrals are evaluated numerically with μ' increasing in 25 MeV steps from 0.4 to 2.0 GeV.)

For the ρ-ω mixing parameter we choose[31]

$$\begin{aligned}\mu_{\rho\omega}^2&=-\langle\rho\,|\,L^{\text{tad}}+L^{\text{em}}\,|\,\omega\rangle\\&=-0.0037\ \text{GeV}^2\ ,\end{aligned} \tag{43}$$

of which $-0.0044\ \text{GeV}^2$ is due to L^{tad} and $+0.0007\ \text{GeV}^2$ is from L^{em}.

For g_ρ and g_ω we use the OBEP result[44,42]

$$\frac{g_\rho g_\omega}{4\pi}=2.80\ , \tag{44}$$

which is compatible with vector meson dominance (VMD) for the electromagnetic charge form factors when the small ϕNN coupling is taken into account.[50]

The relative magnetic couplings K^ρ and K^ω are considerably more uncertain. VMD for the magnetic form factors (which is untested) implies

$$\begin{aligned}K^\rho_{\text{VMD}}&=3.7\ ,\\K^\omega_{\text{VMD}}&=-0.12\ ,\end{aligned} \tag{45}$$

while OBEP fits (when VMD is not assumed) yield considerably different values. (See Nagels *et al.*[44] for a discussion of the validity of VMD.) For example, Nagels *et al.* find

$$\begin{aligned}K^\rho_{\text{OB}}&=6.60\ ,\\K^\omega_{\text{OB}}&=0.655\ .\end{aligned} \tag{46}$$

We will generally present results for both (45) and (46) or with K^ρ and K^ω left as unknown parameters.

C. Summary

Our expressions for the π-η-η' and ρ-ω contributions to the anomalous isospin violating potential are given in (13) and (30), with the ρ width effects incorporated using (42). The parameters are listed in Table I. We consider the mixing masses and (to a lesser extent) $g_\rho g_\omega/4\pi$ to be reasonably well known. g_η, $g_{\eta'}$, K^ρ, and K^ω are very uncertain, however. The two sets of g_η and $g_{\eta'}$ values in Table I both have $g_{\eta'}/g_\eta\sim 1$ to within 10% so the dominant uncertainty in V_{PS} is an overall factor of g_η.

We have presented most results in terms of competing parameter sets, which we call case I, case II, and case III. These are defined as:

(I): Use of the one boson (OB) couplings for both pseudoscalar and vector mesons;

(II): use of the one boson coupling for the pseudoscalar and vector meson dominance (VMD) for the vector meson; and

(III): use of the theoretical (TH) couplings for the pseudoscalar mesons and the one boson (OB) couplings for the vector mesons.

TABLE I. Parameters used in the isospin violating potential. TH, VMD, and OB refer to estimates derived from theory (SU_3, SU_6, Zweig rule), vector meson dominance, and one boson exchange potential fits (to the isospin conserving potential), respectively. We refer frequently to three sets of coupling constants. These are: case I: g_η^{OB}, $g_{\eta'}^{\text{OB}}$, K^ρ_{OB}, K^ω_{OB}; case II: g_η^{OB}, $g_{\eta'}^{\text{OB}}$, K^ρ_{VMD}, K^ω_{VMD}; case III: g_η^{TH}, $g_{\eta'}^{\text{TH}}$, K^ρ_{OB}, K^ω_{OB}.

$\mu_{\pi\eta}^2$	-0.0036 GeV^2
$\mu_{\pi\eta'}^2$	-0.0035 GeV^2
$\chi_{\pi\eta}$	-0.013
$\chi_{\pi\eta'}$	-0.0039
$g_\pi g_\eta^{\text{TH}}/4\pi$	6.01
$g_\pi g_{\eta'}^{\text{TH}}/4\pi$	5.87
$g_\pi g_\eta^{\text{OB}}/4\pi$	10.32
$g_\pi g_{\eta'}^{\text{OB}}/4\pi$	11.12
$\mu_{\rho\omega}^2$	-0.0037 GeV^2
$g_\rho g_\omega/4\pi$	2.80
K^ρ_{VMD}	3.7
K^ω_{VMD}	-0.12
K^ρ_{OB}	6.60
K^ω_{OB}	0.655

Where possible, we have also presented results as a function of K^{ρ} and K^{ω}.

We believe that $V_{\rm CSB} \simeq V_{\rm PS} + V_V$ is a reasonable first approximation to the anomalous isospin violating potential. However, there are several caveats. These include (a) the possible importance of other isospin violating effects (e.g., coupling constants, multibody forces, mixing between scalar or tensor mesons, etc.); (b) uncertainties in the meson-nucleon coupling constants; (c) uncertainties in the $\vec{k}^2$ dependence of the ρ propagator; and (d) the effects of form factors at the meson-nucleon vertices (which we have ignored), of higher order recoil corrections (other than those associated with form factors), and of ambiguities in the nonrelativistic reduction (the last two effects could be resolved by using the full relativistic expressions for T_V and $T_{\rm PS}$, but this does not seem worthwhile given our ignorance of possible form factors).

In this section we have presented a potential which is at least the first step towards a systematic theoretical calculation of anomalous isospin breaking. Neither uncertainties in coupling constants nor the presence of other terms not tested here are likely to affect the qualitative conclusions presented in the following sections.

III. CHARGE SYMMETRY BREAKING IN THE TWO-BODY SYSTEM

In this section we report on the implications of the CSB potential of Sec. II for the two-body scattering system. Our main purpose is to show what should be expected due to anomalous isospin breaking. We are able to explain the absence, to date, of a clear experimental signature of anomalous isospin breaking in the $^1S_0$ scattering length and effective range measurements,[51] and also to offer hope that unambiguous experimental evidence may soon be attainable at finite energies.

The calculations were performed using the following simplifying assumptions. The proton-proton interaction is given by the Reid soft-core potential. The neutron-neutron interaction differs from this by $V_{\rm CSB}$: $V_{nn} = V_{pp} - V_{\rm CSB}$. This is the only difference between neutrons and protons: The electromagnetic interaction is turned off and we set $m_n = m_p$. Phase shifts for the nn and pp systems are then determined by solving the Schrödinger equation.

Since the CSB force is relatively weak, and effects on the phases are approximately linear in the fundamental isospin breaking parameter, r_- of Eq. (4), the contributions to differences between the np and pp phases may be taken to be half those quoted for nn vs pp. Class IV forces have no effect on the nn or pp systems, and so are neglected by this procedure even though they do contribute to the diagonal np phases in second order. For the potential of Eq. (32) these effects are miniscule.[52] In general, of course, there are a number of other well known charge dependent effects which must be added to the CSB effects calculated here before comparison of pp and np experiments is meaningful.

A crucial aspect of the meson mixing contributions to charge symmetry breaking lies in the intrinsic cancellations between contributions with isovector-like range (π or ρ) and those with isoscalarlike ranges. In almost all cases this results in a charge symmetry breaking potential which changes sign at least once as a function of r. This is illustrated in Fig. 2 with plots of $r^2 V_{\rm CSB}$ ($^1S_0$) for cases I and II as defined in Table I. Similar plots may be made for the pseudoscalar and vector meson contributions separately, or for the higher partial waves. (The sign convention on $V_{\rm CSB} = V_{pp} - V_{nn}$ was chosen so that positive $V_{\rm CSB}$ corresponds to additional attraction in the nn system.)

We now consider the pseudoscalar meson contribution to $\Delta\delta(^1S_0) = \delta_{nn}(^1S_0) - \delta_{pp}(^1S_0)$, which is plotted versus laboratory kinetic energy in Fig. 3. At low energies the π-η' mixing dominates the π-η mixing and clearly must be included, whereas at energies around 100 MeV only the π-η contribution is important. This result may be understood by considering the effects of pionlike and the η- or η'-like

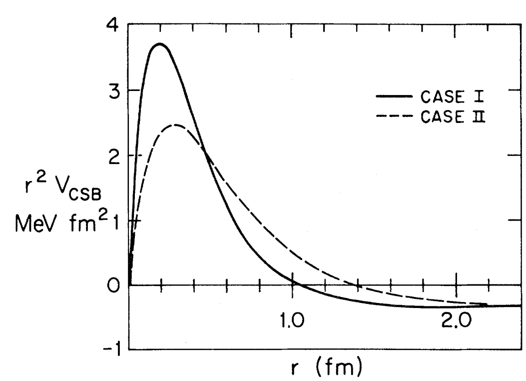

FIG. 2. Plots of $r^2\ V_{\rm CSB}$ for the $^1S_0$ partial wave as a function of r to illustrate both the short range nature of the potential, and the intrinsic cancellations. The solid (case I) and dashed (case II) lines correspond to different ferent sets of coupling constants as described in Table I and in the text.

terms in V_{CSB} separately. The long-range pionlike terms are repulsive whereas the shorter range isoscalar terms are attractive. At low energies the repulsive core of the charge symmetric potential screens off the much stronger η and η' parts fairly effectively, and there are near cancellations in both π-η and π-η' mixing. At zero energy the η-like piece dominates the pionlike piece so the contribution is attractive in the nn system. However in the π-η' mixing the pionlike piece dominates at low energies, ultimately being overtaken by the η'-like contribution at around 50 MeV. At substantially larger energies than we have shown the π-η and π-η' pieces would be roughly equal. The calculations presented in Fig. 3 use the OB coupling constants; similar cancellations result from the use of the TH coupling constants.

In the vector meson contribution to $\Delta\delta(^1S_0)$ there are large cancellations between the K^ρ and K^ω dependent and independent parts of the interaction. The magnetic couplings are quite uncertain, so we present different dependences separately. We have plotted in Fig. 4 $\frac{1}{10}$ the independent term ($A/10$) and the terms proportional to the sum $K^\Sigma=K^\omega+K^\rho$ (B) and the product $K^\pi=K^\rho K^\omega$ (C). The cancellation between K dependent and K independent terms is roughly energy independent, and for reasonable values of the magnetic coupling constants is nearly complete.

This completeness of the cancellation is illustrated in Figs. 5 and 6 in which contour plots of the ρ-ω mixing contribution to the scattering length difference $\Delta a=|a_{nn}|-|a_{pp}|$ and to $\Delta\delta(^1S_0)$ at 200 MeV are presented as a function of K^ρ and K^ω. The full circles indicate the vector dominance and one boson model values. Essentially all published pairs of values of K^ρ and K^ω lie near the line con-

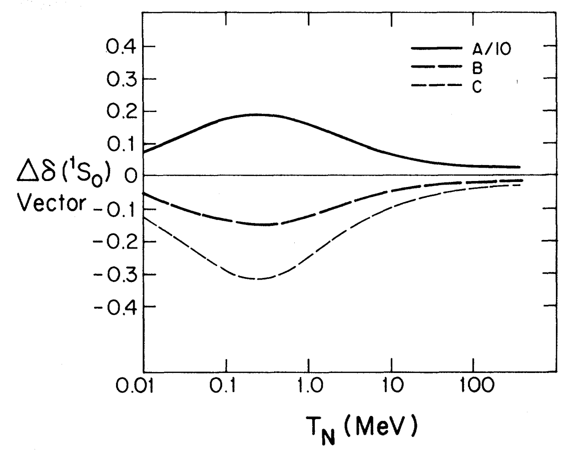

FIG. 4. Vector meson mixing contributions to $\Delta\delta(^1S_0)$ as a function of laboratory kinetic energy. The contributions are broken down into parts independent of the magnetic couplings (A), proportional to the sum $K^\Sigma=K^\rho+K^\omega$ (B), and proportional to the product $K^\pi=K^\rho K^\omega$ (C). Since K^ρ is large and positive we expect substantial cancellations among the three contributions. The final result depends significantly on the magnetic couplings used.

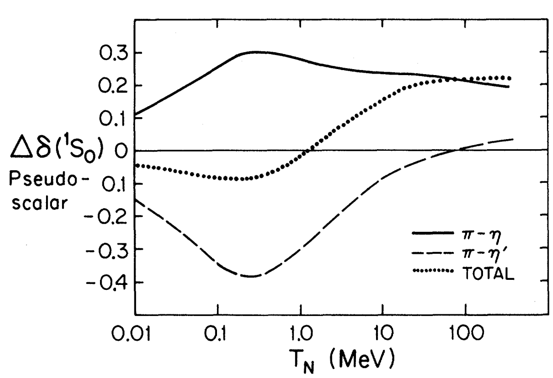

FIG. 3. Pseudoscalar meson mixing contributions to $\Delta\delta(^1S_0)$ in degrees as a function of laboratory kinetic energy. Positive values of $\Delta\delta$ correspond to additional attraction in the nn as opposed to the pp system ($\Delta\delta=\delta_{nn}-\delta_{pp}$). The calculations used the larger OB coupling constants, but the net effect is still rather small. (Use of the TH coupling constants yields a similar curve, scaled down by a factor $\lesssim 2$.)

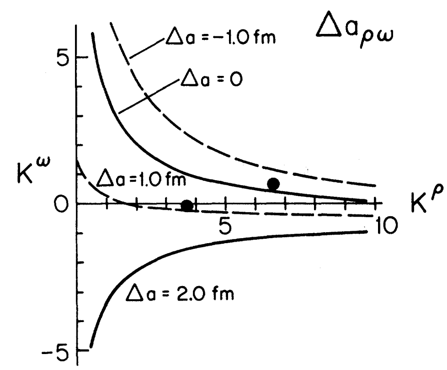

FIG. 5. Contour plot of the vector meson contribution to the difference in neutron and proton scattering lengths, $\Delta a=|a_{nn}|-|a_{pp}|$, showing the dependence on K^ω and K^ρ. The solid circles indicate the OB (upper right) and VMD (lower left) values. Since these are relatively extreme we see that phenomenologically or theoretically motivated coupling constants will lead to near zero contributions to the difference in scattering length.

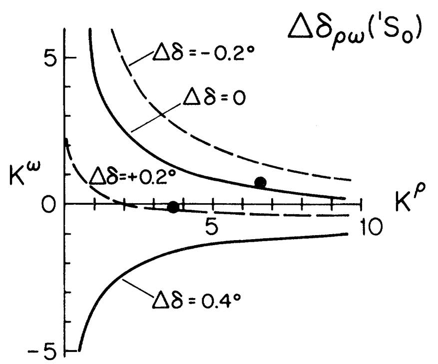

FIG. 6. Same as Fig. 5 but for the $^1S_0$ phase shift difference at 200 MeV.

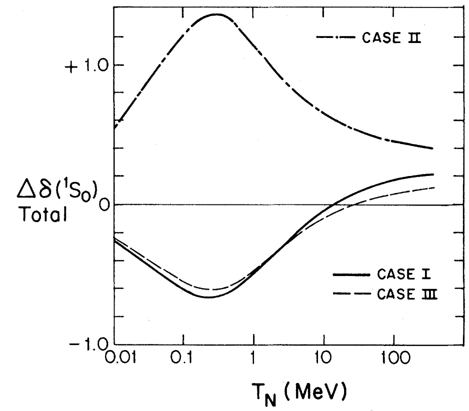

FIG. 7. Total contribution to $\Delta\delta(^1S_0)$ for the three sets of coupling constants defined in Table I. The effects are quite small at medium energies, and very uncertain at low energies.

necting those two relatively extreme points. In other words, the phenomenologically allowed K^ρ and K^ω values cluster around the $\Delta a = 0$ or $\Delta\delta = 0$ lines, but may be found on either side. In Fig. 7 plots of the total contributions to $\Delta\delta(^1S_0)$ for the three sets of coupling constants illustrate the extreme sensitivity to K^ρ and K^ω at low energies, as well as the insensitivity to the pseudoscalar parameters. Since the largest effects are at low energy, the various contributions to Δa and Δr_e are given in Table II.

Because of the cancellations and of the importance of contributions from very short distances we have investigated the sensitivity to the choice of phenomenological potential and to the incorporation of the finite ρ width. By explicitly computing the contributions from $r < 0.5$ fm, and by comparing results using the Reid soft and hard core potentials, we estimate these uncertainties to be of the order of 60%. The hard core potential more effectively screens the short range part of V_{CSB} than does the soft core potential. This leads to decreased attraction or increased repulsion in the nn channel relative to the pp channel (see Fig. 2).

Use of a zero width ρ meson changes the results by 10 to 30% depending on the magnetic couplings used. Modest changes in the ρ width ($\sim$30 MeV) have almost no effect on the final answers.

To summarize these results for the $^1S_0$ partial wave: The effects of pseudoscalar mixing are very

TABLE II. Effects on scattering length and effective range. All lengths are in fm. The experimental determinations of Δa and Δr may be found in Refs. 9, 38, and 51. Note the numbers in the totals section can only be approximately generated from the pseudoscalar and vector numbers, since it is $\Delta a/a$ rather than Δa itself that is linear in V_{CSB}.

		Pseudoscalar (OB)	
	π-η	π-η'	Total
Δa	0.18	−0.24	−0.06
Δr	0.025	0.003	+0.028
		Vector	
	A	B	C
Δa	1.22	−0.096	−0.185
Δr	0.022	−0.0018	−0.003
		Totals	
	Case I	Case II	Case III
Δa	−0.43	+0.86	−0.39
Δr	+0.022	+0.045	+0.010

small at all energies for all reasonable pseudoscalar coupling constants. The contributions from vector meson mixing are also small, except at very low energies. There the theoretical predictions are very uncertain because of strong dependence on both the short range behavior of the wave function and on the magnetic couplings. Experimental efforts to determine the *nn* and *pp* scattering lengths should certainly continue, but any result of the order ± 1 fm for the difference will be consistent with some reasonable choice of coupling constants.

Contributions to *P* and *D* waves are presented in Fig. 8, divided into the OB pseudoscalar contributions (dotted), and vector contributions independent of magnetic couplings (*A*, solid), proportional to K^Σ (*B*, long dash), and proportional to K^π (*C*, short dash). A quick glance shows that the individual contributions to the $^3P_2$ and $^1D_2$ partial waves are sufficiently small that the totals will be small for any reasonable set of coupling constants. For the cases we have examined we find $|\Delta\delta(^3P_2)| < 0.2°$ and $|\Delta\delta(^1D_2)| < 0.1°$, which are too small to be of interest.

Quite large effects are expected for the $^3P_0$ and $^3P_1$ phase shifts, as is shown in Fig. 9. The expected value for $\Delta\delta(^3P_0)$ is dependent on both pseudoscalar and vector meson coupling constants, whereas $\Delta\delta(^3P_1)$ is nearly independent of the vector meson couplings, but dependent on the pseudoscalar meson couplings. These expected large effects are quite promising for efforts to observe anomalous isospin breaking. Furthermore, unlike the situation for the $^1S_0$ phase, there is essentially no contribution from distances inside 0.6 fm. These results will therefore be insensitive to details of the short range part of the charge symmetric nucleon-nucleon interaction. This has been verified by repeating these calculations using the Reid hard core potential.

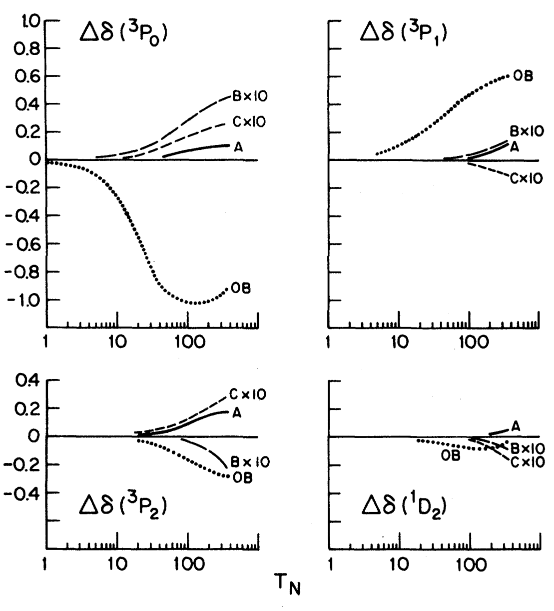

FIG. 8. Contributions of the pseudoscalar (dotted, OB) and vector meson mixing to the *P* and *D* wave $T=1$ phase shifts. The vector contributions are divided into three parts which are independent of magnetic couplings (*A*, solid line), proportional to the sum $K^\Sigma = K^\rho + K^\omega$ (*B*, long dash) or proportional to the product $K^\pi = K^\rho K^\omega$ (*C*, short dash). The curves suggest large effects in the $^3P_0$ and $^3P_1$ phases and small effects in the $^3P_2$ and $^1D_2$ phases.

Isospin breaking may also manifest itself in different coupling constants for neutrons and protons g_{Mpp} and g_{Mnn}. Neither the sign nor magnitude of the coupling constant differences are reliably known, but various approximate calculations suggest[53] that the differences are very small (less than a percent). In order to get a rough idea of the consequences of coupling constant differences, we have calculated with the CSB potential implied by taking

$$V_{nn} = (1+\epsilon)V_{pp} \ . \tag{47}$$

This corresponds to rescaling all coupling constants by the same factor. We have not attempted to treat separately the effects of individual meson exchanges. For $\epsilon = 0.01$, for example, the effects of (47) are only about $\frac{1}{6}$ of those due to meson mixing for all cases considered with the exception of the $^1S_0$ phase shift below 20 MeV. A more careful

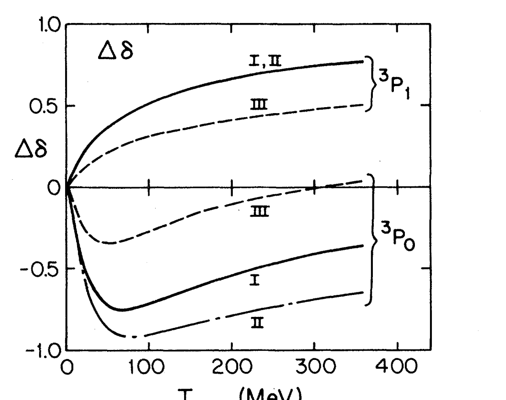

FIG. 9. Totals for $\Delta\delta(^3P_0)$ and $\Delta\delta(^3P_1)$ for the three sets of coupling constants considered here (see Table I) case I solid line, case II long short dashed line, case III short dashed line; for the $^3P_1$ phases the case II and case I are nearly identical, indicating weak dependence on the magnetic couplings of the vector mesons.

treatment of the coupling constants is still an unsolved theoretical problem. Nevertheless, we feel that these calculations indicate that for two-body scattering data the effects are likely to be unimportant.

The important point to emphasize is that we expect large CSB effects, large enough to be readily observable in the near future, in the $^3P_0$ and $^3P_1$ phase shifts. While uncertainties in the coupling constants make it impossible to predict the exact size, a search is certainly warranted, especially since the qualitative results should be unaffected by either coupling constant differences, or the details of the N-N interaction at very short range.

An analysis of the present pp and np data base by Arndt[54] shows some evidence for isospin breaking in the $^3P_0$ and $^3P_1$ channels. Statistical uncertainties are only marginally smaller than the deduced differences, and so more precise data seems to be needed. Experiments planned at the Indiana University Cyclotron Facility cyclotron[55] should provide this data.

Let us review charge symmetry breaking in the two-body system. We have reconciled the existence of a substantial charge symmetry breaking potential with the near zero experimental values for Δa. The importance of including π-η' mixing at low energies has been demonstrated. Finally, CSB effects are calculated to be large enough in the $^3P_0$ and $^3P_1$ channels to be readily observable in the next generation of np and pp elastic scattering experiments.

IV. EFFECTS OF ANOMALOUS ISOSPIN BREAKING IN BOUND STATES

There are basically two ways in which a charge symmetry breaking potential can manifest itself in bound states. Primary attention has focused on differences in ground state binding energies between mirror nuclei over and above the expected Coulomb effects. In a self-conjugate nucleus, charge symmetry breaking forces may lead to mixing between $T=0$ and $T=1$ states of the same spin and parity.

There are many subtle features to the determination of the "anomalous" part of these effects. For this work, we evaluate the contributions of the potential of Sec. II to the ^{3}H-^{3}He mass difference, the Nolen-Shiffer anomalies for ^{17}O-^{17}F and ^{41}Ca-^{41}Sc, and isospin mixing between 1^- states in ^{4}He and 1^+ states in ^{12}C and ^{40}Ca. The evaluations are done using naive structure models simply to show when our anomalous isospin breaking potential will or will not contribute substantially to the matrix elements in question. As a general rule, we find that very large effects are predicted for those cases involving the $^1S_0$ interaction. However, in this channel the very short range parts of the interaction are important. This leads to major uncertainties in the calculation of isospin violating effects in bound states.

Our first estimate is of the ^{3}He-^{3}H binding energy differences. Following Brandenberg, Coon, and Sauer[56] we consider the $^1S_0$ contribution only. However, instead of using the electron scattering form factor we use harmonic oscillator wave functions with a length parameter adjusted to the rms radius.[57] We find that both the pseudoscalar and vector contributions are attractive in the nn states. The pseudoscalar contribution is computed to be 84 (50) keV, using the OB (TH) coupling constants of Table I. The vector meson contribution is around 75 keV for both the one boson (OB) and vector meson dominance (VMD) parameters. This leads to a total increased triton binding energy of between 125 and 160 keV, which is substantially larger than the 80 keV discrepancy quoted by Brandenburg, Coon, and Sauer.[56]

Our main reason for using harmonic oscillator wave functions was the ease with which Jastrow-type short range correlations could be added. Correlations of the form

$$N(1-e^{-r^2/c^2}) , \tag{48}$$

where N preserves the overall normalization, are included in the relative neutron-neutron and proton-proton densities, and the binding energy difference ΔE_B is again computed for pseudoscalar (OB) and vector (OB and TH) potential as well as for the sums. The results as a function of the correlation length c are presented in Fig. 10. Phenomenological correlation lengths are generally in the range 0.7 fm $\leq c \leq 1.2$ fm.[58] However, the figure shows that reductions in ΔE_B of factors of 2 or 3 are achieved by correlation lengths as small as 0.4 fm. The reason for this dramatic sensitivity is evident from a reexamination of the plot of $r^2 V_{\rm CSB}$ in Fig. 2: The volume integral of $V_{\rm CSB}$ is positive, but falls very rapidly if the short range contributions are excluded. Uncertainties of this magnitude arising from the very short range behavior of the wave functions is an unusual if not unprecedented result.

Shlomo[9] has previously investigated these effects in heavier nuclei by comparing charge symmetry breaking computed using either harmonic oscillator or Bethe-Goldstone wave functions. He concludes that the correlations must be included, since results

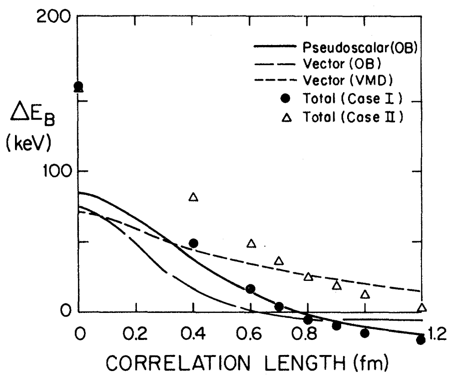

FIG. 10. Meson mixing contributions to the binding energy difference in mass 3 as a function of correlation length. Plotted are the pseudoscalar contribution with the OB coupling constants (solid line), and the vector contributions for both the OB (long dash) and VMD (short dash) couplings. The totals are given as well. The large effects of relatively short correlation lengths should be noted. Corresponding sensitivity will occur in relative $l=0$ contributions in heavier nuclei.

for his short range potentials change by about a factor of 2. However, his estimates were based on positive definite potentials only. Our results, calculated with CSB potentials that change sign as a function of r show much more dramatic change with the inclusion of correlations, leaving even the sign of the effect in doubt. The cancellations are sufficiently sensitive that even in the context of a careful Brueckner-Bethe-Goldstone treatment there will probably remain substantial model dependence on the short range behavior and on the fundamental coupling constants.

We do not know how to remove these uncertainties. We do feel, however, that these will be the dominant uncertainties in any calculation in which the $^1S_0$ phase figures prominently. With this in mind, we conclude that the meson mixing contributions to the mass three binding energy difference could be as large as 100 keV, but the contribution is likely to be under 30 keV and of uncertain sign.

Any reasonable attempt to evaluate binding energy differences due to V_{CSB} for systems heavier than mass four will require matrix elements of the ^3P_J and $^1D_2$ waves as well as the $^1S_0$. We have computed integrals of the form

$$I_{LSJ}^{M}=\frac{\int V_{\text{CSB}}(r,{}^{2S+1}L_J)r^{2L+2n+2}e^{-r^2/b^2}dr}{\int r^{2L+2n+2}e^{-r^2/b^2}dr}\ . \tag{49}$$

These numbers are given for the pseudoscalar, vector, and total contributions using the case I coupling constants in Tables III–VII for the $^1S_0$, ^2P_J, and $^1D_2$ states for values of b corresponding to carbon, oxygen, and calcium radii. These numbers may be exactly converted to matrix elements of harmonic oscillator states for radial quantum number $N_r=0$, 1, and 2. Because of the short range of the V_{CSB} interaction there is almost no contribution for $n>2$, so very accurate numbers can be obtained even for larger N_r.

The numbers in Tables III–VII may be used to compute the contribution of V_{CSB} to a specific analog binding energy difference. We have calculated this for the naive ground states of the ^{17}O-^{17}F and ^{41}Ca-^{41}Sc pairs. By naive ground states we mean a harmonic oscillator single particle model using oscillator length parameters which roughly describe the nuclear size — $b=1.769$ for O-F and $b=2.00$ for Ca-Sc.

TABLE III. Matrix elements of the $^1S_0$ CSB interaction as defined in Eq. (49). ($D-x$ signifies 10^{-x}.)

b	n	Pseudoscalar	Vector	Total
1.5000	1	$0.191D+00$	$0.157D+00$	$0.348D+00$
	2	$-0.233D-01$	$-0.471D-02$	$-0.280D-01$
	3	$-0.198D-01$	$-0.132D-02$	$-0.211D-01$
1.7690	1	$0.108D+00$	$0.941D-01$	$0.202D+00$
	2	$-0.189D-01$	$-0.251D-02$	$-0.214D-01$
	3	$-0.126D-01$	$-0.511D-03$	$-0.131D-01$
2.0000	1	$0.698D-01$	$0.645D-01$	$0.134D+00$
	2	$-0.149D-01$	$-0.152D-02$	$-0.165D-01$
	3	$-0.856D-02$	$-0.244D-03$	$-0.880D-02$

TABLE IV. Matrix elements of the $^3P_0$ CSB interaction as defined in Eq. (49). ($D-x$ signifies 10^{-x}.)

b	n	Pseudoscalar	Vector	Total
1.5000	1	$-0.278D+00$	$0.137D+00$	$-0.141D+00$
	2	$-0.103D+00$	$0.149D-01$	$-0.883D-01$
	3	$-0.489D-01$	$0.221D-02$	$-0.467D-01$
1.7690	1	$-0.163D+00$	$0.649D-01$	$-0.985D-01$
	2	$-0.552D-01$	$0.547D-02$	$-0.497D-01$
	3	$-0.244D-01$	$0.638D-03$	$-0.238D-01$
2.0000	1	$-0.108D+00$	$0.368D-01$	$-0.712D-01$
	2	$-0.338D-01$	$0.254D-02$	$-0.313D-01$
	3	$-0.141d-01$	$0.245D-03$	$-0.139D-01$

Masses 17 and 41 were chosen because of the simple form of the j-j coupled interaction energy for a single particle outside a closed shell (presented in the Appendix of de Shalit and Talmi):

$$\Delta E(j_1^{2j_1+1}, j_2, J=j_2)=(2j_1+1)\overline{\Delta E}(j_1 j_2)\ , \qquad (50)$$

where

$$\overline{\Delta E}(j_1 j_2)=\frac{\sum(2J+1)\Delta E(j_1 j_2 J)}{\sum(2J+1)}\ . \qquad (51)$$

For an l-dependent interaction we want these energies in terms of the L-S coupled matrix elements ΔE_{LSJ}, expressions for which are also given in de Shalit and Talmi

$$\Delta E(j_1^{2j_1+1}, j_2, J=j_2)=(2j_1+1)\sum_{SLJ}(2S+1)(2L+1)(2J+1)\begin{Bmatrix} \frac{1}{2} & l_1 & j_1 \\ \frac{1}{2} & l_2 & j_2 \\ S & L & J \end{Bmatrix}\Delta E_{SLJ}\ . \qquad (52)$$

For given harmonic oscillator single particle states ΔE_{SLJ} may be written:

$$\Delta E_{SLJ}=\sum_{\substack{nlN\Lambda \\ \mathscr{J}}}\langle n_1 l_1 n_2 l, L \mid nlN\Lambda L\rangle^2 (2L+1)(2\mathscr{J}+1)\begin{Bmatrix} l & \Lambda & L \\ J & S & \mathscr{J} \end{Bmatrix}^2 \langle nlS\mathscr{J} \mid V_{\mathrm{CSB}}(r, {}^{2S+1}l_{\mathscr{J}}) \mid nlS\mathscr{J}\rangle\ , \qquad (53)$$

TABLE V. Matrix elements of the $^3P_1$ CSB interaction as defined in Eq. (49). ($D-x$ signifies 10^{-x}.)

b	n	Pseudoscalar	Vector	Total
1.5000	1	$0.151D+00$	$0.217D-01$	$0.172D+00$
	2	$0.615D-01$	$0.347D-02$	$0.650D-01$
	3	$0.308D-01$	$0.587D-03$	$0.314D-01$
1.7690	1	$0.912D-01$	$0.107D-01$	$0.102D+00$
	2	$0.339D-01$	$0.130D-02$	$0.352D-01$
	3	$0.158D-01$	$0.172D-03$	$0.160D-01$
2.0000	1	$0.615D-01$	$0.618D-02$	$0.677D-01$
	2	$0.212D-01$	$0.611D-03$	$0.218D-01$
	3	$0.936D-02$	$0.669D-04$	$0.943D-02$

TABLE VI. Matrix elements of the $^3P_2$ CSB interaction as defined in Eq. (49). ($D-x$ signifies 10^{-x}.)

b	n	Pseudoscalar	Vector	Total
1.5000	1	$-0.208D-01$	$0.156D-01$	$-0.528D-02$
	2	$-0.437D-02$	$0.322D-02$	$-0.116D-02$
	3	$-0.111D-02$	$0.633D-03$	$-0.475D-03$
1.7690	1	$-0.107D-01$	$0.792D-02$	$-0.275D-02$
	2	$-0.175D-02$	$0.124D-02$	$-0.515D-03$
	3	$-0.277D-03$	$0.190D-03$	$-0.872D-04$
2.0000	1	$-0.632D-02$	$0.467D-02$	$-0.164D-02$
	2	$-0.817D-03$	$0.592D-03$	$-0.225D-03$
	3	$-0.446D-04$	$0.749D-04$	$0.303D-04$

where $\langle n_1 l_1 n_2 l_2 L \mid nlN\Lambda L \rangle$ is a Moshinsky Bracket and

$$\begin{Bmatrix} l & \Lambda & L \\ J & S & \mathcal{J} \end{Bmatrix}$$

is a Wigner 6-j symbol. In the above expression we have neglected off diagonal quadrupole contributions, since these involve F waves which we have systematically neglected. Using the above expressions and Tables III–VII, we obtain for the contributions to the Nolen-Shiffer anomaly

$$\Delta E_{\rm NS}(A=17)=295 \text{ keV} ,$$

$$\Delta E_{\rm NS}(A=41)=339 \text{ keV} .$$

These numbers are to be compared with the "experimental" values of 310 keV ($A=17$) (Ref. 9) and 590 keV ($A=41$).[9,59] In Table VIII these numbers are broken down by partial wave. The numbers in parentheses are the $n=0$ contributions.

We note the following general features: The only large contributions are from the $^1S_0$ and $^3P_1$ phases. Each partial wave is dominated by the $n=0$ term; in fact including only the $n=0$ terms would be an excellent approximation. The $^1S_0$ contributions for the two nuclei are nearly equal. This is no accident; already by mass 17 the density of particles with relative $l=0$ with respect to the valence particle is 0.065 fm^{-3}, or nearly $\frac{1}{2}$ nuclear matter density. For ^{41}Ca this density is 0.074 fm^{-3}, and saturation is nearly complete. As a consequence, we expect that saturation will prevent any short range force from explaining the observed increase with A of the Nolen-Shiffer anomaly.

The difficulty of accounting for this A dependence has been previously pointed out by many other workers. The reader is referred to Sec. V A of Shlomo's review article for a discussion.

On the other hand, differences between meson-neutron and meson-proton coupling constants might well be able to explain the A dependence. Since terms of this type seem to have small effects in the two-body system (except for the difference in scattering lengths) it appears possible that charge symmetry breaking in the two-body and A-body systems may have quite different origins.

TABLE VII. Matrix elements of the $^1D_2$ CSB interaction as defined in Eq. (49). ($D-x$ signifies 10^{-x}.)

b	n	Pseudoscalar	Vector	Total
1.5000	1	$-0.198D-01$	$-0.242D-02$	$-0.222D-01$
	2	$-0.126D-01$	$-0.355D-03$	$-0.130D-01$
	3	$-0.815D-02$	$-0.661D-04$	$-0.822D-02$
1.7690	1	$-0.126D-01$	$-0.887D-03$	$-0.134D-01$
	2	$-0.721D-02$	$-0.103D-03$	$-0.732D-02$
	3	$-0.433D-02$	$-0.153D-04$	$-0.435D-02$
2.0000	1	$-0.856D-02$	$-0.412D-03$	$-0.897D-02$
	2	$-0.457D-02$	$-0.398D-04$	$-0.461D-02$
	3	$-0.261D-02$	$-0.498D-05$	$-0.261D-02$

TABLE VIII. Contributions of $V_{\rm CSB}$ in individual relative states to the calculated Nolen-Shiffer anomalies.

	$^1S_0$	$^3P_0$	$^3P_1$	$^3P_2$	$^1D_2$	Total	Discrepancy
^{17}O-^{17}F	233	-14	99	-10	-13	295	310 keV[a]
	(227)	(-14)	(105)	(-10)	(-13)	(295)	
^{41}Ca-^{41}Sc	259	-28	141	-11	-22	339	590 keV[a]
	(249)	(-35)	(169)	(-12)	(-29)	(342)	$260-440$ keV[b]

[a]Reference 9.
[b]Reference 59.

Finally, we return to the biggest uncertainty in these calculations, the neglect of short range correlations. Such correlations will certainly have the effect of significantly reducing the $^1S_0$ contribution, and hence the total. We conclude that forces of this type are too short-ranged in character to reproduce the observed A dependence of the Nolen-Shiffer anomaly. For higher A values the contribution is too small; on the other hand, for light nuclei significant effects may remain even after the expected reduction due to short-range correlations.

We now consider two illustrative cases of isospin mixing within $T=0$ nuclei induced by $V_{\rm CSB}$. The first is between states which are rearrangements within a major shell, such as the 1^+ $T=0$ and $T=1$ states in ^{12}C. For such states in $T_z=0$ nuclei only the $(\vec{\sigma}_A-\vec{\sigma}_B)\cdot\vec{L}$ (or v_-) term described in Eqs. (30) and (32) can contribute. We also consider mixing among the 1^- states in ^{4}He, for which the v_+ potential contributes significantly.

We will estimate the effects of the $(\sigma_A-\sigma_B)$ term in bound states by computing the two-body matrix element relevant for mixing among 1^+ states — the mixing between $^3P_1$ and $^1P_1$ states. For the purposes of this estimate we use the one boson (OB) magnetic couplings, and linearize the potential in $(\mu_\rho$-$\mu_\omega)$, neglecting the ρ-meson width. We obtain for the desired matrix element evaluated with harmonic oscillator functions the values of 14 keV for ^{12}C and 5 keV for ^{40}Ca. These values are much too small to be of interest. Current determinations of the mixing matrix elements between the 1^+ states in ^{12}C obtain values in the range of $100-150$ keV.[60] Recent theoretical calculations[61] show that the Coulomb effects alone are sufficient to explain the mixing. Our results, indicating very small mixing due to the strong interaction support this conclusion.

The situation is very different for single particle promotions across major shell. The CSB interactions among identical particles will be different depending on whether a neutron or a proton is promoted. We consider the specific case of 1^- states[62] in ^{4}He.

There are four one particle-one hole 1^- states in ^{4}He, one ($S=T=0$) of which corresponds to a spurious center of mass excitation. The remaining $T=0$ state can mix with the $T=1$, $S=0$ and $T=1$, $S=1$, $J=1^-$ states. In this simple picture the isospin mixing matrix element can be regarded as the "mass difference" between states with a neutron or a proton promoted to the p shell. In other words, the mixing matrix element is the expectation value of $V_{\rm CSB}$ for two particles in the 0S subtracted from that for two particles one of which is in the 0S and one of which is in the 0P. The Moshinsky tranforms in this case are trivial: For two 0S particles we have no quanta, and so the relative wave function must be 0S

$$M(0S^2)=\langle 0S\,|\,V(^1S_0)\,|\,0S\rangle =\langle V(^1S_0)\rangle\;. \tag{54}$$

For the 0S-0P combination there is one quantum which may be in either the relative motion (if $S=1$) or center-of-mass motion (if $S=0$). Therefore,

$$\begin{aligned} M(0S,0P;S=0,L=1,J=1)&=\langle 0S\,|\,V(^1S_0)\,|\,0S\rangle=\langle V(^1S_0)\rangle\;,\\ M(0S,0P;S=1,L=1,J=1)&=\langle 0P\,|\,V(^3P_1)\,|\,0P\rangle=\langle V(^3P_1)\rangle\;. \end{aligned} \tag{55}$$

This leads to

$$\langle L=1,S=1,T=0;1^-\,|\,V_{\rm CSB}\,|\,L=1,S=1,T=1;1^-\rangle=0 \tag{56}$$

and

$$\langle L=1,S=1,T=0;1^- \mid V_{\rm CSB} \mid L=1,S=0,T=1,1^- \rangle = [\langle V(^1S_0)\rangle - \langle V(^3P_1)\rangle] \,. \tag{57}$$

For a system the size of ^{4}He, we see from Tables III and V that this will lead to a mixing matrix element of around 175 keV. Although this estimate is also plagued by the uncertainties associated with short range correlations, it indicates that large effects on mixing may be expected in odd parity states near shell closures such as the 4^- states in ^{16}O.

To summarize this section, we predict unobservably small mixing associated with the $\Delta\vec{\sigma}\cdot\vec{\mathrm{L}}$ potential. Contributions to binding energy differences in mass 3, 17, and 41 are found to be large, if short range correlations are neglected, and of quite uncertain magnitude (and even sign) if included. Even neglecting correlations, we can explain only half the binding difference in mass 41. Calculations neglecting correlations give large effects for mixing among the 1^- states in ^{4}He.

V. SUMMARY AND CONCLUSIONS

We have constructed a potential representation of the leading contributions to anomalous isospin breaking. These are the effects of pseudoscalar and vector meson mixing due to SU_2 violation intrinsic to the strong interaction and arise from up and down quark mass differences. This potential is presented in Sec. II. We emphasize that the charge symmetry breaking treated here is due to the strong interaction, and is theoretically well founded rather than *ad hoc*. The best theoretical and phenomenological cases are against rather than in favor of an exactly charge symmetric strong interaction.

The potential is used to compute CSB effects in both scattering and bound systems. The distinctive features of the calculations presented here are inclusion of π-η' mixing, the finite width of the ρ meson, and the focus on phase shift differences at finite energies. We obtain the following results.

(i) The amount of charge symmetry breaking in the $^1S_0$ channel is sensitive to both the short range behavior of the wave function and the magnetic couplings of the vector mesons. Nevertheless, for any phenomenologically or theoretically motivated magnetic couplings the differences between *nn* and *pp* scattering lengths and phase shifts are small. We have explained the near zero experimental results for Δa, but beyond that both the experimental and theoretical situations are very ambiguous. We predict small effects in $\Delta\delta(^1S_0)$ at intermediate energies.

(ii) For the $^3P_0$ and $^3P_1$ phase shift differences we predict reasonably large effects, which show some dependence on coupling constants but almost none on the short range behavior. These effects should be readily observable in the analysis of the next generation of *np* and *pp* experiments. In principle, of course, these measurements cannot distinguish charge dependence from charge symmetry breaking. Nevertheless, we feel that the relatively large predicted phase shift differences is an exciting and promising result.

(iii) In contrast, the effects in the $^3P_2$ and $^1D_2$ channels are expected to be very small. The $^3P_2$ phase differences are small in part because of cancellations within the pseudoscalar terms which result from the nature of the pseudoscalar potential. There are further cancellations between the vector and pseudoscalar parts. The $^1D_2$ phase differences appear to be small because of the angular momentum barrier, and may be larger at higher energy.

(iv) A simple calculation of the contribution to the binding energy differences between the mirror nuclei ^{3}He-^{3}H, ^{17}O-^{17}F, and ^{41}Ca-^{41}Sc indicates that the effects are large enough in mass 3 and 17 but probably not in mass 41. (The size of the discrepancy there is disputed.[9,59]) This apparent failure to duplicate the observed A dependence seems to be a consequence of the short range nature of the force. In any case this short range will lead to a very slow A dependence, as has been pointed out by many others.[9]

(v) A more drastic consequence of the short range of the CSB interaction is the sensitivity to short range correlations. Our estimates on the ^{3}He-^{3}H system indicate that even the sign of the $^1S_0$ contribution is uncertain for reasonable values of the correlation length. This uncertainty may be the outstanding remaining conceptual problem in the area of charge symmetry breaking in bound states.

(vi) Isospin mixing in $T_z=0$ nuclei was also investigated, specifically the 1^+ states in ^{12}C and the 1^- states in ^{4}He. There is substantial evidence of isospin mixing matrix elements in the 100 keV range for both these nuclei. For ^{12}C, and for positive parity particle rearrangements in general, we find that only the terms antisymmetric in the isospin operator [v_- of Eq. (32)] contribute, and that the effects are quite small. On the other hand, we estimate large effects (~ 150 keV) for ^{4}He, and generally would expect large effects in one-particle-one-hole states. This result, however, is dependent on the $^1S_0$ contribution, and hence very uncertain due to the large effect of short range correlations.

We have computed anomalous (i.e., nonelectromagnetic) charge symmetry breaking in scattering and bound systems. For the bound systems we generally find naive results neglecting short range correlations are comparable to existing discrepancies; however, inclusion of correlations dramatically reduces $^1S_0$ contributions, and so we feel all these estimates are quite uncertain. In the scattering system we have explained the very small $^1S_0$ scattering length difference. Finally, we predict readily observable charge symmetry breaking effects in the $^3P_0$ and $^3P_1$ *np-pp* phase shift differences.

There are several areas in which further work would be of particular value. On the theoretical side these include incorporating realistic treatments of the short range correlations in the estimates of anomalous charge symmetry breaking contributions to binding energy differences and isospin mixing. There are also ambiguities in the calculation of Coulomb displacement energies which need attention. Although remote, there is some possibility that these taken together would account for the bulk of the remaining discrepancies. Serious investigation of the effects of different coupling constants in the neutral meson-proton-proton and neutral meson-neutron-neutron systems is also needed. On the experimental side, refined measurements of the *n-p* scattering parameters are already being planned. These experiments hold considerable promise for providing unambiguous evidence of charge symmetry breaking.

Note added in proof. We recently received a preprint from S. A. Coon and M. D. Scadron reporting results similar to those we present in Table II.

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[50]A lower value $g_\rho g_\omega/4\pi\simeq 1.6$ is predicted by VMD if g_ϕ is exactly zero. See Ref. 44.

[51]Recent measurements include: $a_{nn}=-16.9\pm0.6$, W. von Witsch *et al.*, Phys. Lett. 80B, 187 (1979); Nucl. Phys. A329, 141 (1979); $a_{nn}=-18.5\pm0.5$, B. Gabioud, *et al.*, Phys. Rev. Lett. 42, 1508 (1979). For the *n-n* effective range, W. von Witsch *et al.*, Phys. Lett. 91B, 342 (1980) obtain $r_{nn}=2.65\pm0.18$. These may be compared with earlier measurements or with the *pp* values quoted in the review articles, $a_{pp}=-17.3\pm1.0$, $r_{pp}=2.83\pm0.2$ (Ref. 9); $a_{pp}=-17.3\pm3.0$, $r_{pp}=2.84\pm0.03$ (Ref. 38).

[52]The primary effects of the class IV interactions due to meson mixing in *np* scattering occur through mixing of $T=0$ and $T=1$ channels; they lead to small but possibly observable effects in asymmetries. See Cheung, Henley, and Miller (Ref. 41) for detailed calculations of these effects. In the diagonal $T=1$ channel phase shifts which we study here the effects are second order in the class IV interaction and are truly minuscule.

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