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Triton form factor from 0.29 to 1.00 fm^{-2}

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Recoil tritons were detected from the elastic electron scattering reaction ^{3}H$(e,t)e'$. From these data charge form factors were determined in the range $0.29 \leq q_\mu{}^2 \leq 1.0$ fm^{-2}. No significant change in the previously reported charge radius for ^{3}H was found.

[NUCLEAR REACTIONS ^{3}H(e,t), $E=77-147$ MeV, measured $\sigma(E,E_r,\theta_r)$ deduced charge form factor, rms charge radius.]

I. INTRODUCTION

The ^{3}H elastic charge form factor was measured by Collard *et al.*[1] in the early 1960's. Their experiment was one in which a high pressure tritium gas cell was used, and the scattered electrons were detected over the range $1.0 \leq q_\mu{}^2 \leq 8.0$ fm^{-2}. There have been questions about the accuracy of the rms charge radius determined from this data in relation to calculations of the observed binding energy difference[2] between ^{3}H and ^{3}He. In our experiment, the charge form factor was extracted for $0.29 \leq q_\mu{}^2 \leq 1.0$ fm^{-2}. The form factor at $q_\mu{}^2 = 1.0$ fm^{-2} is in agreement with Collard's at this square momentum transfer, the only overlapping point that we were able to measure.

II. EXPERIMENT

The magnetic spectrometer facility of the University of Saskatchewan Linear Accelerator Laboratory was used to measure elastically scattered tritons. The experimental apparatus is described in detail elsewhere[3] and only a brief description is given here.

The electron beam was momentum analyzed by a bending magnet slit assembly (the energy resolution was ±0.12%). The beam current was measured with a SLAC type nonintercepting ferrite monitor. Its response was linear and reproducible to ±2% and periodically calibrated with a Faraday cup.

The target was solid Ti^{3}H containing 0.20 mg/cm^2 of ^{3}H. The supporting Ti foil (natural isotopic composition) was 5 μm thick, 1 cm in diameter, and contained about 1.5 Ci of ^{3}H. Heating during manufacture of the target (^{3}H is absorbed by a heated Ti foil) resulted in some wrinkling, about 0.5 – 1.0 mm in depth. Because of the beam spot size of 2 mm and the wrinkling, the actual number of target nuclei for a given setup had an uncertainty of about 7%. This error was determined from analysis of the form factor data (Sec. III). The target was found to have excellent stability. Recoil protons from a similar Ti^{1}H target were observed with 15 μA average currents. No discernable reduction in counting rate (corresponding to a loss of hydrogen from the target) was observed. Average currents during the Ti^{3}H runs were restricted to be less than $\simeq 8$ μA.

Tritons were detected in a spectrometer containing five surface barrier silicon detectors mounted in the focal plane of a 127° double focussing magnet. Calibrations of the spectrometer and details of the system are described in Ref. 3. The systematic errors associated with the spectrometer and the electron beam are given in Table I.

TABLE I. Sources of systematic error.

Item	Uncertainty (%)
Spectrometer solid angle	±2
Momentum acceptance	±2
Incident electron flux	±2
Target nuclei	±7
Total	±8

Amplified signals from the detectors are digitized by 11 bit analog-to-digital converters (ADC's) and read by a PDP 11-55 computer through its CAMAC interface. Because the ratio of Em/q^2 (where E, m, and q are the kinetic energy, mass, and charge of the detected particle) is held constant by the spectrometer for each particle, peaks produced by tritons, deuterons, and protons are easily distinguished as is seen in Fig. 1. Integration of the triton peak gives the total number of counts, C_T. The doubly differential cross section is then

$$\frac{d^2\sigma}{d\Omega_r dE_r}=\frac{C_T}{n_e}\frac{1}{\Delta\Omega\Delta E_r n_t}\ ,$$

where n_e is the number of incident electrons, $\Delta\Omega$ is the spectrometer solid angle, n_t is the number of target nuclei per unit area, and the subscript r denotes the recoil nucleus. The quantity ΔE_r is given by the energy acceptance of each detector ($\Delta E/E=1.17\%$) and the energy of the detected particle.

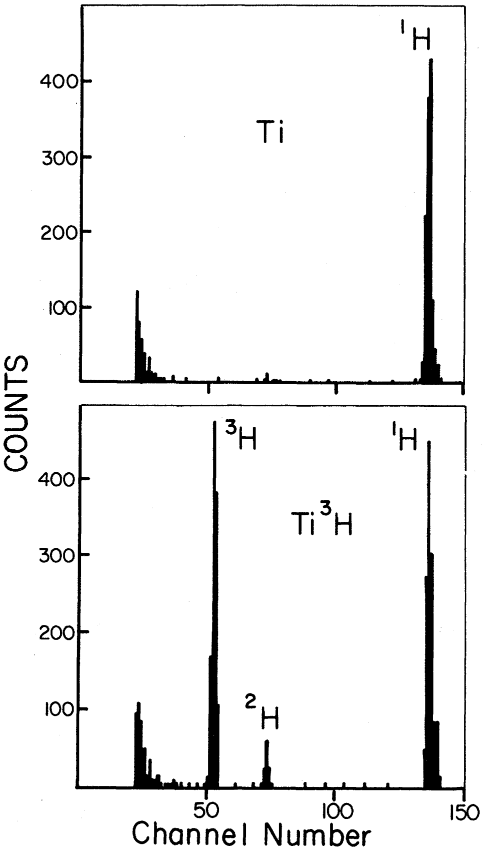

FIG. 1. Pulse height spectrum showing the elastically scattered tritons that were measured in this experiment. Background from a plain Ti target of the same thickness is also shown and it is seen that the Ti contributes very little to the number of counts in the ^{3}H peak region.

III. RESULTS

The elastic charge form factor was determined from the Rosenbluth formula for detection of the recoil particle[4]

$$\frac{d\sigma}{d\Omega_r}=\sigma_{\mathrm{NS}}\left\{G_E{}^2\frac{1}{1+\tau}+\tau G_M{}^2\left[\frac{1}{1+\tau}+2\left[\frac{1}{1+\rho}\right]^2\cot^2\theta_r\right]\right\},$$

where

$$\sigma_{\mathrm{NS}}=\frac{\alpha^2}{E_i{}^2}\frac{(1+\rho)^4\sin^2\theta_r}{\cos^3\theta_r}\frac{1}{1+2\rho+\rho^2\sin^2\theta_r}$$

is the nonspin (Mott) cross section,

$$\tau=\frac{q_\mu{}^2}{4M^2},\ \rho=\frac{E_i}{M}\ ,$$

$q_\mu{}^2$ is the four-momentum transfer squared, and M is the mass of the recoil nucleus. All measurements were made at $\theta_r=45°$ and the largest contribution to the magnetic terms was 0.35% at $q_\mu{}^2=1.0\ \mathrm{fm}^{-2}$. Therefore the scaling approximation $G_M=\mu G_E$ was used to remove the magnetic contribution to the form factor.

Normally, radiative corrections $(1-\delta_{\mathrm{MY}})$ would be applied in integrating the doubly differential cross sections to obtain differential cross sections

$$\frac{d\sigma}{d\Omega}=(1-\delta_{\mathrm{MY}})\int_{E_{\min}}^{\infty}\frac{d^2\sigma}{d\Omega_r dE_r}dE_r\ .$$

The doubly differential cross sections observed in this experiment, however, have low energy tails which cannot be completely accounted for by using the radiative tail (including a correction for the changing form factor as E_r decreases) of Meister and Yennie (MY).[5] An example of such a cross section with the MY correction given on the top scale is shown in Fig. 2. We are able to adequately model the peak shape by taking into account the finite angular opening of the spectrometer, multiple scattering,[6] and Vavilov energy straggling[7] of tritons in the target.

Relative to the main peak, the amount of cross section in the tails in any case is typically $<5\%$. Therefore it was assumed that all tritons detected were scattered from electrons of the measured incident energy. In order to determine the differential cross sections, the data were first sorted into 0.1 MeV bins and then a straight line constrained to pass through the origin was fitted to those data in the "flat" part of the tail (Fig. 2). The doubly differential cross sections were then integrated from $E_r=0$ by summing (cross section) × (bin width) to give the differential cross sections. This procedure was deemed to be the most consistent way of treating the data. The "extrapolated area" is $<2\%$ of the total area for all except for $q_\mu{}^2=0.290\ \mathrm{fm}^{-2}$, where it is approximately 7%.

The cross sections and form factors are listed in Table II. Since $G_E(q_\mu{}^2=0)=1.0$, the number of target nuclei per unit area was determined by least squares fitting of a straight line (Fig. 3) to the measured form factor. When the error in the number of target nuclei per unit area, Δn_t, is not included in the form factor errors, the reduced χ^2 of the fit is large. The errors in G_E were therefore increased by $(\chi_{\nu,\mathrm{exp}}{}^2/\chi_{\nu,\mathrm{th}}{}^2)^{1/2}$ and the additional error was ascribed to uncertainty in n_t. Estimation of the error using this method gives $\Delta n_t/n_t \simeq 7\%$, which dominates the errors associated with the experiment.

The form factor that we measured at $q_\mu{}^2=1.0$ agrees well with that of Collard as shown in Fig. 3. A power series expansion of $G_E(q_\mu{}^2)$ gives information about the moments of the charge distribution.

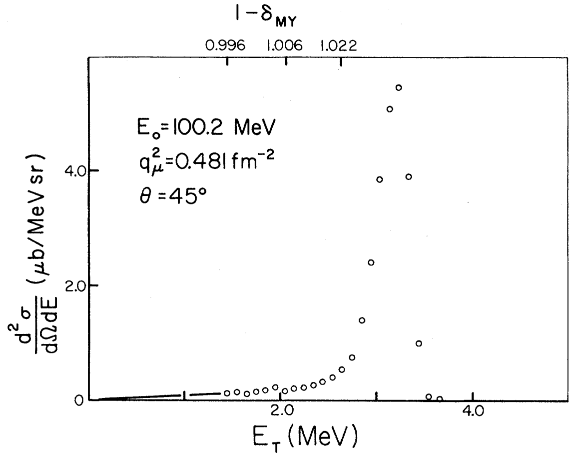

FIG. 2. Typical energy spectrum of elastically scattered tritons. The differential cross section is calculated using

$$\frac{d\sigma}{d\Omega}=(1-\delta_{\mathrm{MY}})\int_{E_{\min}}\frac{d^2\sigma}{d\Omega\,dE}dE\ ,$$

where δ_{MY} is the radiative correction. As can be seen this factor does not change rapidly enough for $d\sigma/d\Omega$ to be completely independent of the cutoff energy ($E_{\min}$). Also shown is the low energy extrapolation of the data used in determining $d\sigma/d\Omega$.

$$G_E(q_\mu{}^2)=1-\frac{1}{3!}q_\mu{}^2\langle r^2\rangle+\frac{1}{5!}q_\mu{}^4\langle r^4\rangle-\dots\ .$$

Three power series fits (that included terms up to r^4, r^6, and r^8, respectively) with the best χ^2 for the Collard data alone give

TABLE II. Experimental results for the reaction $^3\mathrm{H}(e,t)e'$ at $\theta_{\mathrm{lab}}=45°$. $d\sigma/d\Omega$ do not include Δn_t, errors in G_E include Δn_t.

q^2 (fm^{-2})	E_0 (MeV)	$\frac{d\sigma}{d\Omega}$ (μb/sr)	G_E
0.290	77.2	4.52 ±0.03	0.885±0.063
0.386	89.4	3.30 ±0.03	0.863±0.055
0.450	96.8	2.28 ±0.03	0.769±0.069
0.481	100.2	2.26 ±0.02	0.784±0.062
0.600	112.4	1.87 ±0.02	0.790±0.058
0.670	119.0	1.70 ±0.01	0.789±0.033
0.775	128.4	1.43 ±0.02	0.769±0.073
0.850	134.7	1.24 ±0.01	0.743±0.042
0.900	138.8	0.779±0.009	0.603±0.048
0.951	142.9	0.953±0.009	0.682±0.044
1.000	146.7	0.659±0.008	0.579±0.048

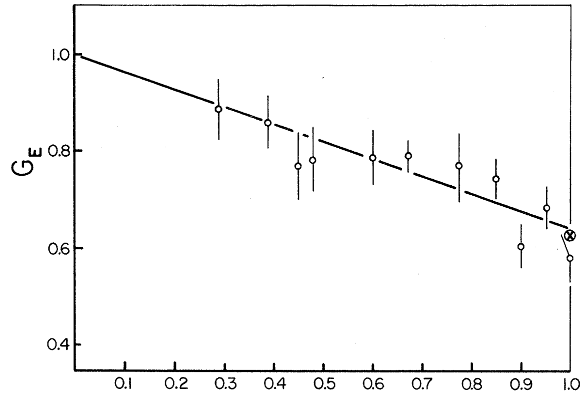

FIG. 3. Charge form factor of ^{3}H with the straight line fit used to determine n_t. The errors shown include Δn_t, as determined from the χ^2 test, in the systematic error. Statistical and systematic errors are added in quadrature. The ⊗ is from Ref. 1.

$$\langle r^2 \rangle^{1/2} = 1.67 \pm 0.06 \text{ fm}$$

and for our data analyzed with Collard's

$$\langle r^2 \rangle^{1/2} = 1.69 \pm 0.05 \text{ fm} .$$

This is a change of only 1% (Collard's best estimate for the radius is 1.70±0.05 fm based on various other fits). Reduction of the radius by 15% from the Collard value as suggested in Ref. 2 is not indicated by this experiment, which agrees better with the calculation of Friar,[8] that suggest a much smaller change is required.

IV. SUMMARY AND CONCLUSIONS

The ^{3}H form factor measurements support the data of Collard *et al.* in that (1) they connect smoothly and (2) no significant change in the radius of ^{3}H determined by Collard was found when both of these data sets were combined in the analysis. However, at low triton energies, energy loss in the target prevented us from measuring the form factor at square momentum transfers less than 0.29 fm^{-2}. Further experiments are required and are planned in which a similar target is used and the scattered electron is detected.

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