Regge-pole analysis of angular distribution for the $^{24}Mg(^{16}O,^{12}C)^{28}Si$ reaction

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A Regge-pole analysis of the backward rise of the angular distribution observed at low incident energy in transfer reaction induced by heavy ions is proposed for a particular example.

[NUCLEAR REACTION Regge-pole analysis of α transfer reaction induced by heavy ions.]

A Regge pole analysis of the $^{24}Mg(^{16}O,^{12}C)^{28}Si$ angular distribution measured between 0° and 180° c.m. at 27.80 MeV center of mass energy, has been performed. The present data are from the Argonne-Brookhaven group.[1] This alpha transfer angular distribution presents a strong backward rise and, furthermore, the excitation function measured in this energy region displays an erratic behavior which is a signature of resonance-like or fluctuating behavior. The aim of this analysis is to try to ascertain if few anomalies in the β_l distorted-wave Born approximation (DWBA) transfer amplitude matrix elements are responsible, in general, for the backward rise of the angular distributions

The present Regge pole analysis is based on a formalism similar to the one developed by Carlson and McVoy.[2] For a zero spin transfer reaction, the DWBA amplitude[3] is given by

$$f(\theta)=\sum_{l=0}^{\infty}(2l+1)\beta_l e^{i(\sigma_l^i+\sigma_l^f)}P_l(\cos\theta) \;.$$

The indices i and f refer, respectively, to the entrance and exit channel, and the quantity σ_l is equal to the Coulomb plus the nuclear phase shift:

$$\sigma_l=\sigma_l^0+\alpha\left[1+\exp\frac{l-l_g}{\Delta}\right]^{-1} \;.$$

The transfer matrix element β_l is

$$\beta_l=\beta_l^0R_l$$

and B_l^0 is the background DWBA transition matrix element given by the Blair-Austern-Hahne relationship[4]

$$\beta_l^0=\frac{1}{2i}\left[E_iE_f\frac{\partial\eta_i}{\partial l}\frac{\partial\eta_f}{\partial l}\right]^{1/2}$$

based on the diffraction model, where E is the center of mass energy and η is the coefficient of reflection provided by a Woods-Saxon form[5]

$$\eta_l=\left[1+\exp\frac{l_g-l}{\Delta}\right]^{-1} \;,$$

where l_g and Δ are, respectively, the grazing partial wave number and the width of the η_l distribution. These two quantities are provided as functions of the radius and diffusivity by the usual semiclassical relationships.[1-5]

Finally, the quantity R_l is the sum of N Regge poles and is equal to

$$R_l=\sum_{k=1}^{N}\frac{ia_ke^{i\phi_k}}{l-l_k-i\Gamma_k/2} \;.$$

The value a_k is the amplitude of the pole resonance, ϕ_k is the corresponding phase, l_k is the resonant partial wave, and Γ_k is the corresponding width.

The transfer cross section in the zero range approximation is then given by

$$\sigma(\theta)\propto|f(\theta)|^2 \;.$$

A small computer code named RAPID[6] has been written to calculate the transfer cross section; furthermore, an automatic search routine STEPIT[7] allows us to determine the Regge pole parameters once the background parameters of β_l^0 have been fixed by analyzing the forward angle data without the Regge pole (R_l=1), which is a slightly arbitrary procedure.

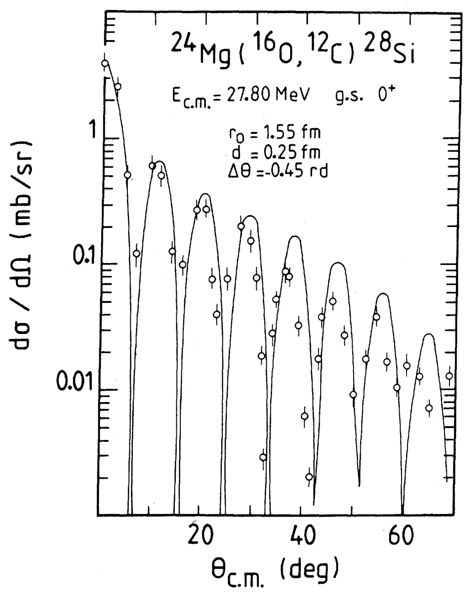

FIG. 1. Standard DWBA fit using the diffraction model of the forward angle angular distribution of $^{24}Mg(^{16}O,^{12}C)^{28}Si$ α-transfer reaction.

These later parameters are the scattering radius, the diffusivity, and the phase angle $\Delta\theta = \alpha/2\Delta$.[3]

In Fig. 1 is presented the analysis for the forward angles alone of the $^{24}Mg(^{16}O,^{12}C)^{28}Si$ alpha transfer angular distribution[1] with the belief that its shape is mainly due to the β_l^0 background transfer matrix elements ($R_l = 1$). The parameters are fairly reasonable: $r_o = 1.55$ fm, $d = 0.250$ fm, and $\Delta\theta = -0.450$ rad. Then, a first analysis of the complete alpha transfer angular distribution has been performed with only one Regge pole and this immediately provided a strong backward rise for the

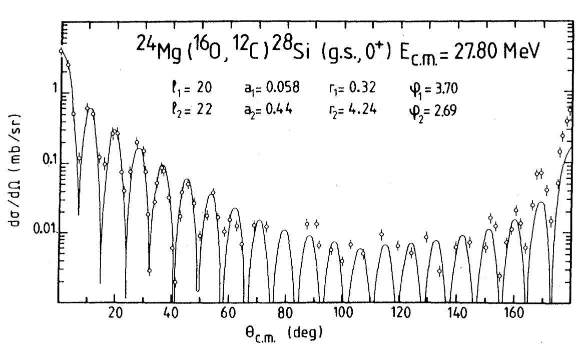

FIG. 2. The two Regge-pole analysis of the alpha transfer reaction angular distribution. The Regge pole parameters are determined by an automatic search (see text). The parameters of the background matrix element are those of Fig. 1.

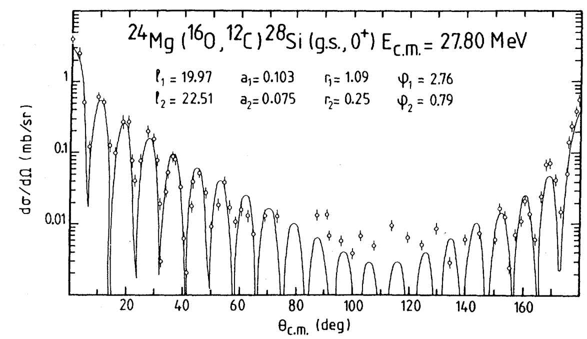

FIG. 3. Same as Fig. 2.

differential cross section. It has turned out that the Regge pole is centered on the grazing wave $l_g = 20$ of the β_l^0 background. The effect of this Regge pole is just to make narrower the β_l^0 background matrix element curve around l_g. Accordingly, the direct surface transfer reaction has to be strongly localized in partial wave number in order to explain the experimental backward rise of the differential cross sections. This analysis is similar to the one of the authors of Ref. 8 (see Fig. 5 of Ref. 8). These authors obtain a good fit by adding to the usual EFR-DWBA amplitude, which reproduces the forward angle angular distribution, a Breit and Wigner resonance term corresponding to $l_R = 20$.

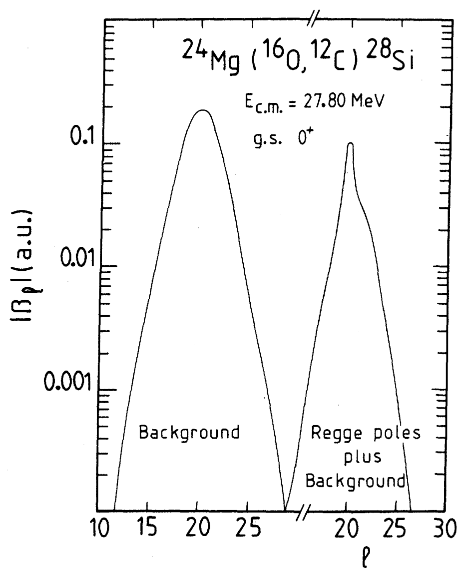

FIG. 4. The DWBA $|\beta_l^0|$ (background) and $|\beta_l| = |\beta_l^0 R_l|$ (two Regge poles plus background) matrix elements plotted versus the partial wave angular momentum.

TABLE I. Regge pole parameters.

Number of poles	a_k (a.u.)	ϕ_k (rad)	l_k[b] ($\hbar$)	Γ_k (a.u.)	χ^2[a]
1	0.200	0.17	19.95	1.38	38
2	0.058	3.70	20	0.32	30
	0.44	2.69	22	4.24	
2	0.103	2.76	19.97	1.09	28
	0.075	0.79	22.51	0.25	
3	2.63	4.40	20	2.54	30
	0.094	0.00	22	0.25	
	2.81	1.38	19.90	2.66	

[a]10% error bars.
[b]The grazing wave for the background β_l^0 is $l_g = 20$.

The quality of the fits can be improved by using a two-pole parametrization; the results are presented in Figs. 2 and 3. From these two figures and Table I, it can be seen that there exists a large ambiguity in the determination of the parameters and this is also unfortunately true for the l resonant angular momenta since it is possible to hesitate between, on one hand, $l = 20$ and 22 and, on the other hand, $l = 20$, 21, and 23. Furthermore, the fits are still qualitative at the backward angle. Another analysis using a three-pole parametrization does not provide better fits as can be seen from the χ^2 mean square deviation value of the theoretical points with respect to the experimental ones (see Table I).

In Fig. 4 are plotted the $|\beta_l^0|$ and total $|\beta_l|$ DWBA transfer matrix elements versus the partial wave angular momentum for the case of two poles localized, respectively, at 20 and 22 $\hbar$. It appears that the β_l total matrix element curve is extremely narrow compared to its corresponding background curve. For the alternate analysis where the two poles are located at 19.97 and 22.52 $\hbar$, the $|\beta_l|$ curve is extremely similar except that the shoulder located for the higher l value is in this latter case a little bit stronger. The semiclassical deflection function is equal to the derivative with respect to the partial wave number of the total phase shift of the DWBA scattering amplitude: Coulomb, nuclear, and Regge pole terms, if any. In Fig. 5 are plotted the two deflection functions of the DWBA amplitude, respectively, for the background matrix element (solid curve) and for the total matrix elements (dashed curve). The solid curve corresponds to the pure Coulomb deflection function except in the grazing wave region. On the other hand, for the dashed curve there is a strong minimum (nuclear rainbow) as soon as the two Regge poles are added (see Table I). As quoted previously, no definite spin assignment, due to the smoothness of the $|\beta_l|$ curve, can be made for the resonant contributions. Similar conclusions about spin assignment were already reached by the authors of Ref. 1; few partial waves resonate and contribute to the fancy shape of the angular distribution. The $^{24}Mg(^{16}O,^{12}C)^{28}Si$ α transfer reaction appears to be extremely localized in the angular momentum space: full width at half maximum (FWHM) = 1$\hbar$ around $l = 20$ (see Fig. 4).

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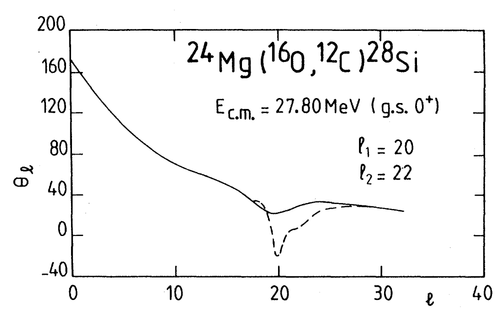

FIG. 5. Semiclassical deflection function: the solid curve is for the background DWBA amplitude which reproduces only the forward angle angular distribution (see Fig. 1); the dashed curve is for the total DWBA amplitude including the two Regge poles (see Fig. 2 and Table I)

[1]M. Pauli, S. J. Sanders, D. F. Geesaman, W. Henning, D. G. Kovar, C. Olmer, J. P. Schiffer, J. Barrette, and M. J. LeVine, Phys. Rev. C 21, 1802 (1980).
[2]B. V. Carlson and K. W. McVoy, Nucl. Phys. A292, 312 (1977); see also K. W. McVoy, Phys. Rev. C 3, 1104 (1971); C. K. Gelbke, J. D. Garrett, M. J. LeVine, and C. E. Thorn, *ibid.* 14, (1976) 127.
[3]M. C. Mermaz, Phys. Rev. C 21, 2356 (1980).
[4]N. Austern and J. S. Blair, Ann. Phys. (N.Y.) 33, 15 (1965); F. J. W. Hahne, Nucl. Phys. A104, 545 (1967).
[5]W. E. Frahn and R. H. Venter, Ann. Phys. (N.Y.) 24, 243 (1963); Nucl. Phys. 59, 651 (1964).
[6]M. C. Mermaz, Saclay, internal report (unpublished).
[7]J. P. Chandler, Indiana University, Dept. of Phys., internal report (unpublished).
[8]S. J. Sanders, M. Paul, I. Cseh, D. F. Geesaman, W. Henning, D. G. Kovar, R. Kozub, C. Olmer, and J. P. Schiffer, Phys. Rev. C 21, 1810 (1980).