2^+ 40 MeV ^{4}He resonance in a $4\hbar\omega$ model space

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Dynamical R-matrix methodology is used to determine the properties of the recently proposed $J^\pi = 2^+$, $\Gamma = 3.5$ MeV, E_x(res) = 40.2 MeV resonance in ^{4}He. The calculations lead to a $J^\pi = 2^+$ state with the following properties: $\Gamma = 6.0$ MeV, E_x(res) = 40.3 MeV, and $T = 0$. The calculations indicate the importance of $4\hbar\omega$ components in the proper description of the 40 MeV state.

[NUCLEAR STRUCTURE Calculated via dynamical R-matrix methodology the following properties for the proposed 40 MeV resonance in ^{4}He: $J^\pi = 2^+$, $T = 0$, $\Gamma = 6.0$ MeV, and E_x (res) = 40.3 MeV.]

I. INTRODUCTION

In a recent Letter, McBroom *et al.*[1] measured the fore-aft asymmetry in the angular distributions for the ^{3}H$(p,\gamma_0)^4$He and ^{4}He$(e,{}^3$H$)pe'$ reactions as a function of proton energies between 17 and 31 MeV. These asymmetry data suggest a new resonance having parameters $J^\pi = 2^+$, $\Gamma = 3.5$ MeV, and E_x (res), the resonance energy position, equal to 40.2 MeV. McBroom *et al.* also argue that this resonance establishes the importance of unusual (lower symmetry) components in the ^{4}He wave function near 40 MeV. These conclusions were based on a consideration of a $2\hbar\omega$ model space and supermultiplet partitions. In view of the importance of the 40 MeV resonance in understanding the structure and decay modes of ^{4}He, additional evaluation seems to be warranted.

This paper will present the results of model calculations which characterize the properties of the 40 MeV state in a $4\hbar\omega$ model space. Theoretical width and resonance predictions will also be compared with those of Ref. 1.

II. BACKGROUND AND FORMALISM

The essential features of the model used in our theoretical analysis have been previously outlined[2-4]. The model is constructed within a $4\hbar\omega$ model space which is considerably larger than the $2\hbar\omega$ spaces usually employed in studies of ^{4}He.[2] The model basis states are completely antisymmetrized and translationally invariant harmonic oscillator eigenfunctions. The eigenenergies are obtained from the solution of the equation[5]

$$\sum_{\lambda'}\left[\langle\lambda|H-E|\lambda'\rangle + \sum_c \gamma_{\lambda c}(b_{\lambda' c}-b_c)\gamma_{\lambda' c}\right]A_{\lambda'} = 0\,, \qquad (1)$$

where $|\lambda\rangle$ are the basis states, A_λ are expansion amplitudes, and all other quantities appearing in Eq. (1) are defined in Refs. 2–4. The sum over channels (c) includes all contributions of $4\hbar\omega$ or less oscillator excitation from all three binary breakup channels—i.e., $p+{}^3$H, $n+{}^3$He, and $d+{}^2$H. The effect of omitting the three-body ($n+p+d$) and four-body ($2n+2p$) breakup channels on the level width is difficult to determine without detailed analysis, but a qualitative estimate can be gained from past ^{4}He, ^{4}Li, and ^{4}H calculations.[2,3] An analysis of these results indicates that three- and four-body breakup channels can have a sizable impact on the calculated widths. In particular, a factor of 2 effect was noted for the ^{4}H and ^{4}Li systems. The addition of three- and four-body breakup channels will likely have a small impact on the level position, since the addition of the three binary breakup channels has a small effect on the shell-model (no channel) level energy.[2,3]

Specific formulas for the resonance positions (E_R^μ) and level widths (Γ_R) may be defined as

$$E_R^\mu = \mathrm{Re}[E_\mu - \xi_\mu(E_R^\mu)] \qquad (2)$$

and

$$\Gamma_R = -2\,\mathrm{Im}[E_\mu - \xi_\mu(E_R^\mu)]\,, \qquad (3)$$

where E_μ is defined as the solution of Eq. (2), and ξ_μ is defined in terms of known R-matrix energies and widths E_μ, $\gamma_{\mu c}$ and standard Coulomb radial functions.[6,7] The reader should refer to Refs. 2–4 for details of the application of Eqs. 1–3 to the ^{4}He system.

Although $4\hbar\omega$ is a significant truncation of the complete $J^\pi = 2^+$ model space, it does provide a 108 component wave function. This degree of wave function detail is significantly larger than the 13 component $2\hbar\omega$ basis configuration for $J^\pi = 2^+$. As noted, the $4\hbar\omega$ truncation introduces a dependency of the model solution on the basis space.[2] This dependency may be partially removed by adjusting the effective interaction to have the properties of the ^{4}He ground state and its breakup clusters. This has been achieved[2-4] with a small (about 15%) modification of the Sussex interaction.[8]

The results of calculations for levels below 35 MeV were summarized in Table 4 of Ref. 2. (This is the $E_R^{\text{unshifted}}$ spectrum of Ref. 3.) This table illustrates a common misconception about the widths of ^{4}He levels below 35 MeV excitation. McBroom *et al.*[1] note that the 1^- and 2^+ states in this energy region are typically 10–15 MeV wide. Table 4 of Ref. 2 indicates that this is not the case, and that widths are more narrow—i.e., typically 2–6 MeV.[9] The 10 MeV claims for $T = 1$ levels are only gross estimates based on single particle widths.[10] In addition, narrow widths near 37 MeV have also been suggested.[11-13] It is, therefore, not surprising that the 40 MeV resonance[1] has a width less than the 10–15 MeV width suggested by McBroom *et al.*

A comment concerning the use of standard $2\hbar\omega$ shell model calculations is also in order. These calculations may provide a reasonably accurate level scheme(if binding properties are ignored), but they do lead to an oversimplified view of the ^{4}He wave function.[2-4] In particular, only components up to $2\hbar\omega$ are examined, and these are overemphasized because higher excitation configurations ($4\hbar\omega$, $6\hbar\omega$, etc.) are omitted. A $4\hbar\omega$ space has similar problems since it omits $6\hbar\omega$, $8\hbar\omega$, etc., but it does allow for about an order of magnitude more detail than the $2\hbar\omega$ space—i.e., 108 versus 13 expansion states.

The oscillator content ($2\hbar\omega$, $4\hbar\omega$, $6\hbar\omega$, etc.) refers to the size of the $|\lambda\rangle$ configuration space. The $|\lambda\rangle$ basis states are defined in terms of radial (N), orbital (L), and spin (S) quantum numbers

$$|\lambda\rangle = \left\{ \left[\prod_{i=1}^{3} R_{N_i L_i}(r_i) Y_{L_i M_i}(\hat{r}_i) \right]^L X(S) \right\}^{J^\pi}, \qquad (4)$$

where the square brackets indicate coupling of internal coordinate angular momenta (L_i) to form the total angular momentum (L) of the system. The curly brackets signify the coupling of L to the total spin (S) to form the total angular momentum (J) and parity (π) of the system.[2,4] The product is over all internal coordinates of the ^{4}He system (see Fig. 1 of Ref. 2): $\vec{r}_1 = \vec{r}_{12}$ (the coordinate connecting the two neutrons), $\vec{r}_2 = \vec{r}_{34}$ (the coordinate connecting the two protons), and $\vec{r}_3 = \vec{r}_A$ (the coordinate connecting the centers of mass of the diproton and dineutron systems). The quantities $Y_{L_i M_i}(\hat{r}_i)$ are spherical harmonics normalized over the unit sphere[14] with M_i being the projection of L_i on the z axis of the $\vec{r}_i$ coordinate. The spin function $X(S)$ represents the spin coupling wave function for the ^{4}He system.[2,4] The radial functions $R_{NL}(r)$ are given by

$$R_{NL}(r) = \left[\frac{2N!}{\Gamma(N + L + \frac{3}{2})} \right]^{1/2} r^L \exp(-\tfrac{1}{2} r^2) \mathcal{L}_N^{L+1/2}(r^2), \qquad (5)$$

where $\mathcal{L}_N^{L+1/2}(r^2)$ is a Laguerre polynomial as defined by Erdélyi *et al.*[15] and is normalized so that

$$\int_0^\infty [R_{NL}(r)]^2 r^2 dr = 1 . \qquad (6)$$

Using the above quantum number definitions permits the number of oscillator quanta (Q) of a basis state to be determined from the relation

$$Q = 2(N_{12} + N_{34} + N_A) + L_{12} + L_{34} + L_A - 6, \qquad (7)$$

where N_i can assume the integer values $1, 2, 3, \ldots$. For example, the first configuration appearing in Table I ($N_{12} = N_{34} = N_A = 1$, $L_{12} = L_A = 0$, and $L_{34} = 2$) contains $2\hbar\omega$ of oscillator excitation.

Finally, a comment concerning the applicability of the present model to the ^{3}H$(p, \gamma_0)^4$He photonuclear experiments, which were used to obtain the 2^+ (40 MeV) data, seems in order. The present model, based on direct transfer mechanisms, was shown to provide a consistent representation of ^{4}He giant dipole resonance results.[16] Therefore, it is expected that the model will also provide a good representation of the data,[1] if the reaction proceeds via a direct $E2$ capture mode. Based on previous results,[16] it appears that the 2^+ (40 MeV) state is formed by a direct capture process in which the entrance channel d-wave proton makes an $E2$ jump to a final bound s state.

III. RESULTS AND DISCUSSION

The model of Refs. 2–4 predicts the second 2^+ state in ^{4}He has a resonance energy of 40.3 MeV and a width of 6.0 MeV. Furthermore, the state is $T = 0$ which was not discussed in Ref. 1. The calculated position is quite close to the McBroom *et al.* prediction of 40.2 MeV, but the model width is about 70% broader. The 6.0 MeV width implies that the 40 MeV state only survives for about 10^{-22} sec. This lifetime suggests that many R-matrix components (see Table I) do not significantly contribute to the scattering process because there is not enough time for these states to be admixed via the $E2$ operator into the initial state. Similar mixing was also noted in the $E1$ transitions of Ref. 16. However, this does not imply that simple $2\hbar\omega$ transitions dominate the scattering overlap and that the 2^+ (40 MeV) wave function is dominated by $2\hbar\omega$ components. As will be noted below, $4\hbar\omega$ states provide a significant contribution to the 40 MeV 2^+ level wave function.

The properties of the first (33 MeV) and second (40 MeV) 2^+ states in the ^{4}He continuum are com-

TABLE I. Amplitudes of basis states for the first two 2^+ states in the ^{4}He system.

N_A	L_{12}	N_{34}	L_{34}	N_{12}	L_A	L	S	Amplitudes 2^+ (33.0)	2^+ (40.2)
1	0	1	2	1	0	2	0	0.102 24	−0.734 90
1	0	2	2	1	0	2	0	−0.031 04	−0.009 22
1	0	1	2	2	0	2	0	−0.010 09	0.213 69
2	0	1	2	1	0	2	0	−0.011 23	0.209 17
1	0	1	0	1	2	2	0	−0.033 78	0.063 24
1	0	2	0	1	2	2	0	0.004 91	0.010 06
1	0	1	0	2	2	2	0	−0.001 22	−0.013 56
2	0	1	0	1	2	2	0	−0.001 36	−0.006 64
1	0	1	2	1	2	2	0	0.003 11	0.015 75
1	2	1	0	1	0	2	0	−0.000 52	−0.002 81
1	2	2	0	1	0	2	0	−0.000 12	−0.010 04
1	2	1	0	2	0	2	0	−0.004 77	−0.005 82
2	2	1	0	1	0	2	0	0.001 60	0.011 60
1	2	1	2	1	0	2	0	−0.003 21	−0.014 78
1	2	1	0	1	2	2	0	0.001 26	0.005 96
1	1	1	2	1	1	2	1	0.000 42	0.003 75
1	1	1	0	1	1	1	1	0.000 15	−0.010 36
1	1	2	0	1	1	1	1	−0.009 66	−0.016 09
1	1	1	0	2	1	1	1	−0.000 04	−0.001 83
2	1	1	0	1	1	1	1	0.000 40	0.007 79
1	1	1	2	1	1	1	1	0.000 25	0.008 00
1	1	1	2	1	1	2	1	−0.003 29	0.001 48
1	1	1	2	1	1	3	1	−0.000 04	−0.000 38
1	1	1	0	1	1	2	1	−0.002 94	0.002 91
1	1	2	0	1	1	2	1	0.001 78	0.001 66
1	1	1	0	2	1	2	1	−0.044 07	0.006 69
2	1	1	0	1	1	2	1	0.043 86	−0.009 46
1	1	1	2	1	1	1	1	−0.000 77	−0.000 71
1	1	1	2	1	1	2	1	−0.043 86	0.008 17
1	1	1	2	1	1	3	1	−0.576 09	−0.033 91
1	1	1	0	1	3	2	1	0.074 45	−0.006 32
1	1	1	0	1	3	3	1	0.132 46	−0.026 44
1	3	1	0	1	1	2	1	−0.001 18	0.033 17
1	3	1	0	1	1	3	1	−0.489 55	−0.183 52
1	0	1	1	1	1	1	1	0.040 65	0.096 69
1	0	2	1	1	1	1	1	0.084 10	−0.030 37
1	0	1	1	2	1	1	1	0.071 64	−0.033 09
2	0	1	1	1	1	1	1	0.105 89	−0.005 88
1	0	1	1	1	1	2	1	−0.538 43	−0.029 71
1	0	2	1	1	1	2	1	0.064 26	−0.006 57
1	0	1	1	2	1	2	1	0.003 92	0.033 19
2	0	1	1	1	1	2	1	0.199 06	−0.027 36
1	0	1	3	1	1	2	1	0.042 54	0.025 32
1	0	1	3	1	1	3	1	0.096 76	−0.006 79
1	0	1	1	1	3	2	1	−0.044 90	0.240 48
1	0	1	1	1	3	3	1	0.004 21	−0.018 86
1	2	1	1	1	1	1	1	0.023 67	−0.148 62
1	2	1	1	1	1	2	1	−0.002 83	0.022 45
1	2	1	1	1	1	1	1	0.011 87	−0.077 98
1	2	1	1	1	1	2	1	0.008 61	−0.063 52
1	2	1	1	1	1	3	1	−0.010 43	0.066 93
1	2	1	1	1	1	2	1	0.002 83	−0.028 28
1	2	1	1	1	1	3	1	0.003 69	−0.007 31
1	1	1	1	1	0	2	0	0.042 36	−0.197 12
1	1	2	1	1	0	2	0	−0.003 34	0.013 35
1	1	1	1	2	0	2	0	0.002 37	−0.020 35
2	1	1	1	1	0	2	0	−0.022 26	0.129 37
1	1	1	3	1	0	2	0	0.010 30	−0.065 36

TABLE I. (*Continued.*)

N_A	L_{12}	N_{34}	L_{34}	N_{12}	L_A	L	S	Amplitudes 2^+ (33.0)	2^+ (40.2)
1	1	1	1	1	2	2	0	0.002 88	−0.028 35
1	1	1	1	1	2	2	0	−0.003 98	0.007 44
1	1	1	1	1	2	2	0	−0.008 54	0.057 93
1	3	1	1	1	0	2	0	−0.010 63	0.065 86
1	1	1	1	1	0	1	1	0.051 85	−0.127 90
1	1	2	1	1	0	1	1	−0.000 64	0.008 82
1	1	1	1	2	0	1	1	−0.013 55	0.029 21
2	1	1	1	1	0	1	1	−0.012 77	0.028 69
1	1	1	1	1	0	2	1	−0.014 53	0.040 39
1	1	2	1	1	0	2	1	−0.005 57	0.020 66
1	1	1	1	2	0	2	1	−0.000 02	−0.000 05
2	1	1	1	1	0	2	1	0.000 67	0.017 82
1	1	1	3	1	0	2	1	−0.013 62	0.040 58
1	1	1	3	1	0	3	1	−0.000 40	0.005 15
1	1	1	1	1	2	1	1	0.000 11	0.000 11
1	1	1	1	1	2	2	1	−0.000 13	−0.018 18
1	1	1	1	1	2	1	1	0.000 25	0.014 84
1	1	1	1	1	2	2	1	0.000 23	0.007 24
1	1	1	1	1	2	3	1	0.000 04	0.000 13
1	1	1	1	1	2	2	1	−0.003 52	0.003 59
1	1	1	1	1	2	3	1	0.000 17	−0.002 19
1	3	1	1	1	0	2	1	−0.000 07	−0.009 24
1	3	1	1	1	0	3	1	−0.005 30	−0.312 78
1	1	1	1	1	0	0	2	−0.054 15	−0.008 51
1	1	2	1	1	0	0	2	0.010 56	0.083 49
1	1	1	3	2	0	0	2	0.009 36	0.081 24
2	1	1	3	1	0	0	2	0.005 21	−0.007 96
1	1	1	1	1	0	1	2	−0.056 49	−0.081 92
1	1	2	1	1	0	1	2	0.000 35	−0.000 21
1	1	1	1	2	0	1	2	−0.013 18	0.061 33
2	1	1	1	1	0	1	2	0.003 93	−0.008 81
1	1	1	1	1	0	2	2	0.000 47	0.006 18
1	1	2	1	1	0	2	2	0.003 68	0.011 46
1	1	1	1	2	0	2	2	0.002 17	0.009 83
2	1	1	1	1	0	2	2	0.001 34	−0.021 40
1	1	1	3	1	0	2	2	0.002 13	0.010 15
1	1	1	3	1	0	3	2	−0.001 45	0.024 92
1	1	1	3	1	0	4	2	−0.003 47	−0.013 88
1	1	1	1	1	2	0	2	−0.000 22	−0.007 48
1	1	1	1	1	2	1	2	−0.004 99	0.003 96
1	1	1	1	1	2	2	2	0.003 16	−0.017 05
1	1	1	1	1	2	1	2	0.000 09	−0.000 05
1	1	1	1	1	2	2	2	−0.004 87	0.003 95
1	1	1	1	1	2	3	2	−0.053 35	0.014 65
1	1	1	1	1	2	2	2	0.002 63	0.000 46
1	1	1	1	1	2	3	2	−0.052 65	0.014 63
1	1	1	1	1	2	4	2	0.052 02	−0.014 61
1	3	1	1	1	0	2	2	0.065 92	0.001 71
1	3	1	1	1	0	3	2	−0.099 51	0.013 41
1	3	1	1	1	0	4	2	0.061 96	0.004 93

pared in Table I. The 40 MeV level is dominantly composed of $2\hbar\omega$ components (61.8%), while the 33 MeV state has a 31.4% $2\hbar\omega$ contribution. Both 2^+ states are also highly fragmented.

Table II summarizes the considerable detail of Table I according to specific LSJ components ($^{2S+1}L_J$)—i.e., $^5S_2$, $^3P_2$, $^5P_2$, $^1D_2$, $^3D_2$, $^5D_2$, $^3F_2$, $^5F_2$, and $^5G_2$. The entries in Table II represent the sum of all squared amplitudes in Table I which have the same LSJ quantum numbers. The quantum num-

TABLE II. LSJ configurations for the first two 2^+ states in ^{4}He.

Configuration	% contribution to the wave function[a] first 2^+ state (E_x = 33.0 MeV)	second 2^+ state (E_x = 40.2 MeV)
$^5S_2$	0.3	1.4
$^3P_2$	2.9	5.8
$^5P_2$	0.3	1.1
$^1D_2$	1.6	70.4
$^3D_2$	32.4	7.3
$^5D_2$	0.4	0.1
$^3F_2$	59.9	13.8
$^5F_2$	1.6	0.1
$^5G_2$	0.7	0.0

[a] Total contribution may not equal 100% due to rounding errors.

bers LSJ were previously defined. The dominant configuration for the 40 MeV state is $^1D_2$, while the 33 MeV state is dominated by the $^3F_2$ configuration. The reader should note that the model basis states are built on internal coordinates, and that traditional shell model coordinates may be obtained from three successive Moshinsky coordinate transformations.[2,4]

The results of Table I also indicate that both 33 and 40 MeV levels are complex and cannot be adequately understood in terms of a $2\hbar\omega$ basis. In fact, $4\hbar\omega$ may not be sufficient, but this larger basis provides a better representation of the 2^+ state. The high degree of fragmentation and the magnitude of the $4\hbar\omega$ components suggests that attempts to describe the 40 MeV state in terms of a $2\hbar\omega$ supermultiplet partition may lead to erroneous conclusions concerning the properties of this state. However, the multiplet approach and the $4\hbar\omega$ approach presented herein suggest that the 40 MeV state is dominated by components from a supermultiplet partition less spatially symmetric than those usually associated with nucleon or electromagnetic channels.

IV. CONCLUSIONS

Dynamical R-matrix calculations lead to predictions for the proposed ^{4}He resonance of McBroom *et al.* which are in qualitative agreement with data. The model (experimental) predictions are as follows: $J^\pi = 2^+ (2^+)$, Γ = 6.0 MeV (3.5 MeV), and E_x (res) = 40.3 MeV (40.2 MeV). The model also predicts a $T = 0$ assignment for this level. The 40 MeV level contains a significant $4\hbar\omega$ component, and predictions based on $2\hbar\omega$ considerations can only lead to a qualitative description of the 40 MeV level.

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